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MATH 1321
MIDTERM EXAM 2022

Student Name:__________ Student Number:__________

Discussion Instructor:__________ Discussion Section:__________

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42 **Question 1** (3 points each) Circle the most correct answer:

$\lim\limits_{b\to\infty}\int_2^b x^{-3} = \lim \frac{x^{-2}}{-2}\Big|_2^b$

$= \lim\limits_{b\to\infty}\left(-\frac{1}{2b^2}+\frac{1}{8}\right)$

$0 + \left(\frac{1}{8}\right)$ conv

1. The integral $\displaystyle\int_2^\infty \frac{2}{x^3-x}\,dx$

   $2 < \frac{1}{x^3}$

   (a) converges by the limit comparison test with $\displaystyle\int_2^\infty \frac{1}{x^3}\,dx$

   (b) diverges by the limit comparison test with $\displaystyle\int_2^\infty \frac{1}{x^3}\,dx$

   (c) converges by the direct comparison test with $\displaystyle\int_2^\infty \frac{1}{x^3}\,dx$

   (d) converges by the limit comparison test with $\displaystyle\int_2^\infty \frac{1}{x}\,dx$

2. The sequence whose nth term is $a_n = \dfrac{n}{\ln n}$ $\quad \frac{1}{\frac{1}{n}} = \frac{1}{0} = \infty$

   (a) converges to 0

   (b) converges to 1

   (c) converges to 2

   (d) diverges

3. The series $\displaystyle\sum_{n=1}^{\infty}(-1)^n\frac{2n^2+1}{n^2-5}$ $\quad$ by AST (3) $\lim \neq 0$ ∴ div by n^{th} term test

   (a) diverges by the nth term test

   (b) converges by the nth term test

   (c) converges absolutely

   (d) converges conditionally

4. If we use S_4 to approximate the sum of the series $\displaystyle\sum_{n=1}^{\infty}\frac{(-1)^{n-1}}{n}$ then the error satisfies

   $S_4 = 1 + \left(\frac{-1}{2}\right) + \frac{1}{3} + \left(\frac{-1}{4}\right)$

   (a) the error is negative and $|\text{error}| < 0.25$

   (b) the error is negative and $|\text{error}| < 0.2$

   (c) the error is positive and $|\text{error}| < 0.1$

   (d) the error is positive and $|\text{error}| < 0.2$

   $= \frac{1}{4} + \frac{1}{3}$

   $\frac{3+4}{12} = \frac{7}{12}$

   error $= L - S_4$

   $=$

5. The series $\sum_{n=2}^{\infty} \frac{(-1)^n}{\ln n}$

$\frac{1}{\ln(n+1)} \cdot \frac{\ln n}{1}$

(a) diverges by the nth term test

(b) converges by the nth term test

(c) converges absolutely

(d) converges conditionally

6. The series $\sum_{n=0}^{\infty} e^{-n}$

$1 + e^{-1} + e^{-2}$ $\quad \frac{e^{-2}}{e^{-1}}$ $\quad e^{-1}$

$\frac{1}{e^2} \cdot \frac{e}{1} = \frac{1}{e} = r$

$\frac{1}{\frac{e-1}{e}}$ $\quad \frac{e}{e-1}$ $\quad \frac{1}{1-\frac{1}{e}}$

(a) converges to $\frac{e}{e-1}$

(b) converges to $\frac{1}{1-e}$

(c) converges to $\frac{e-1}{e}$

(d) diverges

7. The series $\sum_{n=1}^{\infty} \frac{n^5}{5^n}$

$\sqrt[n]{\frac{n^5}{5^n}} = \frac{\sqrt[n]{n^5}}{5} = \frac{(n^{\frac{1}{n}})^5}{5} = \frac{1}{5}$

(a) diverges by the root test

(b) converges by the root test

(c) converges by the direct comparison test with $\sum_{n=1}^{\infty} \frac{1}{5^n}$

(d) diverges by the direct comparison test with $\sum_{n=1}^{\infty} \frac{1}{5^n}$

8. One of the following is true

(a) If $\sum_{n=1}^{\infty} |a_n|$ diverges then $\sum_{n=1}^{\infty} a_n$ diverges

(b) If $\sum_{n=1}^{\infty} |a_n|$ converges then $\sum_{n=1}^{\infty} a_n$ converges

(c) If $\sum_{n=1}^{\infty} a_n$ converges then $\sum_{n=1}^{\infty} |a_n|$ converges

(d) $\sum_{n=1}^{\infty} |a_n|$ and $\sum_{n=1}^{\infty} a_n$ both converge or both diverge

9. The series $\sum_{n=1}^{\infty} \frac{\frac{1}{2}\tan^{-1} n}{n^3+1}$

(a) diverges by the direct comparison test with $\sum_{n=1}^{\infty} \frac{1}{n^3}$

(b) converges by the direct comparison test with $\sum_{n=1}^{\infty} \frac{1}{n^3}$

(c) converges by the limit comparison test with $\sum_{n=1}^{\infty} \frac{1}{n}$

(d) diverges by the limit comparison test with $\sum_{n=1}^{\infty} \frac{1}{n^2}$

10. The sequence whose nth term is $a_n = n\tan^{-1} n$

(a) converges to 0
(b) converges to $\frac{\pi}{2}$
(c) converges to $-\frac{\pi}{2}$
(d) diverges

11. The series $\sum_{n=1}^{\infty} \frac{1}{\sqrt{n}}$

(a) converges by the integral test
(b) diverges by the integral test
(c) diverges by the nth term test
(d) converges by the nth term test

12. The series $\sum_{n=2}^{\infty} \frac{(\ln n)^{35}}{n!}$

$\cancel{\lim \frac{(\ln n+1)^{35}}{(n+1)!} \cdot \frac{n!}{(\ln n)^{35}}}$

$\cancel{\lim \frac{1}{n+1} \left(\frac{\ln n+1}{\ln n}\right)^{35}}$

$\frac{(\ln n)^{35}}{n!} \cdot \cancel{n!} = \infty$ div

(a) diverges by the limit comparison test with $\sum_{n=2}^{\infty} \frac{1}{n!}$

(b) diverges by the nth term test
(c) diverges by the ratio test
(d) converges by the ratio test

13. The series $\sum_{n=1}^{\infty}(x-1)^n$

$a = 0$

$\lim_{n\to\infty} 1^n \qquad e^{\ln 1^n} = e^{\frac{\ln 1}{\frac{1}{n}}} = 1$

$-\frac{3}{2} + \frac{2}{2} = \frac{-1}{2}$

(a) converges absolutely for $0 < x < 2$
(b) converges conditionally for $0 \le x \le 2$
(c) converges conditionally for $0 < x < 2$
(d) converges absolutely for $0 \le x \le 2$

14. The integral $\int_1^2 \frac{dx}{(x-1)^{\frac{3}{2}}}$

$\lim_{a\to 1^+} \int_a^2 \frac{dx}{(x-1)^{\frac{3}{2}}} = \lim (x-1)^{-\frac{3}{2}} = \lim \frac{(x-1)^{-\frac{1}{2}}}{-\frac{1}{2}}$

$= \lim_{a\to 0^+} \frac{-2}{(x-1)^{\frac{1}{2}}} = \frac{-2}{\sqrt{-1}}$

(a) converges to 0
(b) converges to 1
(c) converges to $-\frac{1}{2}$
(d) diverges

15. If $\sum a_n$ is a convergent series of positive terms, then the series $\sum (a_n)^n$ converges

(a) True
(b) False

16. Consider the series $\sum_{n=1}^{\infty}(-1)^n \frac{1}{n^2+19}$. The least number of terms that are needed to estimate the sum of the series with an error of less than 0.01 is

(a) fifteen terms
(b) nine terms
(c) ten terms
(d) five terms

Error $< u_{n+1}$

$u_n < u_{n+1}$

$\frac{1}{n^2+19} < 0.01$

$\frac{1}{n^2+19} < \frac{1}{100}$

$n^2+19 > 100$

$n^2 > 81$

$n = 9$

17. The series $\displaystyle\sum_{n=2}^{\infty} \frac{7}{n(n+1)}$

(a) converges to $\frac{7}{2}$
(b) converges to $\frac{1}{2}$
(c) converges to $-\frac{1}{2}$
(d) diverges

18. The integral $\displaystyle\int_2^{\infty} \frac{dx}{\sqrt{x^2-1}}$

$\frac{1}{\sqrt{x^2-1}} < \frac{1}{\sqrt{x^2}} = \frac{1}{x}$ dx

$\lim_{x \to \infty} \frac{x}{\sqrt{x^2-1}} = \infty$

(a) diverges by the direct comparison test with $\displaystyle\int_2^{\infty} \frac{dx}{x}$

(b) diverges by the limit comparison test with $\displaystyle\int_2^{\infty} \frac{dx}{x^2}$

(c) converges by the direct comparison test with $\displaystyle\int_2^{\infty} \frac{dx}{x}$

(d) converges by the limit comparison test with $\displaystyle\int_2^{\infty} \frac{dx}{x^2}$

19. The series $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n\sqrt{n^2+1}}$

conv

$\frac{1}{n\sqrt{n^2+1}} \cdot \frac{n}{1}$ 0

(a) diverges by the limit comparison test with $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n}$

(b) converges by the limit comparison test with $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n^2}$

(c) converges by the limit comparison test with $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n}$

(d) diverges by the limit comparison test with $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n^2}$

20. The series $\displaystyle\sum_{n=1}^{\infty} (-1)^n \frac{5}{3^n}$

$-\frac{5}{3} + \frac{5}{3^2} - \frac{5}{3^3}$

$-\frac{1}{3}$ r

$\frac{-\frac{5}{3}}{1+\frac{1}{3}} = \frac{-\frac{5}{3}}{\frac{4}{3}}$

$-\frac{5}{4}$

(a) converges to $-\frac{5}{4}$
(b) converges to $\frac{15}{4}$
(c) converges to $-\frac{5}{2}$
(d) diverges

21. The sequence whose nth term is $a_n = 1 - \cos(\frac{1}{n})$

(a) converges to $1 - \frac{\pi}{2}$
(b) converges to 1
(c) converges to 0
(d) diverges

22. The series $\sum_{n=1}^{\infty} \frac{(\sin n)^2}{n^{\frac{5}{2}}}$

$\frac{(\sin n)^2}{n^{\frac{5}{2}}}$

(a) converges by the nth term test
(b) diverges the limit comparison test with $\sum_{n=1}^{\infty} \frac{1}{n}$
(c) converges by the limit comparison test with $\sum_{n=1}^{\infty} \frac{1}{n^3}$
(d) converges by the limit comparison test with $\sum_{n=1}^{\infty} \frac{1}{n^2}$

23. The sequence whose nth term is $a_n = \sqrt[n]{4^n n}$ $= \sqrt[n]{4^n} \sqrt[n]{n}$

$4 \quad 1$

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(a) converges to 0
(b) converges to 4
(c) converges to 2
(d) diverges

24. The series $\sum_{n=1}^{\infty} (1 - \frac{1}{2n})^n$ $e^{-\frac{1}{2}}$ $\frac{1}{e^{\frac{1}{2}}}$

$\sqrt[n]{(1 - \frac{1}{2n})^n}$

(a) diverges by the nth term test
(b) diverges by the root test
(c) converges by the root test
(d) converges by the nth term test

$\lim 1 - \frac{1}{2n}$

$1 - 0$

1

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Question 2 (10 points) Evaluate the integral $\int_0^2 \frac{dx}{(x-1)^{\frac{2}{3}}}$.

$$\int_0^2 \frac{dx}{(x-1)^{\frac{2}{3}}} = \int_0^1 \frac{dx}{(x-1)^{\frac{2}{3}}} + \int_1^2 \frac{dx}{(x-1)^{\frac{2}{3}}} = 3+3 = \boxed{6}$$

Converges

$$\Rightarrow \int_0^1 \frac{dx}{(x-1)^{\frac{2}{3}}} = \lim_{a\to 1^-} \int_0^a \frac{dx}{(x-1)^{2/3}}$$

$$= \lim_{a\to 1^-} \int_0^a (x-1)^{-2/3}$$

$$= \lim_{a\to 1^-} 3(x-1)^{1/3} \Big|_0^a$$

$$= 3\lim_{a\to 1^-} \left[(a-1)^{1/3} - (-1)^{1/3}\right]$$

$$= 3\left[(-1)^{1/3}\right] = \boxed{3} \text{ Converges}$$

تعوض فوق

$$\Rightarrow \int_1^2 \frac{dx}{(x-1)^{2/3}} = \lim_{b\to 1^+} \int_b^2 \frac{dx}{(x-1)^{2/3}}$$

$$= \lim_{b\to 1^+} 3(x-1)^{\frac{1}{3}} \Big|_b^2$$

$$= 3\lim_{b\to 1^+} \left[1 - (b-1)^{\frac{1}{3}}\right]$$

$$= \boxed{3} \text{ converge.}$$

تعوض فوق

$$\int_0^2 = 3+3 = 6$$

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Question 3 (14 points) Consider the power series $\sum_{n=1}^{\infty} \frac{(x-3)^n}{n\,2^n}$. Answer the following questions:

1. For what values of x does the series converge absolutely?
2. Find the radius of convergence.
3. For what values of x does the series converge conditionally?
4. Find the interval of convergence.

$\sum_{n=1}^{\infty} \frac{(x-3)^n}{n\,2^n}$

Applay Ratio test:

$\lim_{n\to\infty} \left| \frac{\cancel{(x-3)}^{n+1}}{(n+1)\,2^{n+1}} \cdot \frac{n\,2^n}{\cancel{(x-3)^n}} \right|$

$= |x-3| \lim_{n\to\infty} \left| \frac{n}{(n+1)\,2^n} \right|$

~~H'Lopital rule~~ $= \frac{1}{(n+1)\,2^n \ln 2 + 2^n}$

اللى يجوز أيضاً كلى

Ratio test

$\sqrt{\frac{x-3}{2^n}}$

$\left|\frac{x-3}{2}\right| < 1$

$-2 < \frac{x-3}{2} < 2$

$1 < x < 5$

Applay Root test

$\lim_{n\to\infty} \sqrt[n]{\frac{(x-3)^n}{n\,2^n}}$

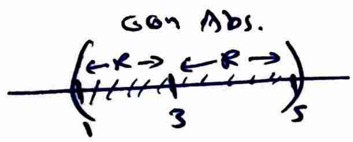
$\lim_{n\to\infty} \left| \frac{x-3}{\sqrt[n]{n}\;2} \right|$

$\left|\frac{x-3}{2}\right| \boxed{<} 1$

$-1 < \frac{x-3}{2} < 1$

$\boxed{1 < x < 5}$

$a = 3 \to R = 2$



$x = 1 \to \sum \frac{(-2)^n}{n\,2^n} = \sum \frac{1}{n}(-1)^n$

harmonic Alternating

$\therefore$ Converges

Cond.

$x = 2 \to (-1)^n \frac{1}{n\,2^n}$

RT $\lim \left| \frac{1}{(n+1)\,2^{n+1}} \cdot n\,2^n \right|$

$: \frac{1}{2} \quad \frac{n}{n+1}$

$\frac{1}{2}$ Converges

$\boxed{R = 2}$

values Converges absolutely

$x \in (1\ \ 5)$

8 values converges Conditionaly

$[1\ \ 5]$

$\therefore \sum \frac{(x-3)^n}{n\,2^n}$ converges on $[1\ \ 5]$

Question 4 (10 points) Find the Taylor series generated by $f(x) = \frac{1}{x^2}$ at $a = 2$. (Write the final answer using the sigma notation).

$$\sum_{n=0}^{\infty} \frac{f^{(n)}(a)}{n!}(x-a)^n = f(0) + f'(1)(x-a)^1$$

$$f(x) = \frac{1}{x^2} = \frac{1}{4} \qquad f'(x) = \frac{-2}{x^3} \quad \frac{-2}{8}$$

$$f''(x) = \frac{3x}{x^6} = \frac{3}{x^5} = \frac{3}{32}$$

$$\frac{6}{x^4}$$

$$\boxed{f(x) = \frac{1}{4} + -\frac{2}{8}(x-2) + \frac{3}{32}\frac{(x-2)^2}{2!} + \cdots}$$

$$\sum$$

x