Birzeit University

Department of Mathematics

Math 132. Summer 2016

Student name: **Student ID no.:** **Section :.....**

Q#1 (a) Describe mathematically the upper hemisphere of radius 1centred at the origin

4 $x^2+y^2+z^2=0 \quad , \quad z \geq 0$

(b) Find the unit vector in the direction of

$\vec{v} = 2i + 3j - k$

3 $\dfrac{\vec{v}}{|v|} = \dfrac{2i+3j-k}{\sqrt{14}}$

(c)find an equation for the set of all points that are equal distance

from $y = 3$ and $y = -1$

3 $y = 1$

Q#2 Consider the vectors

$$v = 2i + j - 3k$$

$$u = i - 2j + k$$

(a) Find the angle between the two vectors v and u

(b) Find the proj $_u^v$

(c) Find two vector $\vec{w}$ and $\vec{z}$

Such that $\vec{v} = \vec{w} - \vec{z}$

Where w is parallel to u and $\vec{z}$ perpendicular to $\vec{u}$

(a) $\theta = \cos^{-1}\left(\frac{u \cdot v}{|u||v|}\right)$

$= \cos^{-1}\left(\frac{-3}{\sqrt{6}\sqrt{14}}\right)$ 1

$= \cos^{-1}\left(\frac{-3}{\sqrt{84}}\right)$

$\cancel{(b)\ \text{Proj}_u v = \frac{u \cdot v}{|u|^2}\vec{u}}$

$\cancel{= \frac{-3}{6} \cdot (3i - 2j + 0k)}$

(b) $\text{Proj}_{\vec{u}} V = \frac{v \cdot u}{|u|^2} u = \frac{-3}{6}(i - 2j + k)$

$= -\frac{1}{2} i + j - \frac{1}{2} k$ 3

(c) $\vec{V} = \text{Proj}_{\vec{u}} v + (\vec{v} - \text{Proj}_u v)$ 2

$= \left(-\frac{1}{2} i + j - \frac{1}{2} k\right) + (2i + j - 3k) - \left(-\frac{1}{2} i + j - \frac{1}{2} k\right)$

$= \underbrace{\left(-\frac{1}{2} i + j - \frac{1}{2} k\right)}_{w \parallel \vec{u}} + \underbrace{\left(\frac{5}{2} i - \frac{5}{2} k\right)}_{-z \perp u}$

1 $w = -\frac{1}{2} i + j - \frac{1}{2} k$

$z = -\frac{5}{2} i + \frac{5}{2} k$

Q#3 let $\vec{u} = 2i + j - k$

$\vec{v} = i - 2j + k$

(a) Find the area of the parallegram determined by $\vec{u}$ and $\vec{v}$

$$\|\vec{u} \times \vec{v}\| = \begin{vmatrix} i & j & k \\ 2 & 1 & -1 \\ 1 & -2 & 1 \end{vmatrix}$$

$$u \times v = \begin{vmatrix} 1 & -1 \\ -2 & 1 \end{vmatrix} i - \begin{vmatrix} 2 & -1 \\ 1 & 1 \end{vmatrix} j + \begin{vmatrix} 2 & 1 \\ 1 & -2 \end{vmatrix} k$$ 3

$$|u \times v| = |-i - 3j - 5\hat{k}| = \sqrt{1 + 9 + 25}$$

$$= \sqrt{35}$$ 2

(b) Find the volume of the box formed by $\vec{u}$ and $\vec{v}$ and $w = i + j + 2k$

$$|(u \times v) \cdot \vec{w}|$$ 2

$$= |(-i - 3j - 5k) \cdot (i + j + 2k)|$$ 2

$$= |-1 - 3 - 10| = 14$$ 1

Q#4(a) Find the equation of the line passing through the points (1,-1,2) (4,0,1)

$\vec{v} = 3i + j - k$

$x = 1 + 3t$

$y = -1 + t$

$z = 2 \cancel{+} t$

(b)Find the distance from the line

$x = 2 + 3t$

$y = -1 + t$

$z = 1 - 2t$

To the points $s = (3,1,2)$

$P = (2,-1,1)$, $V = 3i + j - 2k$

$PS = i + 2j + k$ ✓

The distance from S to the line

$= \frac{|PS \times V|}{|V|}$

$PS \times V = \begin{vmatrix} i & j & k \\ 1 & 2 & 1 \\ 3 & 1 & -2 \end{vmatrix}$

$= \begin{vmatrix} 2 & 1 \\ 1 & -2 \end{vmatrix} i - \begin{vmatrix} 1 & 1 \\ 3 & -2 \end{vmatrix} j + \begin{vmatrix} 1 & 2 \\ 3 & 1 \end{vmatrix} k$

$= -5i + 5j - \cancel{5}k$

$= \frac{\sqrt{25 + 25 + 25}}{\sqrt{9 + 1 + 4}}$

4

$= \sqrt{\frac{75}{14}}$

Q#5(a) Find the equation of the plane which contains the points

$$P(2,4,5), Q(1,5,7), R(-1,6,8)$$

$PQ = -i + j + 2k$ (1)

$PR = -3i + 2j + 3k$

$\vec{n} = PQ \times PR = \begin{vmatrix} i & j & k \\ -1 & 1 & 2 \\ -3 & 2 & 3 \end{vmatrix}$

$= \begin{vmatrix} 1 & 2 \\ 2 & 3 \end{vmatrix} i - \begin{vmatrix} -1 & 2 \\ -3 & 3 \end{vmatrix} j + \begin{vmatrix} -1 & 1 \\ -3 & 2 \end{vmatrix} k$ (2)

$= -i - 3j + k$

$-x - 3y + z = -1(2) - 3(4) + 1(5) = -9$ (2)

$\boxed{x + 3y - z = 9}$

b)Find the equation of the line that intersect the two planes

$$3x - 6y - 2z = 3$$

$$2x + y - z = 2$$

$z = 0$

$3x - 6y = 3$

$12x + 6y = 12$

$15x = 15$

$x = 1$

$y = 0$

$(1, 0, 0)$ (2)

$n_1 = 3i - 6j - 2k$

$n_2 = 2i + j - k$

$n_1 \times n_2 = \begin{vmatrix} i & j & k \\ 3 & -6 & -2 \\ 2 & 1 & -1 \end{vmatrix}$

$= \begin{vmatrix} -6 & -2 \\ 1 & -1 \end{vmatrix} i - \begin{vmatrix} 3 & -2 \\ 2 & -1 \end{vmatrix} j + \begin{vmatrix} 3 & -6 \\ 2 & 1 \end{vmatrix} k$

$= 8i - j + 15k$ (2)

$x = 1 + 8t, \; y = -t, \; z = 15t$ (1)