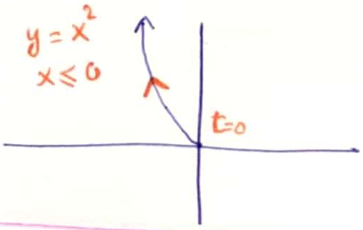
11.1 Prametrization of plane curves.

Identify the particl's path by finding a Cartesian equation. Graph the Cartisian equation. Indicate the portion of the graph traced by the particle & the direction of motion.

$\boxed{2}$ $x = -\sqrt{t}$, $y = t$, $t \geqslant 0$

$x = -\sqrt{y}$ or $y = x^2$, $x \leqslant 0$, $y \geqslant 0$

(Initial Point) IP: $t=0$
$\Rightarrow (x,y) = (0,0)$.

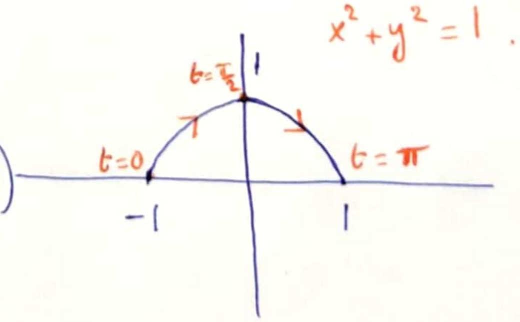


$\boxed{6}$ $x = \cos(\pi - t)$, $y = \sin(\pi - t)$, $0 \leqslant t \leqslant \pi$

$x^2 + y^2 = \cos^2(\pi - t) + \sin^2(\pi - t) = 1$

$\Rightarrow x^2 + y^2 = 1$

IP: when $t=0$
$\Rightarrow (x,y) = (\cos(\pi), \sin(\pi))$
$= (-1, 0)$.



TP: when $t = \pi$.
$\Rightarrow (x,y) = (\cos 0, \sin 0)$
$= (1, 0)$

To check the direction:
$\Rightarrow t = \frac{\pi}{2}$
$\Rightarrow (x, y) = (\cos\frac{\pi}{2}, \sin\frac{\pi}{2})$
$= (0, 1)$

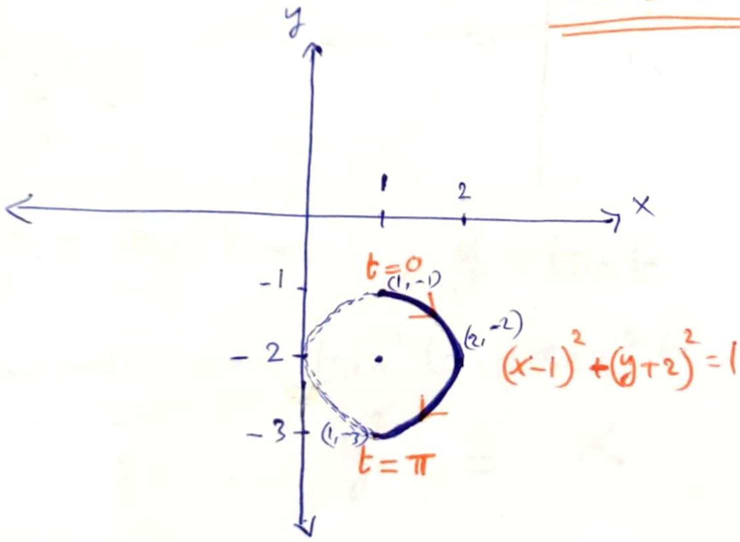
10 $x = 1+\sin t$, $y = \cos t - 2$, $0 \leq t \leq \pi$

$x = 1+\sin t \Rightarrow \sin t = x-1 \Rightarrow \boxed{\sin^2 t = (x-1)^2}$

$y = \cos t - 2 \Rightarrow \cos t = y+2 \Rightarrow \boxed{\cos^2 t = (y+2)^2}$

$\sin^2 t + \cos^2 t = (x-1)^2 + (y+2)^2 = 1$

$(x-1)^2 + (y+2)^2 = 1$ * (it is a circle equation with center $(1,-2)$ & a radius of 1).



Note that → we know

$-1 \leq \cos t \leq 1.$

$-3 \leq \cos t - 2 \leq -1$

$-3 \leq y \leq -1$

So, $\boxed{y < 0}$

& $-1 \leq \sin t \leq 1.$

$0 \leq 1+\sin t \leq 2.$

$0 \leq x \leq 2.$

So, $\boxed{x \geq 0}$.

★ IP : $t=0$

$(x,y) = (1+\sin(0), \cos(0)-2)$
$= (1, -1)$

when $y<0$ & $x \geq 0$
⇒ the curve in the fourth quadrent.

★ TP: $t = \pi$

$(x,y) = (1+\sin(\pi), \cos(\pi)-2)$
$= (1+0, -1-2) = (1,-3)$

★ $t = \frac{\pi}{2} \Rightarrow (x,y) = (2, -2)$

$\boxed{14}$ $x = \sqrt{t+1}$, $y = \sqrt{t}$, $t \geq 0$

$y = \sqrt{t} \Rightarrow y^2 = t$

$\Rightarrow x = \sqrt{y^2+1}$, $y \geq 0$.

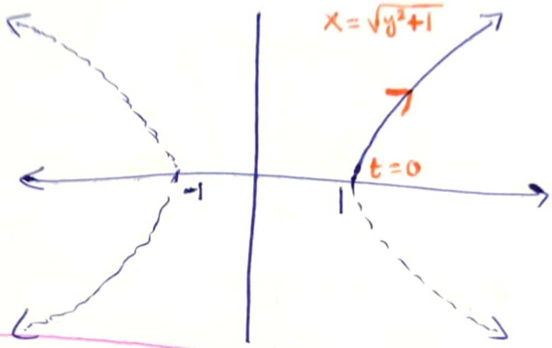
$\Rightarrow x^2 = y^2 + 1$

$\Rightarrow x^2 - y^2 = 1$

it's a hyperbola equation with $y \geq 0$ & $x \geq 1$

* when $t=0$
IP $(x,y) = (1,0)$

* No TP.



$t=0$ $(1,0)$

$t=1$ $(\sqrt{2},1)$

$\boxed{15}$ $x = \sec^2 t - 1$, $y = \tan t$, $-\frac{\pi}{2} < t < \frac{\pi}{2}$.

remember: $\tan^2 t = \sec^2 t - 1$

so, $y^2 = x$

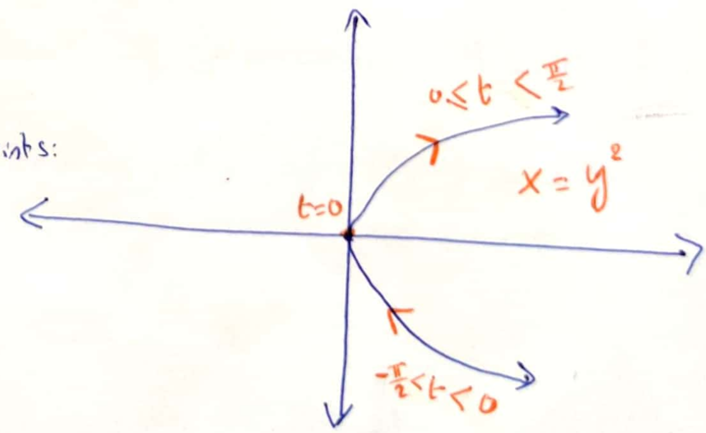
* No IP & No TP.

$\Rightarrow$ To check the direction, take two points:

* when $t=0$
$\Rightarrow (x,y) = (0,0)$

* when $t = \frac{\pi}{4}$
$\Rightarrow (x,y) = (1,1)$

since $x = y^2 \Rightarrow x \geq 0$.



18 $x = 2\sinh t \quad , y = 2\cosh t \quad , -\infty < t < \infty .$

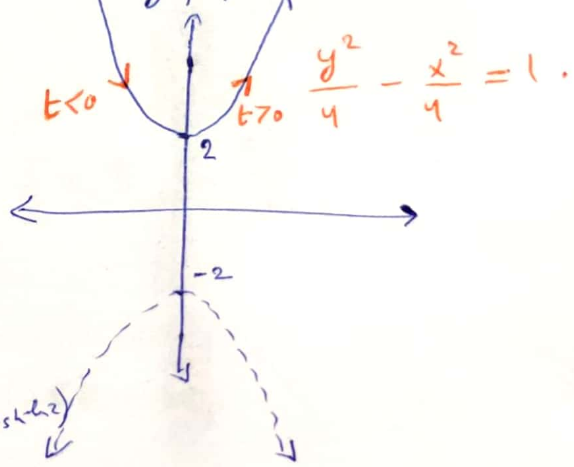
$\sinh t = \frac{x}{2}$

$\cosh t = \frac{y}{2} \quad , y > 0 \quad (y \geq 2).$

$\cosh^2 t - \sinh^2 t = 1.$

$\left(\frac{y}{2}\right)^2 - \left(\frac{x}{2}\right)^2 = 1.$

$\frac{y^2}{4} - \frac{x^2}{4} = 1$



☆ No IP & No TP.

when $t = 0$
$\Rightarrow (x, y) = (0, 2)$

☆ when $t = \ln 2$
$\Rightarrow (x, y) = (2\sinh \ln 2, 2\cosh \ln 2)$
$= \left(\frac{3}{2}, \frac{5}{2}\right).$

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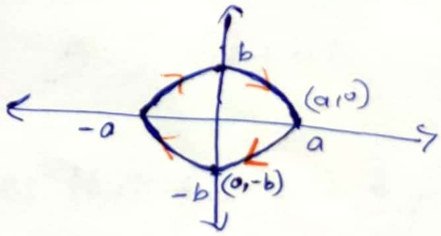
20 Find Parametric equations & parametric interval for the motion of a particle that starts at $(a, 0)$ & traces the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1.$

a) Once clockwise

$\frac{x}{a} = \sin t \Rightarrow x = a \sin t$

$\frac{y}{b} = \cos t \Rightarrow y = b \cos t.$



$\frac{\pi}{2} \leq t \leq \frac{5\pi}{2}$

☆ IP when $t = \frac{\pi}{2} \Rightarrow (x, y) = (a, 0)$

☆ TP when $t = \frac{5\pi}{2} \Rightarrow (x, y) = (a, 0)$

☆ To check the direction:
take $t = \pi$.
$x = a \sin \pi = 0.$
$y = b \cos \pi = -b$ $\Big\} (0, -b)$

(b) Once Counterclockwise.

$$\frac{x}{a} = \cos t \Rightarrow x = a\cos t.$$

$$\frac{y}{b} = \sin t \Rightarrow y = b\sin t. \qquad 0 \leq t \leq 2\pi$$

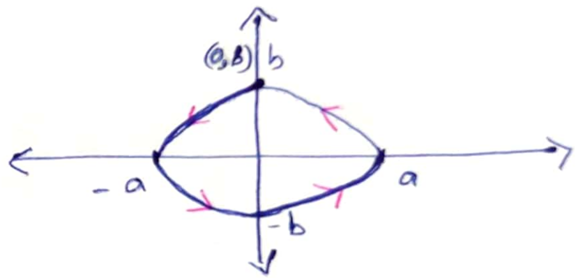
IP: $t=0 \Rightarrow (x,y) = (a,0)$

TP: $t=2\pi \Rightarrow (x,y) = (a,0)$

To check the direction, take $t = \frac{\pi}{2}$.

$x = a\cos\frac{\pi}{2} = 0$

$y = b\sin\frac{\pi}{2} = b$ $\Big\} (0,b)$



(c) Twice clockwise.

$x = a\sin t$

$y = b\cos t \qquad \frac{\pi}{2} \leq t \leq \frac{9\pi}{2}$

d) Twice counterclockwise.

$x = a\cos t.$

$y = a\sin t \qquad 0 \leq t \leq 4\pi.$

22) Find a parametrization of the curve:

The line segment with end points $(-1, 3)$ & $(3, -2)$.
(x_1, y_1) (x_2, y_2)

$$m_{\text{slope}} = \frac{-2-3}{3--1} = -\frac{5}{4}$$

$$y - y_1 = m(x - x_1).$$

$$y - 3 = -\frac{5}{4}(x+1) \Rightarrow y = -\frac{5}{4}x + \frac{7}{4}.$$

Let $x = t$, $-1 \leq t \leq 3$.
so, $y = -\frac{5}{4}t + \frac{7}{4}$

another possible:

Let $t = \frac{x+1}{4}$, when $x = -1 \Rightarrow t = 0$
$x = 3 \Rightarrow t = 1$

$\Rightarrow x = 4t - 1$, $0 \leq t \leq 1$.
$y = -5t + 3$

or

Let $t = x + 1$
$x = t - 1$, $0 \leq t \leq 4$.
$y = -\frac{5}{4}t + 3$

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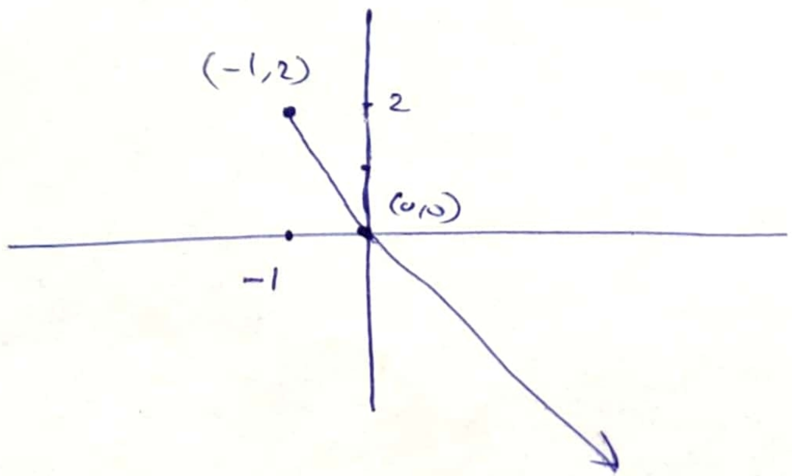
(26) Find a parametrization for the ray with initial point $(-1, 2)$ that passes through the point $(0,0)$.

$$m = \frac{2-0}{-1-0} = -2.$$

$$y - 0 = -2(x-0) \Rightarrow y = -2x.$$

let $x = t$ $\qquad t \geq -1$

$\Rightarrow y = -2t$



11.2 :- Calculus with Parametric Curves.

7) If $x = \sec t$, $y = \tan t$, $t = \frac{\pi}{6}$

Find the tangent line & $\frac{d^2y}{dx^2}$.

when $t = \frac{\pi}{6}$: $x = \sec\frac{\pi}{6} = \frac{2}{\sqrt{3}}$

$y = \tan\frac{\pi}{6} = \frac{1}{\sqrt{3}}$

The point is $\left(\frac{2}{\sqrt{3}}, \frac{1}{\sqrt{3}}\right)$.

slope $= \frac{dy}{dx}$

$$\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}} = \frac{\sec^{\cancel{2}} t}{\cancel{\sec t}\tan t} = \frac{\sec t}{\tan t}.$$

$$= \frac{1}{\cancel{\cos t}} \cdot \frac{\cancel{\cos t}}{\sin t}$$

$$= \csc t.$$

So, $\left.\frac{dy}{dx}\right|_{t=\frac{\pi}{6}} = \csc\frac{\pi}{6} = 2.$

The tangent line is :-

$$y - \frac{1}{\sqrt{3}} = 2\left(x - \frac{2}{\sqrt{3}}\right)$$

$$\boxed{y = 2x - \sqrt{3}}$$

$$\Rightarrow \frac{d^2y}{dx^2} = \left.\frac{\frac{dy'}{dt}}{\frac{dx}{dt}}\right|_{t=\frac{\pi}{6}} = \left.\frac{-\csc t \cot t}{\sec t \tan t}\right|_{t=\frac{\pi}{6}}.$$

$$= \frac{-2 \cdot \sqrt{3}}{\frac{2}{\sqrt{3}} \cdot \frac{1}{\sqrt{3}}}$$

$$= \frac{-2\sqrt{3}}{\frac{2}{3}} = -2\sqrt{3} \cdot \frac{3}{2} = \boxed{-3\sqrt{3}}$$

T4) Find the equation of the line tangent to the curve $x = t + e^t$, $y = 1 - e^t$, $t = 0$. & Find $\frac{d^2y}{dx^2}$ at this point.

at $t = 0 \Rightarrow x = 0 + e^0 = 1$

$y = 1 - e^0 = 1 - 1 = 0$ $\Big\}$ (x, y) $(1, 0)$

$$\frac{dy}{dx} = \left.\frac{\frac{dy}{dt}}{\frac{dx}{dt}}\right|_{t=0} = \left.\frac{-e^t}{1+e^t}\right|_{t=0} = \frac{-1}{1+1} = \frac{-1}{2}.$$

So, The line is :- $\boxed{y = -\frac{1}{2}x + \frac{1}{2}}$.

$$\frac{d^2y}{dx^2} = \frac{\frac{dy'}{dt}}{\frac{dx}{dt}}$$

$$\frac{dy'}{dt} = \frac{(1+e^t)(-e^t) + e^t e^t}{(1+e^t)^2}$$

$$= \frac{-e^t - \cancel{e^{2t}} + \cancel{e^{2t}}}{(1+e^t)^2} = \frac{-e^t}{(1+e^t)^2}$$

So, $\left.\frac{d^2y}{dx^2}\right|_{t=0} = \left.\frac{\frac{-e^t}{(1+e^t)^2}}{1+e^t}\right|_{t=0} = \left.\frac{-e^t}{(1+e^t)^3}\right|_{t=0}$

$$= \frac{-1}{(1+1)^3} = \boxed{\frac{-1}{8}}$$

20) Find the slope of the curve $x = f(t)$, $y = g(t)$ at $t=0$ & the tangent line

$t = \ln(x-t)$ & $y = te^t$.

To find $\frac{dx}{dt} \Rightarrow$ implicit derivative

$$\Rightarrow 1 = \frac{1}{x-t}\left(\frac{dx}{dt} - 1\right).$$

$$x - t = \frac{dx}{dt} - 1$$

$$\frac{dx}{dt} = x - t + 1$$

but : when $t=0 \Rightarrow 0 = \ln(x-0) \Rightarrow x = e^0 = 1$
$y = 0e^0 = 0$ $\Big\}$ (x, y) $(1, 0)$

So, $\frac{dx}{dt} = 1 - t + 1 = \boxed{2-t}$

$$\frac{dy}{dt} = \boxed{te^t + e^t}$$

$$\left.\frac{dy}{dx}\right|_{t=0} = \left.\frac{\frac{dy}{dt}}{\frac{dx}{dt}}\right|_{t=0} = \left.\frac{te^t + e^t}{2-t}\right|_{t=0}$$

$$= \frac{0+1}{2-0} = \boxed{\frac{1}{2}}.$$

So, the tangent line is

$$\boxed{y = \frac{1}{2}x - \frac{1}{2}}.$$

22 Find the area enclosed by the y-axis & the curve $x = t - t^2$, $y = 1 + e^{-t}$.

$$\int x \, dy.$$

$x = t - t^2$.

$dy = -e^{-t} dt$.

$$\int_0^1 \left| (t - t^2) \cdot -e^{-t} \, dt \right|$$

$$= \int_0^1 \left| (t^2 - t) e^{-t} \, dt \right|$$

$$= \left| \left(-e^{-t}(t^2 - t) - e^{-t}(2t-1) - 2e^{-t} \right) \Big|_0^1 \right|$$

$$= \left| -e^{-1}(0) - e^{-1}(-1) - 2e^{-1} - \left(0 - e^0(-1) - 2(e^0)\right) \right|$$

$$= \left| -\frac{3}{e} + 1 \right|$$

y-axis: $x = 0$

& $x = t - t^2$

Find the intersection points:

$t - t^2 = 0$

$t(1-t) = 0$

$\boxed{t = 0, 1}$

$t^2 - t$	$+$	e^{-t}
$2t-1$	$-$	$-e^{-t}$
2	$+$	e^{-t}
		$-e^{-t}$

25\ Find the length of the curve $x = \cos t$, $y = t + \sin t$. $0 \le t \le \pi$

$$L = \int_0^{\pi} \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}\, dt.$$

$$\Rightarrow \frac{dx}{dt} = -\sin t \Rightarrow \left(\frac{dx}{dt}\right)^2 = (-\sin t)^2 = \sin^2 t.$$

$$\Rightarrow \frac{dy}{dt} = 1 + \cos t \Rightarrow \left(\frac{dy}{dt}\right)^2 = (1 + \cos t)^2 = 1 + 2\cos t + \cos^2 t.$$

$$\Rightarrow \left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2 = \sin^2 t + 1 + 2\cos t + \cos^2 t.$$

$$= 1 + 1 + 2\cos t.$$

$$= 2 + 2\cos t.$$

$$= 2(1 + \cos t).$$

$$\Rightarrow L = \int_0^{\pi} \sqrt{2(1+\cos t)}\, dt = \sqrt{2} \int_0^{\pi} \sqrt{1+\cos t \left(\frac{1-\cos t}{1-\cos t}\right)}\, dt.$$

$$= \sqrt{2} \int_0^{\pi} \sqrt{\frac{1-\cos^2 t}{1-\cos t}}\, dt.$$

$$= \sqrt{2} \int_0^{\pi} \frac{|\sin t|}{\sqrt{1-\cos t}}\, dt.$$

$$= \sqrt{2} \int_0^{\pi} \frac{\sin t}{\sqrt{1-\cos t}}\, dt.$$

since $0 \le \sin t \le 1$ when $0 \le t \le \pi$

let $u = 1 - \cos t$

$du = \sin t\, dt$

$$= \sqrt{2} \int \frac{du}{\sqrt{u}}$$

$$= \sqrt{2} \cdot 2\sqrt{u}$$

$$= \sqrt{2}\; 2\sqrt{1-\cos t} \Big|_0^{\pi}$$

$$= 2\sqrt{2}(\sqrt{2} - 0) = \boxed{4}$$

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$\boxed{27}$ Find the length of the curve

$$x = \frac{t^2}{2}, \quad y = \frac{(2t+1)^{3/2}}{3}, \quad 0 \le t \le 4.$$

$$\Rightarrow \frac{dx}{dt} = \frac{2t}{2} = t \Rightarrow \left(\frac{dx}{dt}\right)^2 = t^2.$$

$$\Rightarrow \frac{dy}{dt} = \frac{\frac{3}{2}(2t+1)^{\frac{1}{2}}}{3} \cdot 2 = \frac{\cancel{3} \cdot \cancel{2}(2t+1)^{\frac{1}{2}}}{\cancel{3} \cdot \cancel{2}}$$

$$= (2t+1)^{\frac{1}{2}}$$

$$\left(\frac{dy}{dt}\right)^2 = \left((2t+1)^{\frac{1}{2}}\right)^2 = |2t+1| = 2t+1 \qquad 0 \le t \le 4.$$

$$\Rightarrow \left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2 = t^2 + 2t + 1$$

$$= (t+1)^2.$$

$$L = \int_0^4 \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2} = \int_0^4 \sqrt{(t+1)^2}\, dt.$$

$$= \int_0^4 (t+1)\, dt$$

$$= \left(\frac{t^2}{2} + t\right)\Big|_0^4$$

$$= \frac{16}{2} + 4 - 0$$

$$= 8 + 4$$

$$= \boxed{12}$$

11.3 : Polar Coordinates.

1) which polar Coordinate pairs lable the same point.

a) $(3,0)$ b) $(-3,0)$ c) $\left(2,\frac{2\pi}{3}\right)$ d) $\left(2,\frac{7\pi}{3}\right)$

e) $(-3,\pi)$ f) $\left(2,\frac{\pi}{3}\right)$ g) $(-3,2\pi)$ h) $\left(-2,-\frac{\pi}{3}\right)$

Remember :- $(r,\theta) = (r, \theta + 2\pi n)$ ❀

$= (-r, \theta + (2n+1)\pi)$ ❀❀ $\quad n = 0, \mp1, \mp2, \mp3, \ldots$

a) = e)

$(3,0) = (-3,\pi)$ (Compare with ❀❀ with $n=0$)

b) = g)

$(-3,0) = (-3,2\pi)$ (❀ with $n=1$).

c) = h)

$\left(2,\frac{2\pi}{3}\right) = \left(-2,-\frac{\pi}{3}\right)$ (❀❀ with $n=-1$)

d) = f)

$\left(2,\frac{\pi}{3}\right) = \left(2,\frac{7\pi}{3}\right)$ (❀ with $n=1$).

6) Find the Cartesian coordinates of the following points :-

$$(r,\theta) \longrightarrow (x,y) \qquad x = r\cos\theta \qquad y = r\sin\theta.$$

a) $\left(\sqrt{2}, \frac{\pi}{4}\right)$ $\Rightarrow$ $x = \sqrt{2}\cos\frac{\pi}{4} = \sqrt{2}\cdot\frac{1}{\sqrt{2}}$

$= 1$

$y = \sqrt{2}\sin\frac{\pi}{4} = \sqrt{2}\cdot\frac{1}{\sqrt{2}} = 1$

$(x,y) = (1,1)$

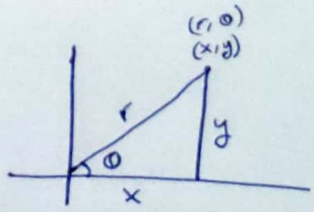
b) $(1,0)$ $\Rightarrow$ $\left.\begin{array}{l} x = r\cos 0 = 1 \\ y = 1\sin 0 = 0 \end{array}\right\}$ $(x,y) = (1,0)$

c) $\left(0, \frac{\pi}{2}\right)$ $\Rightarrow$ $\left.\begin{array}{l} x = 0\cos\frac{\pi}{2} = 0 \\ y = 0\sin\frac{\pi}{2} = 0 \end{array}\right\}$ $(x,y) = (0,0)$

d) $\left(-\sqrt{2}, \frac{\pi}{4}\right)$ $\Rightarrow$ $\left.\begin{array}{l} x = -\sqrt{2}\cos\frac{\pi}{4} = -1 \\ y = -\sqrt{2}\sin\frac{\pi}{4} = -1 \end{array}\right\}$ $(x,y) = (-1,-1)$

e) $\left(-3, \frac{5\pi}{6}\right)$ $\Rightarrow$ $\left.\begin{array}{l} x = -3\cos\left(\frac{5\pi}{6}\right) = -3\left(-\frac{\sqrt{3}}{2}\right) = \frac{3\sqrt{3}}{2} \\ y = -3\sin\left(\frac{5\pi}{6}\right) = -3\cdot\frac{1}{2} = -\frac{3}{2} \end{array}\right\}$ $(x,y) = \left(\frac{3\sqrt{3}}{2}, -\frac{3}{2}\right)$

f) $\left(5, \tan^{-1}\left(\frac{4}{3}\right)\right)$ $\Rightarrow$ $\left.\begin{array}{l} x = 3 \\ y = 4 \end{array}\right\}$ $(x,y) = (3,4)$



$\tan\theta = \frac{y}{x}$

$\tan^{-1}\left(\frac{y}{x}\right) = \theta$

g) $(-1, 7\pi) \Rightarrow x = -1 \cos(7\pi) = (-1)(-1) = 1$
$y = -1 \sin(7\pi) = -1(0) = 0$
$(x,y) = (1,0)$

7) Find the polar coordinates, $0 \leq \theta \leq 2\pi$ & $r \geq 0$.

$(x,y) \longrightarrow (r,\theta)$: $r^2 = x^2 + y^2$, $\theta = \tan^{-1}\left(\frac{y}{x}\right)$
$\cos\theta = \frac{x}{r}$ & $\sin\theta = \frac{y}{r}$

a) $(1,1) = (x,y) \Rightarrow r^2 = x^2 + y^2 = 1^2 + 1^2 = 2 \Rightarrow r = \sqrt{2}$
$\cos\theta = \frac{1}{\sqrt{2}}$
$\sin\theta = \frac{1}{\sqrt{2}}$
$\theta = \frac{\pi}{4}$.
$(r,\theta) = \left(\sqrt{2}, \frac{\pi}{4}\right)$

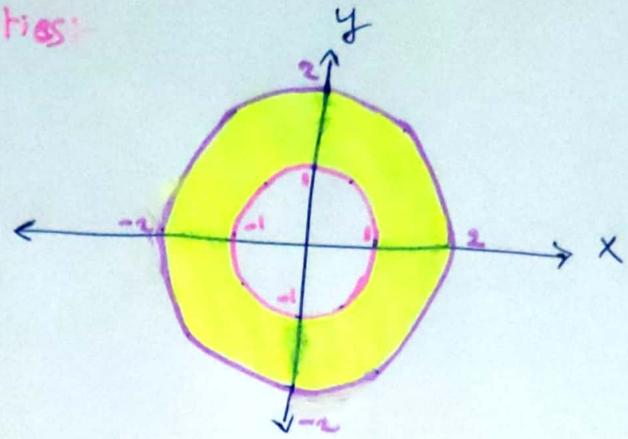
b) $(-3,0) \Rightarrow r = \sqrt{(-3)^2 + (0)^2} = 3$
$\cos\theta = -\frac{3}{3} = -1$
$\sin\theta = \frac{0}{3} = 0$
$\theta = \pi$
$(r,\theta) = (3,\pi)$

c) $(\sqrt{3}, -1) \Rightarrow r = \sqrt{(\sqrt{3})^2 + (-1)^2} = \sqrt{4} = 2$.
$\cos\theta = \frac{\sqrt{3}}{2}$
$\sin\theta = \frac{-1}{2}$
in the 4^{th} quadrant
$\theta = \frac{11\pi}{6}$
$(r,\theta) = \left(2, \frac{11\pi}{6}\right)$

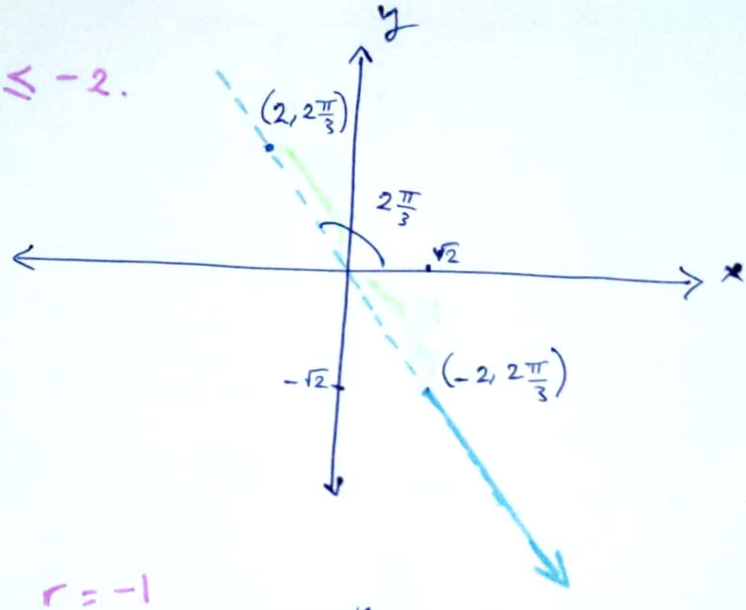
d) $(-3,4) \Rightarrow r = \sqrt{(-3)^2 + (4)^2} = 5$.
$\cos\theta = -\frac{3}{5}$
$\sin\theta = \frac{4}{5}$
in the 2^{nd} quadrant
$\theta = \pi - \tan^{-1}\left(\frac{4}{3}\right)$
$(r,\theta) = \left(5, \pi - \tan^{-1}\left(\frac{4}{3}\right)\right)$

Graph the sets of points whose polar coordinates satisfy the equations & inequalities:

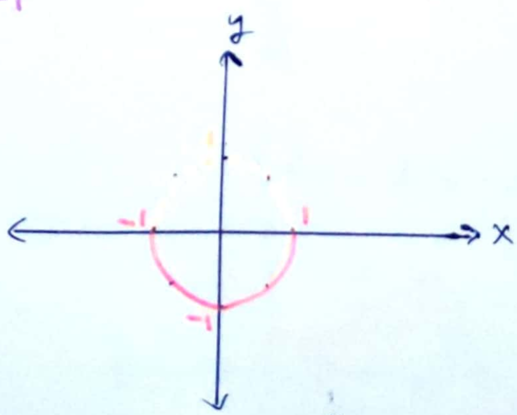
14) $1 \leq r \leq 2$.



16) $\theta = \frac{2\pi}{3}$, $r \leq -2$.



22) $0 \leq \theta \leq \pi$, $r = -1$



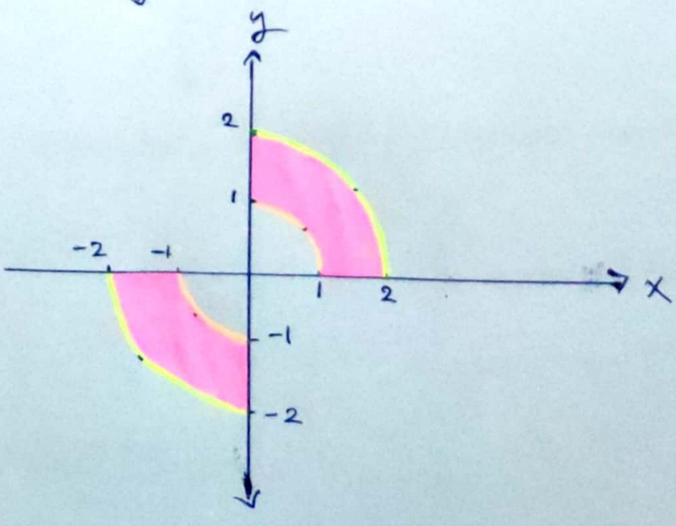
26) $0 \leq \theta \leq \frac{\pi}{2}$, $1 \leq |r| \leq 2$.

which mean

$1 \leq r \leq 2$.

or $-1 \leq -r \leq -2$

$\Rightarrow -2 \leq r \leq -1$.



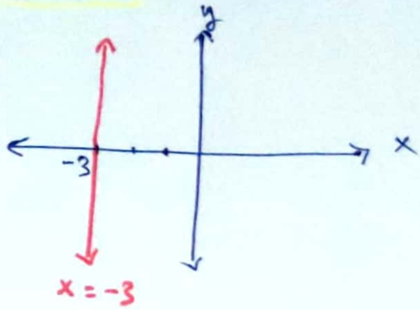
Replace the polar equations with equivalent Cartesian equations.

Then describe the graph.

32 $r = -3 \sec \theta$.

$$r = -3 \cdot \frac{1}{\cos\theta} \Rightarrow r\cos\theta = -3.$$

$\boxed{x = -3}$ (vertical line through $(-3,0)$).



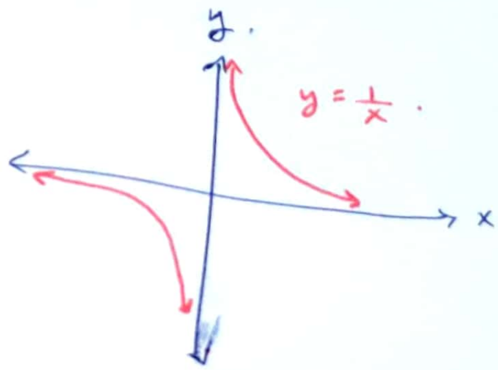
38 $r^2 \sin(2\theta) = 2$.

$$r^2 \cdot 2\sin\theta\cos\theta = 2.$$

$$r\sin\theta \cdot r\cos\theta = 1$$

$$y \quad x \quad = 1$$

$$\boxed{y = \frac{1}{x}}$$

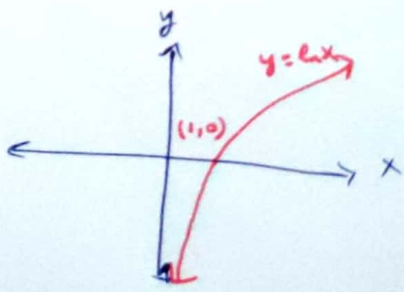


42 $r\sin\theta = \ln r + \ln(\cos\theta)$

$(\ln a + \ln b = \ln(ab))$

$$r\sin\theta = \ln(r\cos\theta).$$

$$\boxed{y = \ln x}.$$



52 $r\sin\left(\frac{2\pi}{3} - \theta\right) = 5$.

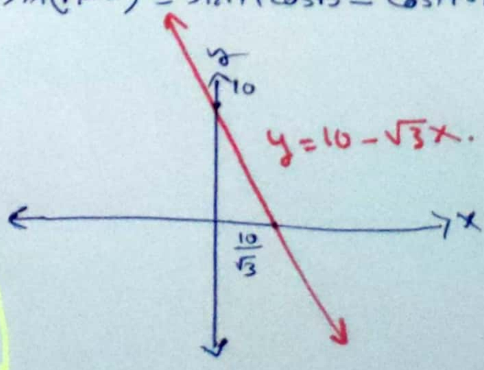
*Remember: $\sin(A-B) = \sin A\cos B - \cos A\sin B$.

$$r\left(\sin\left(\frac{2\pi}{3}\right)\cos\theta - \cos\left(\frac{2\pi}{3}\right)\sin\theta\right) = 5.$$

$$r\left(\frac{\sqrt{3}}{2}\cos\theta - -\frac{1}{2}\sin\theta\right) = 5$$

$$\frac{\sqrt{3}}{2} r\cos\theta + \frac{1}{2} r\sin\theta = 5.$$

$$\frac{\sqrt{3}}{2} x + \frac{1}{2} y = 5 \Rightarrow \boxed{y = 10 - \sqrt{3}\,x}$$



62. $x^2 + xy + y^2 = 1$. Find polar equation.

$$x^2 + y^2 + xy = 1.$$

$$r^2 + (r\cos\theta)(r\sin\theta) = 1.$$

$$r^2 + r^2\cos\theta\sin\theta = 1.$$

$$r^2\left(1 + \frac{1}{2}\sin 2\theta\right) = 1.$$