10.10

2] $(1+x)^{\frac{1}{3}} = 1 + \sum_{k=1}^{\infty} \binom{\frac{1}{3}}{k} x^k$

$= 1 + \frac{1}{3}x - \frac{1}{9}x^2 + \frac{5}{81}x^3 - \dots$

25] $f(x) = \int_0^x \sin t^2 \, dt \quad , [0,1] \qquad E < 10^{-3}$

$= \int x^2 - \frac{x^{3^2}}{3!} + \frac{x^{5^2}}{5!} - \frac{x^{7^2}}{7!} + \dots dx$

$= \int x^2 - \frac{x^6}{3!} + \frac{x^{10}}{5!} - \frac{x^{14}}{7!} + \dots dx$

$= \frac{x^3}{3!} - \frac{x^7}{7.3!} + \frac{x^{11}}{11.5!} - \frac{x^{15}}{15.7!} + \dots$

$\approx \frac{x^3}{3!} - \frac{x^7}{7.3!} + \frac{x^{11}}{11.5!}$

$|E| < \frac{1}{15.7!} \approx 0.000013$

Haytham Shehadeh

10] $\frac{x}{\sqrt[3]{1+x}}$

$= x(1+x)^{-\frac{1}{3}}$

$= x\left[1 + \sum_{k=1}^{\infty} \binom{-\frac{1}{3}}{k} x^k\right]$

$= x\left[1 + -\frac{1}{3}x + \frac{2}{9}x^2 - \frac{14}{81}x^3 + \dots\right]$

$= x - \frac{x^2}{3} + \frac{2x^3}{9} - \frac{14x^4}{81} + \dots$

16]
$$\int_0^{0.2} \frac{e^{-x}-1}{x}\,dx$$

$$\int_0^{0.2} \frac{1}{x}\left[1-x+\frac{x^2}{2!}-\frac{x^3}{3!}+\frac{x^4}{4!}-\cdots-1\right]dx$$

$$\int_0^{0.2}\left[-1+\frac{x}{2}-\frac{x^2}{6}+\frac{x^3}{24}-\cdots\right]dx$$

$$= -x+\frac{x^2}{4}-\frac{x^3}{18}+\frac{x^4}{96} \qquad |E| \leq \frac{(0.2)^4}{96} \approx 0.00002$$

$$\approx -x+\frac{x^2}{4}-\frac{x^3}{18}+\cdots\Big|_0^{0.2} \approx -0.19044$$

26]
$$f(x)=\int_0^x t^2 e^{-t^2}\,dt\ ,\ [0,1] \qquad E<10^{-3}$$

$$e^t=\sum_{n=0}^{\infty}\frac{t^n}{n!} \rightarrow e^{-t^2}=\sum_{n=0}^{\infty}\frac{(-t^2)^n}{n!}=\sum_{n=0}^{\infty}\frac{(-1)^n t^{2n}}{n!}$$

$$t^2e^{-t^2}=t^2\sum_{n=0}^{\infty}\frac{(-1)^n t^{2n}}{n!}=\sum_{n=0}^{\infty}\frac{(-1)^n t^{2n+2}}{n!}=t^2-t^4+\frac{t^6}{2!}-\frac{t^8}{3!}+\frac{t^{10}}{4!}-\frac{t^{12}}{5!}+\cdots$$

$$\int_0^x t^2e^{-t^2}\,dt$$

$$=\frac{x^3}{3}-\frac{x^5}{5}+\frac{x^7}{7.2!}-\frac{x^9}{9.3!}+\frac{x^{11}}{11.4!}-\frac{x^{13}}{13.5!}$$

$$E<\left|\frac{x^{13}}{13.5!}\right|=\frac{|x|^{13}}{13.5!}<\frac{1}{13.5!}=6.41\times10^{-4}<10^{-3}$$

$$f(x)=\frac{x^3}{3}-\frac{x^5}{5}+\frac{x^7}{7.2!}-\frac{x^9}{9.3!}+\frac{x^{11}}{11.4!}$$

30] $\lim_{x\to 0} \frac{e^x - e^{-x}}{x}$

$\lim_{x\to 0} \frac{1}{x}\left[1+x+\frac{x^2}{2!}+\frac{x^3}{3!}+\dots -1+x-\frac{x^2}{2!}+\frac{x^3}{3!}-\dots\right]$

$\lim_{x\to 0} \frac{1}{x}\left[2x+\frac{2x^3}{3!}+\dots\right]$

$\lim_{x\to 0} \left[2+\frac{2x^2}{3!}+\dots\right] = 2$

33] $\lim_{y\to 0} \frac{y-\tan^{-1}y}{y^3}$

$\lim_{y\to 0} \frac{1}{y^3}\left[y-\left[y-\frac{y^3}{3}+\frac{y^5}{5}-\frac{y^7}{7}+\dots\right]\right]$

$\lim_{y\to 0} \frac{1}{y^3}\left[y-y+\frac{y^3}{3}-\frac{y^5}{5}+\frac{y^7}{7}-\dots\right]$

$\lim_{y\to 0} \left[\frac{1}{3}-\frac{y^2}{5}+\frac{y^4}{7}-\dots\right] = \frac{1}{3}$