10.2

1

37] $\sum_{n=1}^{\infty} (\ln\sqrt{n+1} - \ln\sqrt{n})$

$S_n = \ln\sqrt{2} + \cancel{\ln\sqrt{3}} - \cancel{\ln\sqrt{2}} + \ln 2 - \cancel{\ln\sqrt{3}}$

$S_n = \ln\sqrt{n+1}$

$\lim_{n\to\infty} S_n = \infty$

Hence, $\sum_{n=1}^{\infty} (\ln\sqrt{n+1} - \ln\sqrt{n})$ div by n^{th} Partial Sum test

51] $\sum_{n=1}^{\infty} (-1)^{n+1} \frac{3}{2^n} = \frac{3}{2} - \frac{3}{4} + \frac{3}{8} - \frac{3}{16}$

$r = \frac{-\cancel{3}}{2\cancel{4}} \cdot \frac{\cancel{2}}{\cancel{3}} \Rightarrow r = \frac{-1}{2} \in (-1,1)$ conv

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$\text{Sum} = \frac{a}{1-r} = \frac{\cancel{3}}{2} \cdot \frac{\cancel{2}}{\cancel{3}} = 1$

Hence, $\sum_{n=1}^{\infty} (-1)^{n+1} \frac{3}{2^n}$ conv by Geometric Series

64] $\sum_{n=1}^{\infty} \frac{2^n + 4^n}{3^n + 4^n}$

$\lim_{n\to\infty} \frac{2^n + 4^n}{3^n + 4^n} = \lim_{n\to\infty} \frac{(\frac{1}{2})^n + 1}{(\frac{3}{4})^n + 1} = \frac{0+1}{0+1} = 1 \neq 0$ div by n^{th} term test

75] $\sum_{n=0}^{\infty} (-1)^n (x+1)^n$

$S_n = 1 - (x+1) + (x+1)^2 - (x+1)^3 + \dots$

$r = -x-1 \in (-1,1) \Rightarrow$ conv

$\text{Sum} = \frac{a}{1-r} = \frac{1}{1+x+1} = \frac{1}{2+x}$

$-1 < -x-1 < 1$
$+1 \quad\quad +1 \quad +1$

$0 < -x < 2$

$\boxed{0 > x > -2}$

6] $\frac{5}{1\cdot 2}+\frac{5}{2\cdot 3}+\frac{5}{3\cdot 4}+\cdots+\frac{5}{n(n+1)}+\cdots$

$5\sum \frac{\cancel{5}1}{n(n+1)}$ $\quad\quad \frac{\cancel{5}1}{n(n+1)}=\frac{A}{n}+\frac{B}{n+1}$

$5\sum\left(\frac{1}{n}-\frac{1}{n+1}\right)$ $\quad\quad =\cancel{5}\frac{1}{n}-\frac{1}{n+1}$

$S_n=5\left(1-\cancel{\tfrac{1}{2}}+\cancel{\tfrac{1}{2}}-\cancel{\tfrac{1}{3}}+\cancel{\tfrac{1}{3}}-\tfrac{1}{4}\right)$

$S_n=5\left(1-\frac{1}{n+1}\right)$

Hence, $\sum\frac{5}{n(n+1)}$ conv by n^{th} Partial Sum Test

$\lim_{n\to\infty} S_n = 1.5 = 5$

14] $\sum_{n=0}^{\infty}\left(\frac{2^{n+1}}{5^n}\right)=\sum_{n=0}^{\infty}2\cdot\left(\frac{2}{5}\right)^n = 2+\frac{4}{5}+\frac{8}{25}$

$r=\frac{2}{5}\in(-1,1)$ conv

$Sum=\frac{a}{1-r}=2\cdot\frac{5}{3}=\frac{10}{3}$

Hence, $\sum_{n=0}^{\infty}\left(\frac{2^{n+1}}{5^n}\right)$ conv by Geometric Series

18] $\left(\frac{-2}{3}\right)^2+\left(\frac{-2}{3}\right)^3+\left(\frac{-2}{3}\right)^4+\left(\frac{-2}{3}\right)^5+\left(\frac{-2}{3}\right)^6+\cdots$

$\sum_{n=1}^{\infty}\left(\frac{-2}{3}\right)^{n+1}$

$r=\frac{-2}{3}\in(-1,1)$ conv

$Sum=\left(\frac{-2}{3}\right)^2\cdot\frac{3}{5}=\frac{4}{\cancel{9}_3}\cdot\frac{\cancel{3}}{5}=\frac{4}{15}$

Hence, $\sum_{n=1}^{\infty}\left(\frac{-2}{3}\right)^{n+1}$ conv by Geometric Series

24] $1.\overline{414} = 1.414414414\ldots$

$$= 1 + \left(\frac{414}{1000} + \frac{414}{1000\,000} + \cdots\right)$$

$r = \frac{1}{1000} \in (-1,1)$ conv

$$\text{Sum} = 1 + \left(\frac{414}{\cancel{1000}} \cdot \frac{\cancel{1000}}{999}\right) = \frac{414}{999} + 1 = \frac{1413}{999}$$

32] $\sum_{n=0}^{\infty} \frac{e^n}{e^n + n}$

$\lim_{n\to\infty} \frac{e^n}{e^n + n} = \lim_{n\to\infty} \frac{e^n}{e^n + 1} = \lim_{n\to\infty} \frac{e^n}{e^n} = 1 \neq 0$ div by n^{th} term test

38] $\sum_{n=1}^{\infty} (\tan(n) - \tan(n-1))$

$S_n = \cancel{\tan 1} - \tan 0 + \cancel{\tan 2} - \cancel{\tan 1} + \tan 3 - \cancel{\tan 2} + \ldots$

$S_n = \tan(n)$

$\lim_{n\to\infty} S_n = \infty$ Hence, $\sum_{n=1}^{\infty} (\tan(n) - \tan(n-1))$ div by Partial Sum test

44] $\sum_{n=1}^{\infty} \frac{2n+1}{n^2(n+1)^2} = \sum_{n=1}^{\infty} \frac{1}{n^2} - \frac{1}{(n+1)^2}$

$S_n = 1 - \cancel{\frac{1}{4}} + \cancel{\frac{1}{4}} - \cancel{\frac{1}{9}} + \cancel{\frac{1}{9}} - \frac{1}{16} + \ldots$

$S_n = 1 - \frac{1}{(n+1)^2}$

$\lim_{n\to\infty} S_n = 1$ Hence, $\sum_{n=1}^{\infty} \frac{2n+1}{n^2(n+1)^2}$ conv by n^{th} Partial Sum test

54] $\sum_{n=0}^{\infty} \frac{\cos n\pi}{5^n}$

$\cos n\pi \Rightarrow (-1)^n$

$\sum_{n=0}^{\infty} (-1)^n \frac{1}{5^n} = 1 - \frac{1}{5} + \frac{1}{25} - \frac{1}{100}$

$r = -\frac{1}{5} \in (-1,1)$ conv

Sum $= 1 \cdot \frac{5}{6} = \frac{5}{6}$

Hence, $\sum_{n=0}^{\infty} \frac{\cos n\pi}{5^n}$ conv by Geometric Series

62] $\sum_{n=1}^{\infty} \frac{n^n}{n!}$

$n < \frac{n^n}{n!}$

$\sum_{n=1}^{\infty} n$

$\lim_{n\to\infty} n = \infty$

Hence. div

63] $\sum_{n=1}^{\infty} \frac{2^n + 3^n}{4^n}$

$= \sum_{n=1}^{\infty} \left(\frac{1}{2}\right)^n + \sum_{n=1}^{\infty} \left(\frac{3}{4}\right)^n$

$r = \frac{1}{2} \in (-1,1)$ conv

$r = \frac{3}{4} \in (-1,1)$ conv

Sum $= \frac{1}{2} \cdot \frac{2}{1} = 1$

Sum $= \frac{3}{4} \cdot \frac{4}{1} = 3$

$\sum_{n=1}^{\infty} \frac{2^n + 3^n}{4^n} = 1 + 3 = 4$

Hence. conv by Geometric Series

78] $\sum_{n=0}^{\infty} (\ln x)^n = 1 + \ln x + (\ln x)^2 + (\ln x)^3 + \cdots$

$r = \ln x \in (-1, 1) \rightarrow$ conv

$-1 < \ln x < 1$

$\text{Sum} = \frac{a}{1-r} = \frac{1}{1-\ln x}$

$e^{-1} < e^{\ln x} < e^{1}$

$\boxed{\frac{1}{e} < x < e}$

90] $1 + e^{b} + e^{2b} + e^{3b} + \cdots\cdots = 9$

$\sum_{n=0}^{\infty} e^{nb} = 9$

$\text{Sum} = \frac{1}{1-e^{b}} = 9$

$\frac{1}{9} = 1 - e^{b}$

$e^{b} = \frac{8}{9}$

$b = \ln\left(\frac{8}{9}\right)$