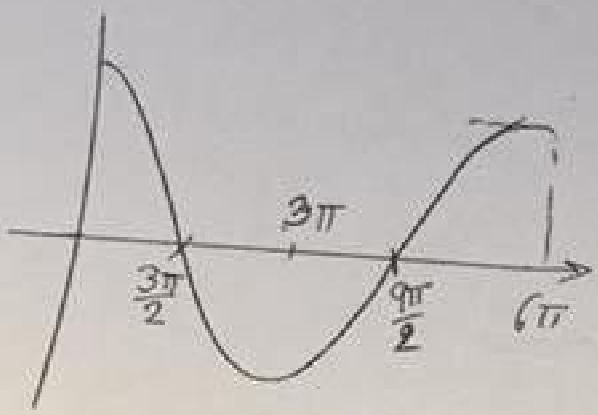
Question 3.(6 points) Consider the polar curve $r = \cos\left(\frac{\theta}{3}\right)$.

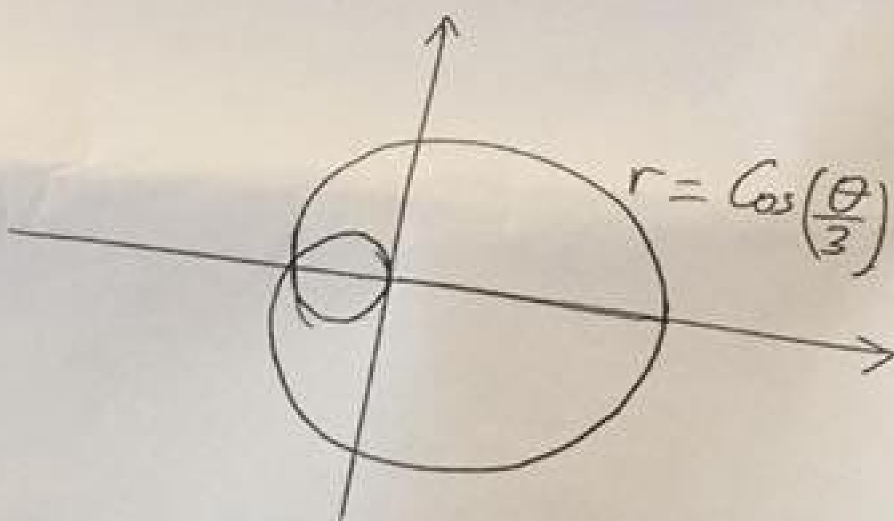
(a) Show that the curve is symmetric about the x−axis.

$r = \cos\left(\frac{\theta}{3}\right) \qquad (r, \theta)$

$r = \cos\left(\frac{-\theta}{3}\right) : (r, -\theta)$

(b) Graph the curve. Hint: graph the curve in the interval $[0, \frac{3\pi}{2}]$ then use symmetry.





... polar curve $r = 1 + \cos(2\theta)$

1. Show it is symmetric about the x-axis.

symmetric about x-axis that means point $(r, \theta) = (r, -\theta)$

the point

$r = 1 + \cos 2x \theta$

$r = 1 + \cos -2\theta$ ⇒ Knowing that cos is an even function so $\cos -\theta = \cos\theta$

$r = 1 + \cos 2\theta = 1 + \cos 2\theta$ so it symmetric about x-axis

2. Show it is symmetric about the pole.

symmetric about pole $(r, \theta) = (-r, \theta)$ or $\boxed{(r, \pi + \theta)}$

$r = 1 + \cos 2(\pi + \theta)$

$r = 1 + \cos(2\pi + 2\theta) = r = 1 + \cos 2\pi \cos 2\theta + \sin 2\pi \sin 2\theta$

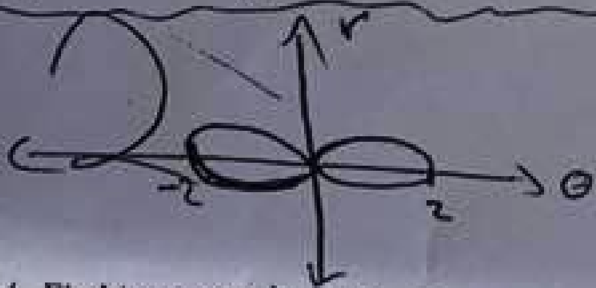
$r = 1 + 1 \times \cos 2\theta + \therefore \times \sin 2\theta$

$r = 1 + \cos 2\theta + \therefore$

$r = 1 + \cos 2\theta$ so its symmetric about the origin

3. Graph the curve

Knowing that the graph is symmetric about x, origin so it symmetric about y-axis



r	θ
2	0
1.5	$\frac{\pi}{6}$
$\frac{1}{2}$	$\frac{\pi}{3}$
∴	$\frac{\pi}{2}$
$\frac{1}{2}$	$\frac{2\pi}{3}$

$\cos 60 = \sin$

$\cos 2 \times \frac{\pi}{6} = \cos \frac{\pi}{3} =$

$\cos 2\frac{\pi}{3} =$

$= \cos 120$

$= \cos 60$

$= -\frac{1}{2}$

$1 - \frac{1}{2} =$

4. Find its slope $\left(\frac{dy}{dx}\right)$ at $\theta = \frac{\pi}{6}$

$$\text{slope} = \frac{r' \sin\theta + r\cos\theta}{r'\cos\theta - r\sin\theta} \qquad r' = \frac{dr}{d\theta}$$

$$= \frac{\frac{-2\sqrt{3}}{2} \times \frac{1}{2} + \frac{3}{2} \times \frac{\sqrt{3}}{2}}{\frac{-2\sqrt{3}}{2} \times \frac{\sqrt{3}}{2} - \frac{3}{2} \times \frac{1}{2}}$$

$$\frac{\frac{-2\sqrt{3}}{4} + \frac{3\sqrt{3}}{4}}{\frac{2 \times 3}{4} - \frac{3}{4}} = \frac{\sqrt{3}}{4} \div \frac{-9}{4} = \frac{\sqrt{3}}{4} \times \frac{-4}{9} = \frac{-\sqrt{3}}{9}$$

$r = 1 + \cos 2\theta$

$r\big|_{\frac{\pi}{6}} = 1 + \cos 2 \times \frac{\pi}{6}$

$r = 1 + \cos\frac{\pi}{3}$

$r = 1 + \frac{1}{2}$

$r = \frac{3}{2}$

$r' = -\sin 2\theta \times 2$

$r' = -2\sin 2\theta$

$\cos\frac{\pi}{6} =$

$\boxed{\cos 30 = \frac{\sqrt{3}}{2}}$

$\boxed{\sin 30 = \frac{1}{2}}$

$\cos$

$2x$

$2\frac{\pi}{3}$

$y = r\sin$

$r'\sin\theta + r\cos\theta$

$2, \frac{-\pi}{3}+\pi$ $\quad 2, -\frac{\pi}{3}+\frac{3\pi}{3}$ $\quad 2, \frac{2\pi}{3}$ $\quad \frac{2\times 180}{3}$ 120

$\frac{2\times 180^{60}}{3} = 120$

BIRZEIT UNIVERSITY
MATHEMATICS DEPARTMENT

Excellent

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Test 2 Math 132 Fall 2016/2017

Name: DaouD abdo BZU#: 1141267 Section#:

Circle the correct answers.

1. Which polar coordinate pairs label the same point as $(2, \frac{-\pi}{3})$

(a) $(2, \frac{-\pi}{3}+\pi)$

(b) $(2, \frac{-\pi}{3}-\pi)$

(c) $(-2, \frac{2\pi}{3})$

(d) $(-2, \frac{-2\pi}{3})$

$x=2$
$r^2 = x^2+y^2$
$4+\frac{\pi^2}{4}$

$\frac{-\pi}{3}-\frac{3\pi}{3}$ $\quad \frac{-\pi}{3}+\frac{3\pi}{3}$

$-\frac{4\pi}{3}$ $\quad \frac{2\pi}{3}$

$\frac{2\times 180}{3}$ $\quad -\frac{4\times 180^{60}}{3}$ 240

2. The polar equation $r^2\sin(2\theta) = 2$ is equivalent to which of the following cartesian equations.

(a) $y = x$

(b) $xy = 1$

(c) $y^2 = x^2$

(d) $xy = 0$

$r^2 2\sin\theta\cos\theta = 2$
$xy = 1$

$r^2 2\sin\theta\cos\theta$
$2xy = 2$

$\frac{2\times 180^{60}}{3}$ 120

$-\frac{2\times 180}{3}$ -120

$-1 - -1 \quad -1+1$

$-1 - -1 \quad -1+1=0 \quad -2 \quad -1$

If $v = <-1, 0, 1>$, and $u = <-1, 2, 0>$, then a unit vector in the direction of $v - u$ is

$4+1 \quad \sqrt{5}$

(a) $< \frac{-2}{3}, \frac{-2}{3}, \frac{1}{3} >$

(b) $< \frac{-2}{3}, \frac{2}{3}, \frac{-1}{3} >$

(c) $< \frac{2}{3}, \frac{2}{3}, \frac{1}{3} >$

(d) $< \frac{2}{3}, \frac{-2}{3}, \frac{1}{3} >$

(e) $< \frac{1}{3}, \frac{-2}{3}, \frac{-2}{3} >$

(f) None of the above

$v - u = <$

$v - u = -1 - -1$
$-1+1=0$
$2-0=2$

$-1 \quad -1$

$-1 - -1$

$-2 \quad -1$

$-1 - -1$

$\frac{-2}{\sqrt{5}}, \frac{-1}{\sqrt{5}}$

$\sqrt{4+1} = \sqrt{5}$

$-1+1=0$

5) Sketch the graph

$r = \frac{1}{2} + \sin\theta$

symmetric about y-axis

θ	$r = \frac{1}{2} + \sin\theta$
0	$\frac{1}{2}$
$\frac{\pi}{6}$	1
$\frac{\pi}{4}$	1.21
$\frac{\pi}{3}$	1.37
$\frac{\pi}{2}$	1.5
$-\frac{\pi}{2}$	$\frac{1}{2} - \frac{1}{2} = 0$
$\frac{\pi}{4}$	$\frac{1}{2} - 0.71 = -0.21$
$\frac{\pi}{3}$	$0.5 - 0.87 = -0.37$
$\frac{\pi}{2}$	$\frac{1}{2} - 1 = -0.5$

4) (a) Replace the polar equation with equivalent Cartesian equation

$r^2 \sin 2\theta = 1$

$r^2 2\sin\theta\cos\theta = 1$

$r\sin\theta \, r\cos\theta = \frac{1}{2}$

$\boxed{xy = \frac{1}{2}}$

(b) Find the first three terms of the Binomial expansion of

$\frac{1}{\sqrt[3]{1-\frac{x}{3}}}$

$(1+x)^m = 1 + mx + \frac{m(m-1)}{2}x^2 + \cdots$

$\left(1+\left(\frac{-x}{3}\right)\right)^{-\frac{1}{3}} = 1 - \frac{1}{3}\left(\frac{-x}{3}\right) + \frac{-\frac{1}{3}\left(-\frac{4}{3}\right)}{2}\left(\frac{-x}{3}\right)^2 + \cdots$

$= 1 + \frac{x}{9} + \frac{2}{81}x^2 + \cdots$

3) Estimate $\int_0^{0.1} e^{-x^2}\,dx$ with $|\text{error}| \le 10^{-8}$

$$e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \cdots$$

2 $$e^{-x^2} = 1 - x^2 + \frac{x^4}{2!} - \frac{x^6}{3!} + \frac{x^8}{4!} - \frac{x^{10}}{5!} + \cdots$$

$$\int_0^{0.1} e^{-x^2} = \int_0^{0.1} 1 - x^2 + \frac{x^4}{2!} - \frac{x^6}{3!} + \frac{x^8}{24} - \frac{x^{10}}{120} - \cdots$$

2 $$= x - \frac{x^3}{3} + \frac{x^5}{5.2!} - \frac{x^7}{7(3!)} + \frac{x^9}{9(24)} - \frac{x^{11}}{11(120)} \cdots \Big|_0^{0.1}$$

2 $$= 0.1 - \frac{(0.1)^3}{3} + \frac{(0.1)^5}{5.2} - \frac{(0.1)^7}{7(6)} + \frac{(0.1)^9}{9(24)}$$

since $\frac{(0.1)^7}{42} < 10^{-8}$

but $\frac{(0.1)^5}{10} = \frac{1}{10^6} \not< \frac{1}{10^8}$

So $$\int_0^{0.1} e^{-x^2}\,dx \approx 0.1 - \frac{(0.1)^3}{3} + \frac{(0.1)^5}{10}$$

with $|\text{error}| \le \frac{(0.1)^7}{42} < \frac{1}{(10^7)42} < \frac{1}{1\ldots}$

(2)Find Taylor series expansion of $f(x) = \frac{1}{x}$, $a = 3$
and find the interval of convergence of the series and the sum of the series where it converges.

$$f(x) = f(a) + f'(a)(x-a) + \frac{f''(a)}{2!}(x-a)^2 + \frac{f'''(a)}{3!}(x-a)^3 \cdots$$

$f(x) = \frac{1}{x}$ $\quad f(3) = \frac{1}{3}$

$f'(x) = \frac{-1}{x^2}$ $\quad f'(3) = \frac{-1}{9}$

3 $\quad f''(x) = \frac{2}{x^3}$ $\quad f''(3) = \frac{2}{27}$

$f^{(4)}(x) = \frac{-6}{x^4}$ $\quad f'''(3) = \frac{-6}{81}$

$$\frac{1}{x} = \frac{1}{3} - \frac{1}{9}(x-3) + \frac{2}{(27)(2)}(x-3)^2 - \frac{6}{(81)(6)}(x-3)^3 + \cdots$$

$$= \frac{1}{3} - \frac{1}{9}(x-3) + \frac{1}{3^3}(x-3)^2 - \frac{1}{3^4}(x-3)^3 + \cdots$$

3 $$= \frac{1}{3}\left[1 - \frac{x-3}{3} + \frac{(x-3)^2}{3^2} - \frac{(x-3)^3}{3^3} + \cdots\right]$$

$$= \frac{1}{3}\sum_{n=0}^{\infty} \frac{(-1)^n (x-3)^n}{3^n} = \frac{1}{3}\sum_{n=0}^{\infty} (-1)^n \left(\frac{x-3}{3}\right)^n$$

ibsolutly,

Geometric Series converges if $|r| < 1$

2 $$\left|\frac{x-3}{3}\right| < 1 \Rightarrow |x-3| < 3$$

$$\Rightarrow -3 < x-3 < 3 \Rightarrow \boxed{0 < x < 6}$$

and it converges to $\frac{a}{1-r}$

2 $$= \frac{1/3}{1 - \frac{x-3}{3}} = \frac{1/3}{\frac{3-(x+3)}{3}} = \frac{1}{x}$$

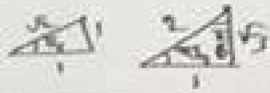
52/60

Birzeit University- Mathematics Department
Math 231-Calculus III

First Hour Exam
Name: Majd I. Abdullah
Instructor: Marwan AL Aqeeli

Spring 2015/2016
Number: 1142357
Section: 11:00 → 12:20

Question 1. (42 points) Circle the correct answer:



(1) One of the following is the point $(x, y) = (-1, -\sqrt{3})$ in polar coordinates

$\boxed{\text{(a) } (2, \frac{\pi}{3})}$
(b) $(-2, \frac{\pi}{3})$
(c) $(-2, \frac{4\pi}{3})$
(d) $(-2, \frac{\pi}{6})$

$r = \sqrt{x^2 + y^2} = \sqrt{(-1)^2 + (-\sqrt{3})^2} = \sqrt{1+3} = \sqrt{4} = 2$

$\theta = \tan^{-1}\left(\frac{-\sqrt{3}}{-1}\right) = \tan^{-1}\left(\frac{\sqrt{3}}{1}\right) = \frac{\pi}{3}$

(2) The equation of the line through $(2,2,1)$ perpendicular to the vectors $\mathbf{u} = \mathbf{j} + \mathbf{k}$, $\mathbf{v} = \mathbf{i} - \mathbf{j}$ is

(a) $x = 2 + t, y = 2 - t, z = 1 + t, t \in (-\infty, \infty)$
(b) $x = 2 - t, y = 2 - t, z = 1 - t, t \in (-\infty, \infty)$
$\boxed{\text{(c) } x = 2 + t, y = 2 + t, z = 1 - t, t \in (-\infty, \infty)}$
(d) $x = 2 + t, y = 2 + t, z = 1 + t, t \in (-\infty, \infty)$

$u \times v = \begin{vmatrix} i & j & k \\ 0 & 1 & 1 \\ 1 & -1 & 0 \end{vmatrix} = 1i + 1j - 1k$

$\vec{n} = i + j - k$

$P(2,2,1)$

$x = 2 + t$
$y = 2 + t$
$z = 1 - t$

(3) The polar curve $r = -\sin^2(2\theta)$ is symmetric about

(a) the x-axis.
(b) the y-axis.
(c) the origin.
(d) all of the above.

(4) The center and radius of the sphere $x^2 + y^2 + z^2 - 2x + 2y = 2$ are

(a) $(1, 1, 0)$, 4
$\boxed{\text{(b) } (1, -1, 0), 2}$
(c) $(-1, 1, 0)$, 2
(d) $(-1, -1, 0)$, 2

$(x^2 - 2x + 1) + (y^2 + 2y + 1) + z^2 = 2 + 1 + 1$

$(x-1)^2 + (y+1)^2 + z^2 = 4$

Center $(1, -1, 0)$ $r = 2$

(5) The vector projection of $\mathbf{u} = \mathbf{i} + \mathbf{k}$ onto $\mathbf{v} = \mathbf{j} + \mathbf{k}$ is

(a) $2\mathbf{v}$
$\boxed{\text{(b) } \frac{1}{2}\mathbf{v}}$
(c) $\frac{1}{\sqrt{2}}\mathbf{v}$
(d) $\mathbf{v}$

$\text{proj}_v u = \left(\frac{u \cdot v}{v \cdot v}\right) v = \frac{1}{2}(j + k) = \frac{1}{2} v$

$y^2 + 2y + 1$

$x^2 - 2x + 1$

$u \cdot v = (1)(0) + (0)(1) + (1)(1) = 1$

$v \cdot v = (0)(0) + (1)(1) + (1)(1) = 2$

(14)

3

QUESTION TWO: [30 points]

(a) Sketch the graph of $r = 2\cos\theta + 1$.

(b) Find the slope of $r = 2\cos\theta + 1$ at the origin.

(0,3)

1- when we substude $-\theta \Rightarrow r = 2\cos(-\theta)+1$

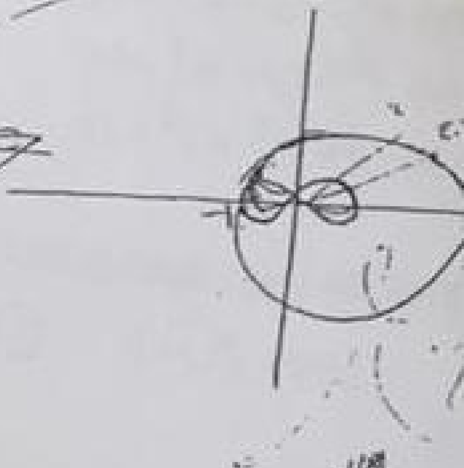
$r = 2\cos\theta + 1$

$\Rightarrow$ symmetry about x-axis

$r = 2\cos(\pi-\theta)+1$

$r = -2\cos\theta + 1 \Rightarrow$ not symmetry about y-axis

θ	r
0	3
$\frac{\pi}{4}$	2.7
$\frac{\pi}{3}$	2
$\frac{\pi}{2}$	1
π	-1



2- slop= $\dfrac{f'(\theta)\sin\theta + f(\theta)\cos\theta}{f'(\theta)\cos\theta - f(\theta)\sin\theta}$, $r' = -2\sin\theta$

$r(0) = 2+1$

$= 3$ at (0,

$= \dfrac{0+3(1)}{0-0} = \dfrac{3}{0}$ not define

$\Rightarrow$ ~~slope at the origin~~ $= \tan(\ $

$= 0$

$(2,-2)$ is ... coordinates

(a) $(2\sqrt{2},\frac{\pi}{4})$

(b) $(2\sqrt{2},-\frac{\pi}{4})$

(c) $(2\sqrt{2},\frac{3\pi}{4})$

(d) $(-2\sqrt{2},\frac{\pi}{4})$.

7. The curve whose polar equation is given by $r=\theta$ is symmetric about

(a) the x-axis

(b) the y-axis

(c) the origin

(d) None of the above.

8. Find a Cartesian equation for the curve $r=8\cos\theta$.

(a) $(x-4)^2+y^2=16$

(b) $x^2+(y+4)^2=16$

(c) $x^2+(y-4)^2=16$

(d) $(x+4)^2+y^2=16$

$r^2 = 8r\cos\theta$

$x^2+y^2 = 8x$

$\Rightarrow x^2+y^2-8x=0$

$t^2 \quad t^3 \quad t=0$?

$|u \cdot v| = |u||v| \cos\theta \le |u||v|$

6. If $|u| = 3$ and $|v| = 4$, then
(a) $v \cdot u = 12$.
(b) u and v are parallel.
(c) $v \cdot u \le 12$.
(d) $v \cdot u > 12$.

$\frac{dr}{d\theta} = \cos\theta$

$\frac{\cos\theta \sin\theta + r\cos\theta}{\cos\theta\cos\theta - r\sin\theta}$ $(-1, 0)$

$\frac{-1}{1} = -1$

7. The slope of the curve $r = \sin\theta - 1$ at $\theta = 0$ is
(a) -1.
(b) 1.
(c) -2.
(d) undefined.

8. The area of the surface generated by revolving the curve $r = 2\cos\theta$, , about the y-axis is
(a) 4Π.
(b) 8Π.
(c) $4\Pi^2$
(d) $8\Pi^2$.

$dr = -2\sin\theta$

$= 4\sin^2\theta$

$= 4\cos^2\theta$

$\int 2\pi r \cos\theta \sqrt{r^2 + \left(\frac{dr}{d\theta}\right)^2}\, d\theta$

$\int 4\pi \cos^2\theta \sqrt{4}$

$2(8\pi)\int_0^{\frac{\pi}{2}} \left(\frac{1}{2} + \frac{1}{2}\cos 2\theta\right)$

$\left(\frac{\pi}{2} - 0\right)$

4π

9. The graph of the polar curve $r^2 = \sin\theta$ is
(a) Symmetric about the x-axis, the y-axis and the origin.
(b) Symmetric about the y-axis only.
(c) Symmetric about the x-axis and the y-axis only.
(d) Symmetric about the origin.

Spring 2007/2008
Kh. Nazzal.

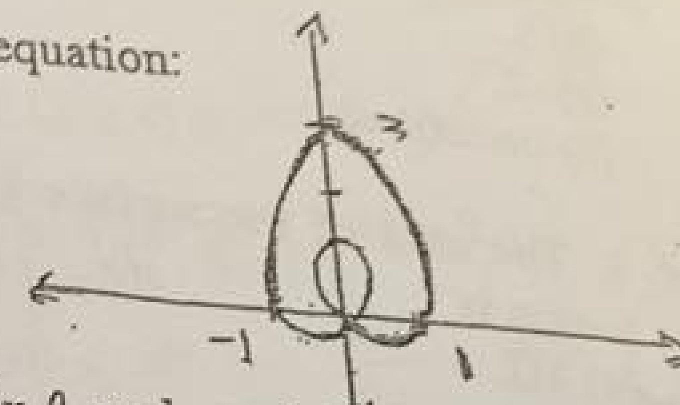
Student Name: Mustafa Salem

Number: 1061002 Section 2 TR 11:00 - 12:30

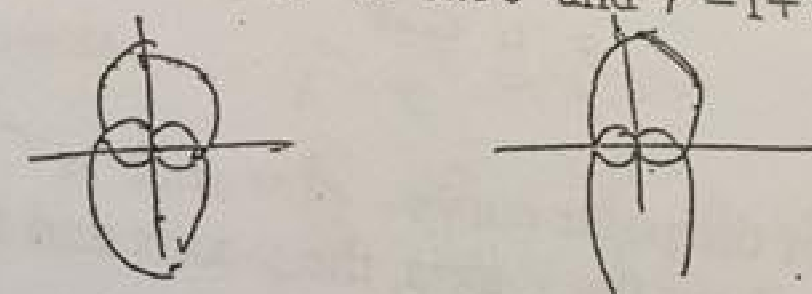
Q1 :(80 points) Choose the correct answer.

1. The polar curve in the opposite figure is represented by the equation:
   (a) $r = \sin\theta + 2$.
   (b) $r = 2\sin\theta + 1$.
   (c) $r = 1 - \sin\theta$.
   (d) $r = 2\sin\theta - 2$.

2 + sin θ   2 - sin θ   -2(1 - sin



2. The number of points of intersection of the curves $r = 1 - \sin\theta$ and $r = 1 + \sin\theta$ is
   (a) 0.
   (b) 2.
   (c) 4.
   (d) 3.



3. The set of points in the plane whose polar coordinates satisfy the equation $r = \sec\theta$ is represented by
   (a) the line $y = -x$
   (b) a vertical line through (1,0).
   (c) the line $y = x$.
   (d) a horizontal line through (1,0).

$r = \frac{1}{\cos}$

$r\cos = 1$

$x = 1$

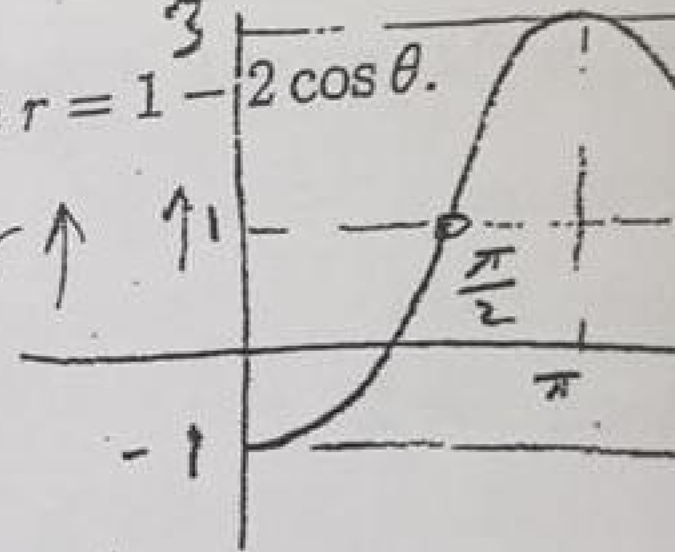
4. The equation $x = 2$ in the space represents

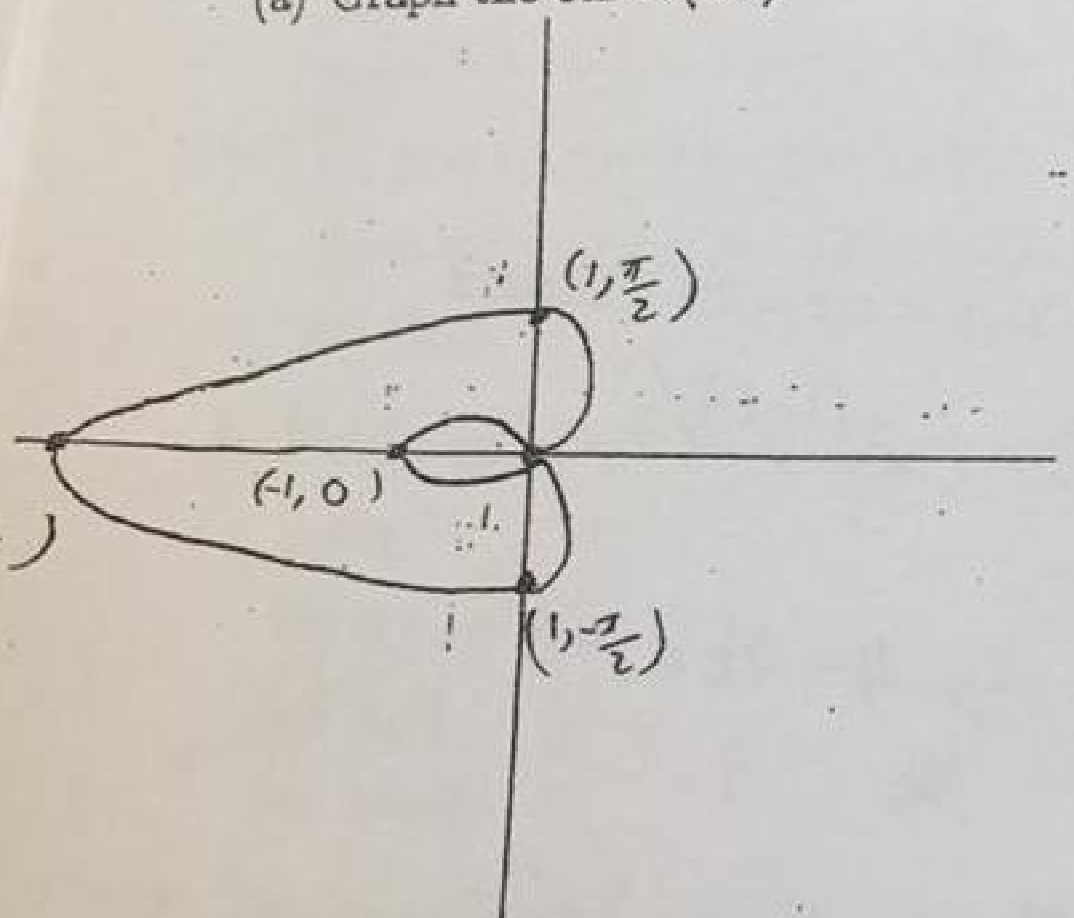
$x = 2$

(14)

Question 3.(14%) Consider the polar curve $r = 1 - 2\cos\theta$.

(a) Graph the curve.(6%)





θ	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$
r	-1	$1-\sqrt{3}$	$1-\sqrt{2}$	0

(b) Find the area inside the inner loop of the curve.(8%)

$$2\int_0^{\pi/3} \frac{1}{2}(1-2\cos\theta)^2$$

$$\int_0^{\pi/3} (1 - 4\cos\theta \ldots$$

(b) (1, 2, 3).

(c) (3, 2, 1).

(d) They do not intersect.

$2 - t = s$

$2 - t = (1+t)/2 \Rightarrow 4 - 2t$

$\Rightarrow$

15. The point $(x, y) = (1, \sqrt{3})$ has polar coordinates

(a) $(2, \pi/3)$.

(b) $(-2, \pi/3)$.

(c) $(-2, 4\pi/3)$.

(d) $(2, 4\pi/3)$.

$\tan\theta = \frac{\sqrt{3}}{1} = \frac{\frac{\sqrt{3}}{2}}{\frac{1}{2}}$

$\Rightarrow \theta =$

$x = r\cos\theta$

$2 = r$

16. The slope of the polar curve $r = 1 + \cos\theta$ at $\theta = \frac{\pi}{2}$ is

(a) 1.

(b) −1.

(c) 2.

(d) −2.

$r' = -\sin\theta$

slope $= \frac{-\sin\theta \cdot \sin\theta + (1+\cos\theta)}{-\sin\theta\cos\theta - (1+\cos\theta}$

(c) $x = 2t, y = -3t, z = 2t.$

(d) None.

$x = 1 + 2t \; , \; y = $

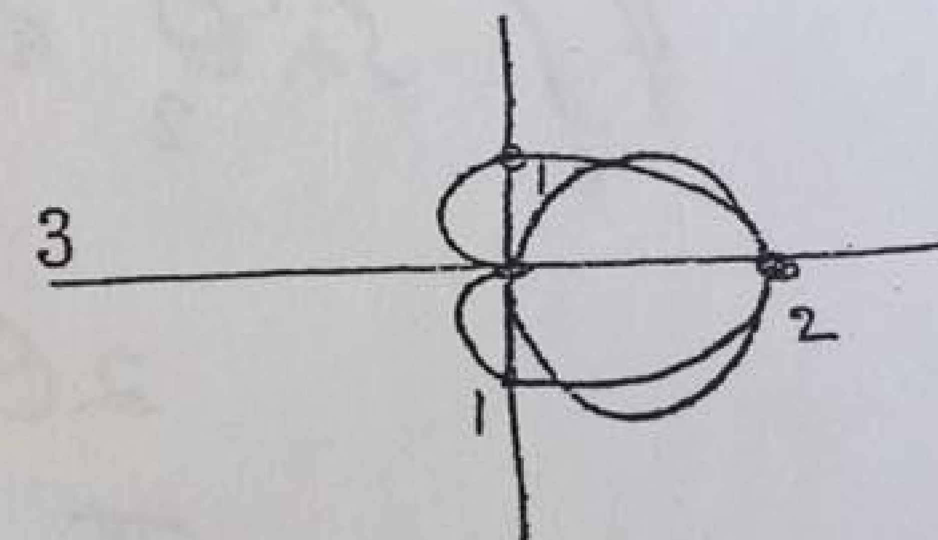
12. The polar curves $r = 1 + \cos\theta$ and $r = 2\cos\theta$ intersect

(a) Only at the point $(2, 0)$.

(b) Only at the origin.

(c) At the origin and at $(2, 0)$.

(d) The curves do not intersect.



$1 + \cos\theta = 2\cos\theta$

$1 = \cos\theta$

Question 2.(72%) Circle the correct answer:

1. The polar curve $r = \sin\left(\frac{\theta}{2}\right)$ is symmetric about

   (a) x-axis.

   (b) y-axis.

   (c) origin.

   (d) All the above.

$(r,\theta) \to (r,-\theta)$ $(\frac{1}{2}, 135)$

2. The length of the polar curve $r = 1 - \cos\theta$, $0 \leq \theta \leq \frac{\pi}{2}$ is

   (a) $2\sqrt{2}$.

   (b) $2\sqrt{2} - 2$.

   (c) $2\sqrt{2} - 1$.

   (d) $\cancel{4} - 2\sqrt{2}$.

$\int \sqrt{1 - 2\cos\theta + \cos^2\theta + \sin^2\theta}$

$\int \sqrt{2 - 2\cos\theta} = \int \sqrt{2(1-\cos\theta} = \sqrt{2.2}$

$\int_0^{\pi/2} 2\sqrt{\sin}$

3. One of the following polar equations represents a line

   (a) $r = 1$.

   (b) $\theta = \pi/4$.

   (c) $r \sec\theta = 1$.

   (d) $r \csc\theta = 1$.

$\int 2\sin\theta$

(d) None of the above

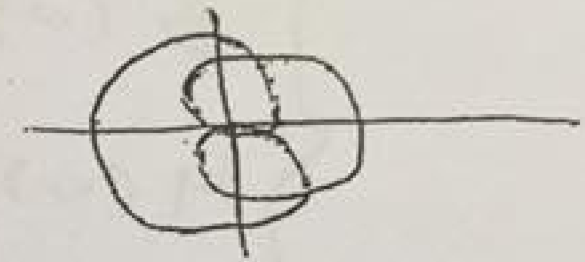
(17) The curves $r = 1 + \cos\theta$ and $r = 1 - \cos\theta$ intersect in

(a) one point

(b) two points

(c) three points

(d) four points



(18) The slope of the curve $r = 1 + \cos\theta$ at $\theta = \frac{\pi}{2}$ is

$r = 1 +$

(a) 0

(b) 1

$x = (1 + \cos\theta)\cos\theta$

$y = (1 + \cos\theta)\sin\theta$

(c) -1

(d) $\frac{1}{2}$

(19) The distance from the point $(2, -3, 4)$ to the plane $x + 2y + 2z = 13$ is

(a) 0

5 (5, 2, 3)

(3

(b) 3

(16) One of the following points lies on the curve $r^2 = \sin\theta$

(a) $(-1, 0)$ x

(b) $(1, 0)$ y

(c) $(-1, -\frac{\pi}{2})$

(d) None of the above

(17) The curves $r = 1 + \cos\theta$ and $r = 1 - \cos\theta$ intersect in

(a) one point

(10) Let $\vec{u}, \vec{v}$ be two

(a) the area of the parallelogram determined

(b) a vector orthogonal to $\vec{u}$ and $\vec{v}$

(c) the volume of the parallelpiped determined by $\vec{u}$

(d) A vector in the plane of $\vec{u}$ and $\vec{v}$

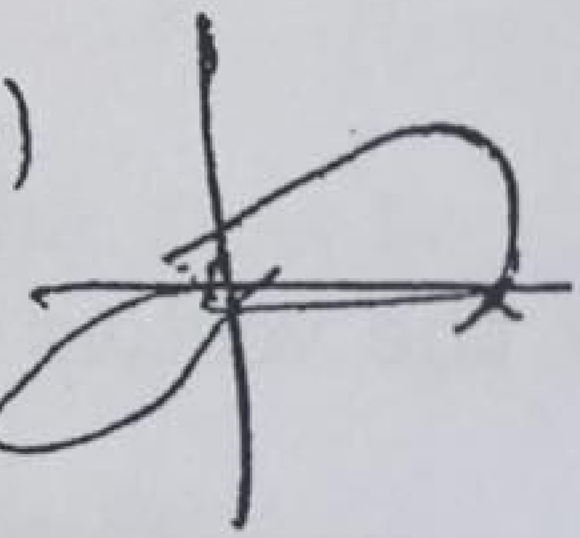
(11) The curve $r^2 = 4\cos(2\theta)$ is symmetric about the

(a) x-axis

(b) y-axis

(c) origin

(d) all of the above



(12) The vector projection of $\vec{B} = 6\text{i} + 3\text{j} + 2\text{k}$ onto $\vec{A}$ =

(a) $-\frac{4}{9}\text{i} + \frac{8}{9}\text{j} + \frac{8}{9}\text{k}$

B

(a) $2i + j - k$.

(b) $\frac{4}{3}(2i + j - k)$. ✓

(c) $4(2i + j - k)$.

(d) $\frac{1}{3}(2i + j - k)$.

Question 2.(8%) Consider the polar curve $r = 2 + \sin(2\theta)$. Show that the curve is symmetric about the origin and then plot the curve.

about the origin $(r, \theta) = (-r, \theta)$ or $(r, \pi + \theta)$.

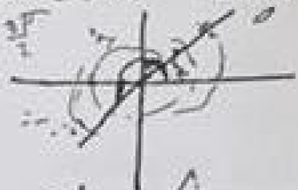
$-r = 2 + \sin(2\theta)$

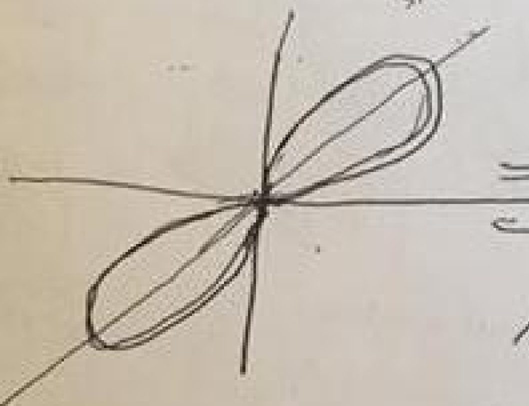
$r \stackrel{?}{=} 2 + \sin(2\pi + 2\theta) =$

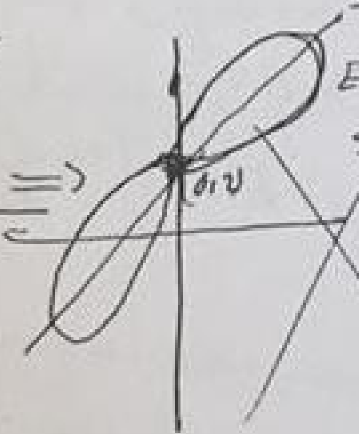
$r \stackrel{?}{=} 2 + \sin 2(\pi + \theta)$

$r = 2 + \sin 2(\theta)$ ✓

$\Rightarrow$ ~~$\sin(\pi + 2\theta) = \sin 2\theta$~~).







$\sin 2(\pi + \theta) = \sin 2\theta$

Ex.

$\sin 2(\pi + \frac{\pi}{2}) = \sin 2($

$\sin(3\pi) = \sin \pi$

$0 = 0$

$\sin 2(\frac{6\pi}{5} + \pi) = \sin 2($

$\sin(\frac{7\pi}{6}) = \sin$

$\sin \frac{7\pi}{3}$

$r = 2 + \sin(2\theta)$

$r = 2 + 2\sin\theta\cos\theta$

$r = 2(1 + \sin\theta\cos\theta)$

(b) $\sec(t)$

(c) $\tan(t)$

(d) $\tan(t)$

(17) The polar curve $r = \sin\left(\frac{\theta}{2}\right)$ is symmetric about

(a) x-axis.

(b) y-axis.

(c) origin.

(d) All of the above.

(1, 1, 0)

(18) The distance between the plane $2x - y + 3z =$

$\frac{1}{2}+\cos(\frac{\pi}{3}) = \frac{1}{2}+\frac{1}{2} = 1$ | $\frac{1}{2}\cos(\pi) = -0.5$

$\frac{1}{2}+\cos(\frac{\pi}{2}) = \frac{1}{2}$

$\frac{1}{2}+\cos(\frac{3\pi}{4}) = \frac{1}{2} - \frac{\sqrt{2}}{2} = \frac{1-\sqrt{2}}{2}$

Cod

Question 2.(8 points) Consider the polar curve $r = \frac{1}{2} + \cos(2\theta)$.

(a) Show that the curve is symmetric about the x-axis, y-axis and the origin.

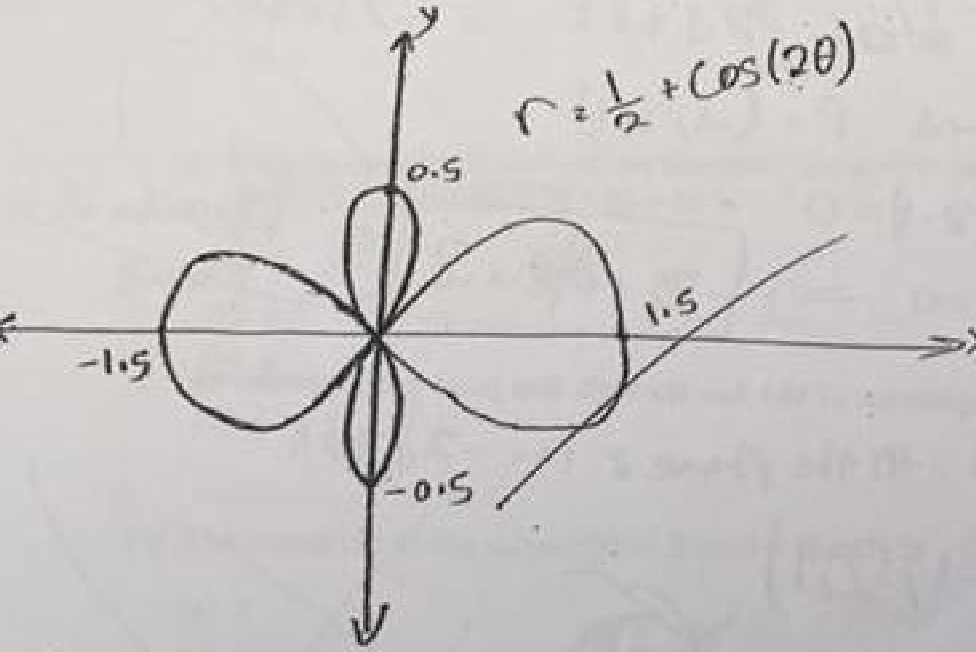
$r = \frac{1}{2} + \cos(2\theta)$ — (r, θ) on the graph

- Symmetry about X-axis:
  take $-\theta$:
  $\frac{1}{2} + \cos(2(-\theta))$
  $= \frac{1}{2} + \cos(-2\theta)$
  $= \frac{1}{2} + \cos(\theta) = r$
  $\therefore (r, -\theta)$ on the graph
  $\therefore$ symmetry about x-axis

- Symmetry about the origin:
  ~~take~~ take 1 period 2π:
  $\frac{1}{2} + \cos(2\theta + 2\pi)$
  $= \frac{1}{2} + \cos(2(\pi+\theta))$
  $= r$
  $\therefore (r, \pi+\theta)$ on the graph
  $\therefore$ Symmetry about origin

- Symmetry about y-axis:
  take $\pi - \theta$:
  $\frac{1}{2} + \cos(2(\pi-\theta))$
  $= \frac{1}{2} + \cos(2\pi - 2\theta)$
  $= \frac{1}{2} + \cos(2\theta) = r$
  $\therefore (r, \pi-\theta)$ on the graph
  symmetry about y-axis

(b) Graph the curve. Hint: use symmetry.



θ	r
0	1.5
$\frac{\pi}{6}$	1
$\frac{\pi}{4}$	0.5
$\frac{\pi}{3}$	0
$\frac{\pi}{2}$	-0.5
	-1
	-1.5

Using symmetry

(a) the z–axis

(b) the xy–plane

(c) the xz–plane

(d) the line $x+y=1$ in the xy–plane.

$$\begin{vmatrix} i & j & k \\ 1 & 1 & 1 \\ 1 & 1 & -1 \end{vmatrix} = -2($$

(19) The cartesian equation of the polar curve $r\cos\left(\theta-\frac{\pi}{4}\right)=1$ is

(a) $x+y=1$

(b) $x-y=\sqrt{2}$

(c) $x+y=\sqrt{2}$

(d) $x-y=1$

$r\left(\cos\theta\cos\left(\frac{\pi}{4}\right)+\sin(\theta)\sin\left(\frac{\pi}{4}\right)\right)$

$\frac{r}{\sqrt{2}}\left(\cos\theta+\sin\theta\right)=1$

$x+y=\sqrt{2}$

(20) The volume of the box determined by the vectors $\mathbf{u}=\mathbf{i}-\mathbf{j}-\mathbf{k}$, $\mathbf{v}=2$

$\mathbf{w}=\mathbf{i}-\mathbf{j}$ is

(a) 5

(b) 7

(c) 6

(d) 4

$(u\times v)\cdot w = \begin{vmatrix} i & j & k \\ 1 & -1 & 0 \\ 1 & -1 & -1 \\ 2 & 3 & 1 \end{vmatrix} = 1\begin{vmatrix} -1 \\ 3 \end{vmatrix}$

$21(2$