46) $a_n = \frac{\sin^2 n}{2^n}$ {10.1} $\dfrac{1}{\backslash}$

$$0 \leq \frac{\sin^2 n}{2^n} \leq \frac{1}{2^n}$$

$$\downarrow$$

$$\lim_{n\to\infty} \frac{\sin^2 n}{2^n} = 0 \quad \text{conv} \qquad \lim_{n\to\infty} \left(\frac{1}{2}\right)^n = 0$$

59) $a_n = \frac{\ln n}{n^{\frac{1}{n}}}$

$$\lim_{n\to\infty} \frac{\ln n}{n^{\frac{1}{n}}} = \frac{\infty}{1} = \infty \quad \text{div}$$

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68) $a_n = \ln\left(1+\frac{1}{n}\right)^n$

$$\ln\left(\lim_{n\to\infty} \left(1+\frac{1}{n}\right)^n\right) = \ln e^1 = 1 \quad \text{conv}$$

84) $a_n = \sqrt[n]{n^2+n}$

$$\lim_{n\to\infty} e^{\frac{\ln(n^2+n)}{n}} = e^{\lim_{n\to\infty} \frac{2n+1}{n^2+n}} = e^0 = 1 \quad \text{conv}$$

6) $a_n = \frac{2^n - 1}{2^n}$

$\cancel{a_n = \left(\frac{1}{2}\right)^n}$

$a_1 = \frac{2-1}{2} = \frac{1}{2}$

$a_2 = \frac{3}{4}$ $\qquad a_4 = \frac{15}{16}$

$a_3 = \frac{7}{8}$

$a_1 = -2$, $a_{n+1} = \frac{n a_n}{(n+1)}$

10)

$a_2 = \frac{-2}{2} = -1$

$a_3 = \frac{-2}{3}$

$a_4 = \frac{-2}{4} = \frac{-1}{2}$

$a_5 = \frac{-2}{5}$

$a_6 = \frac{-2}{6} = \frac{-1}{3}$

$a_7 = \frac{-2}{7}$

$a_8 = \frac{-2}{8} = \frac{-1}{4}$

$a_9 = \frac{-2}{9}$

$a_{10} = \frac{-2}{10} = \frac{-1}{5}$

22) $2, 6, 10, 14, 18, \ldots$

$a_n = 4n - 2$, $n = 1, 2, \ldots$

26) $0, 1, 1, 2, 2, 3, 3, 4, \ldots$

$a_n = \lfloor \frac{n}{2} \rfloor$ $n = 1, 2, 3, \ldots$

31) $a_n = \frac{1 - 5n^4}{n^4 + 8n^3}$

$$\lim_{n\to\infty} \frac{1-5n^4}{n^4+8n^3} = \frac{5}{1} = 5 \quad \text{conv}$$

35) $a_n = 1 + (-1)^n$

$\lim_{n\to\infty} 1 + (-1)^n$

does not exist

div

$a_1 = 0$

$a_2 = 1+1 = 2$

$a_3 = 1-1 = 0$

$a_4 = 1+1 = 2$

$a_5 = 1-1 = 0$

36) $a_n = (-1)^n \left(1 - \frac{1}{n}\right)$

$a_1 = 0$

$a_2 = \frac{1}{2}$

$a_3 = -\frac{2}{3}$

$a_4 = \frac{3}{4}$

$\lim_{n\to\infty} (-1)^n \left(1 - \frac{1}{n}\right)$

does not exist

div

41) $a_n = \sqrt{\frac{2n}{n+1}}$

$= \sqrt{\lim_{n\to\infty} \frac{2n}{n+1}} = \sqrt{2}$ conv

44) $a_n = n\pi \cos(n\pi)$

$a_n = n\pi (-1)^n$

$\lim_{n\to\infty} n\pi (-1)^n$ does not exist

div

48) $a_n = \frac{3^n}{n^3}$

$\lim_{n\to\infty} \frac{3^n \ln 3}{3n^2} = \lim_{n\to\infty} \frac{3^n (\ln 3)^2}{6n} = \lim_{n\to\infty} \frac{3^n (\ln 3)^3}{6} = \infty$ div

50) $a_n = \frac{\ln n}{\ln 2n}$

$\lim_{n\to\infty} \frac{1}{\cancel{x}} \cdot \frac{2\cancel{x}}{2} = 1$ conv

$a_n = \left(1-\frac{1}{n}\right)^n$

57]

$$\lim_{n\to\infty} \left(1-\frac{1}{n}\right)^n = e^{-1} = \frac{1}{e}$$

58] $a_n = (n+4)^{\frac{1}{(n+4)}}$

$$\lim_{x\to\infty} x^{\frac{1}{x}} = 1 = \lim_{n\to\infty} (n+4)^{\frac{1}{(n+4)}} = 1 \text{ conv}$$

60] $a_n = \ln n - \ln(n+1)$

$$\lim_{n\to\infty} \ln\left(\frac{n}{n+1}\right) = \ln\left(\lim_{n\to\infty} \frac{n}{n+1}\right) = \ln 1 = 0 \quad \text{conv}$$

63] $a_n = \frac{n!}{n^n}$

$$\frac{n!}{n^n} \leq \frac{1}{n}$$

$$\lim_{n\to\infty} \frac{1}{n} = 0$$

$$\therefore \lim_{n\to\infty} \frac{n!}{n^n} = 0 \quad \text{conv}$$

70] $a_n = \left(\frac{n}{n+1}\right)^n$

$u = n+1$

$$\lim_{n\to\infty} \left(1 - \frac{1}{n+1}\right)^n$$

$$\lim_{u\to\infty} \left(1 - \frac{1}{u}\right)^{u-1}$$

$$\lim_{u\to\infty} \left(1-\frac{1}{u}\right)^u \left(1 - \frac{1}{u}\right)^{-1} = e^{-1} = \frac{1}{e}$$

or

$$e^{\lim_{n\to\infty} \frac{\ln\left(\frac{n}{n+1}\right)}{\frac{1}{n}}}$$

$$e^{\lim_{n\to\infty} \frac{\frac{\frac{1}{(n+1)^2}}{\frac{n}{n+1}}}{\frac{-1}{n^2}}}$$

$$e^{\lim_{n\to\infty} \frac{n}{-n-1}} = e^{-1} = \frac{1}{e} \quad \text{conv}$$

72] $a_n = \left(1-\frac{1}{n^2}\right)^n$

$u = n^2$

$\lim_{u\to\infty} \left(1-\frac{1}{u}\right)^{u^{\frac{1}{2}}}$

$\lim_{u\to\infty} \left(1-\frac{1}{u}\right)^{u}$

$e^{\lim_{n\to\infty} \frac{\ln\left(1-\frac{1}{n^2}\right)}{\frac{1}{n}}} = e^{\lim_{n\to\infty} \frac{2}{\cancel{n^3}} \cdot \frac{\cancel{n^2}}{n^2-1} \cdot \frac{\cancel{n^2}}{-1}} = e^{\lim_{n\to\infty} \frac{2n}{-n^2+1}} = e^0 = 1$ conv

76] $a_n = \sinh(\ln n)$

$a_n = \frac{e^{\ln n} - e^{-\ln n}}{2}$

$a_n = \frac{n - \frac{1}{n}}{2}$

$\lim_{n\to\infty} a_n = \infty$ div

82] $a_n = \frac{1}{\sqrt{n}} \tan^{-1} n$

$\lim_{n\to\infty} a_n = \lim_{n\to\infty} 0 \cdot \frac{\pi}{2} = 0$ conv

86] $a_n = \frac{(\ln n)^5}{\sqrt{n}}$

$\lim_{n\to\infty} \frac{\frac{5}{\cancel{n}}}{\sqrt{n}} \cdot \frac{2\sqrt{\cancel{n}}}{1} = \lim_{n\to\infty} \frac{10}{\sqrt{n}} = 0$ conv

92] $a_1 = -1$, $a_{n+1} = \frac{a_n+6}{a_n+2}$

$a_2 = 5$

$a_3 = \frac{11}{7}$

$a_4 =$

$\lim_{n\to\infty} a_n = L \Rightarrow$ conv

$\lim_{n\to\infty} \frac{a_n+6}{a_n+2}$

$\lim_{n\to\infty} \frac{L+6}{L+2} = L$

$L(L+2) = L+6$

$L^2+2L-L-6=0$

$L^2+L-6=0$

$(L-2)(L+3)=0$

$L=2$

or

$L=-3$

$a_n > 0 \quad \therefore L=2$

111] $a_n = \frac{3n+1}{n+1}$

$a_1 = \frac{4}{2} = 2$

$a_2 = \frac{7}{3}$

$a_3 = \frac{10}{4} = \frac{5}{2}$

$a_4 = \frac{13}{5}$

$M = 3, 4, 5$

$m = 1, 0, -1$

non decreasing

bounded from above

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