{ 10.3 }

$\frac{1}{}$

**8]** $\sum_{n=2}^{\infty} \frac{\ln(n^2)}{n}$ $\qquad f(x) = \frac{\ln x^2}{x}$

$= \frac{\ln 2^2}{2} + \sum_{n=3}^{\infty} \frac{\ln(n^2)}{n}$ $\qquad f'(x) = \frac{x \cdot \frac{2}{x} - \ln x^2}{x^2} = \frac{2 - \ln x^2}{x^2}$

$= \ln 2 + \sum_{n=3}^{\infty} \frac{\ln(n^2)}{n} = \ln 2 + \infty = \infty$

$2 = 2\ln x$

$\ln x = 1$

$e^{\ln x} = e^1 \Rightarrow \boxed{x = e}$

$2 \quad + + \quad e \quad - - \quad \infty$

$u = \ln n$

$du = \frac{1}{n} dn$ $\qquad \int 2u \, du = (\ln n)^2$

$\sum_{n=3}^{\infty} \frac{\ln(n^2)}{n} = \lim_{b \to \infty} \left[ 2\ln b^3 - 2\ln 3 \right] = \infty$

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Hence, $\sum_{n=2}^{\infty} \frac{\ln(n^2)}{n}$ div by Integral test

**26]** $\sum_{n=1}^{\infty} \frac{1}{\sqrt{n}(\sqrt{n}+1)}$ $\qquad f(x) = \frac{1}{\sqrt{x}(\sqrt{x}+1)}$

$\int_1^{\infty} \frac{1}{\sqrt{x}(\sqrt{x}+1)}$ $\qquad u = \sqrt{x}+1$, $\quad du = \frac{1}{2\sqrt{x}} dx$

Hence, $\sum_{n=1}^{\infty} \frac{1}{\sqrt{n}(\sqrt{n}+1)}$ div by IT

$\int \frac{1}{u} \cdot 2 \, du = 2\ln(\sqrt{x}+1)$

$2 \lim_{b \to \infty} \left[ \ln(\sqrt{b}+1) - \ln 2 \right] = \infty$

$\sum_{n=1}^{\infty} n \tan \frac{1}{n}$

34]

$\lim_{n\to\infty} \frac{\tan\frac{1}{n}}{\frac{1}{n}}$ $u = \frac{1}{n}$

$\lim_{u\to 0^+} \frac{\tan u}{u} = \lim_{u\to 0^+} \sec^2 u = 1 \neq 0$ div by $n^{th}$ term test

6] $\sum_{n=2}^{\infty} \frac{1}{n(\ln n)^2}$ $f(x) = \frac{1}{x(\ln x)^2}$

$f'(x) = \frac{-2(\ln x)^3}{(x(\ln x)^2)^2}$

$\int_2^{\infty} \frac{1}{x(\ln x)^2}$ $u = \ln x$

$du = \frac{1}{x}dx$

$\int \frac{du}{u^2} = \frac{-1}{u} = \frac{-1}{\ln x}$

$\lim_{b\to\infty} \frac{-1}{\ln b} + \frac{1}{\ln 2} = \frac{1}{\ln 2}$ conv by IT

13] $\sum_{n=1}^{\infty} \frac{n}{n+1}$ $\lim_{n\to\infty} \frac{n}{n+1} = 1 \neq 0$ div by $n^{th}$ term test

20] $\sum_{n=2}^{\infty} \frac{\ln n}{\sqrt{n}}$ $\int u e^{\frac{u}{2}} du$

$\int_2^{\infty} \frac{\ln x}{\sqrt{x}}$

$u = \ln x$

$du = \frac{1}{x}dx$ $= 2ue^{\frac{u}{2}} - 4e^{\frac{u}{2}}$

$e^{\ln x} = e^u$ $= 2\ln x\, e^{\frac{\ln x}{2}} - 4e^{\frac{\ln x}{2}}$

$x = e^u$

$x^{\frac{1}{2}} = e^{\frac{u}{2}}$

| | |
|---|---|
| $u$ | $e^{\frac{u}{2}}$ |
| $1$ (+) | $2e^{\frac{u}{2}}$ |
| $0$ (−) | $4e^{\frac{u}{2}}$ |

$\lim_{b\to\infty} \left[ [2be^{\frac{b}{2}} - 4e^{\frac{b}{2}}] - [2\ln 2\, e^{\frac{\ln 2}{2}} - 4e^{\frac{\ln 2}{2}}] \right] = \infty$ div by Integral test

22] $\sum_{n=1}^{\infty} \frac{5^n}{4^n+3}$

$\lim_{n\to\infty} \frac{5^n \ln 5}{4^n \ln 4} = \lim_{n\to\infty} \left(\frac{5}{4}\right)^n \frac{\ln 5}{\ln 4} = \infty \neq 0$ div by $n^{th}$ term test

28] $\sum_{n=1}^{\infty} \left(1+\frac{1}{n}\right)^n$

$\lim_{n\to\infty} \left(1+\frac{1}{n}\right)^n = e^1 \neq 0$ div by $n^{th}$ term test

32] $\sum_{n=1}^{\infty} \frac{1}{n(1+\ln^2 n)}$ $u = \ln x$ $du = \frac{1}{x}dx$ $f(x) = \frac{1}{x(1+\ln^2 x)}$

$\int \frac{1}{1+u^2} = \tan^{-1} u$

$\lim_{b\to\infty} \left[\tan^{-1} b - \tan^{-1} 0\right] = \frac{\pi}{2}$ conv by IT

38] $\sum_{n=1}^{\infty} \frac{n}{n^2+1}$ $u = x^2+1$ $du = 2x\,dx$ $f(x) = \frac{x}{x^2+1}$

$\lim_{b\to\infty}\left[\frac{1}{2}\ln(b^2+1) - \frac{1}{2}\ln 2\right]$ $\int \frac{\cancel{x}du}{2u} = \frac{1}{2}\ln(x^2+1)$

$= \infty$ div by IT

40] $\sum_{n=1}^{\infty} \text{sech}^2 n$

$\lim_{b\to\infty} \int_1^b \text{sech}^2 x\,dx = \lim_{b\to\infty} \tanh x \Big|_1^b = \lim_{b\to\infty}\left[\tanh b - \tanh 1\right] = 1 - \tanh 1$

conv by IT

42) $\sum_{n=3}^{\infty} \left( \frac{1}{n-1} - \frac{2a}{n+1} \right)$

$$\lim_{b\to\infty} \left[ \ln \left| \frac{(n-1)}{(n+1)^{2a}} \right| \right]_3^b = \lim_{b\to\infty} \left[ \ln \frac{b-1}{(b+1)^{2a}} - \ln \left( \frac{2}{4^{2a}} \right) \right]$$

conv $\to \ln 2$

$$\lim_{b\to\infty} \frac{b-1}{(b+1)^{2a}} = \lim_{b\to\infty} \frac{1}{2a\,(b+1)^{2a-1}} = \begin{cases} 1, & a = \frac{1}{2} \\ \infty, & a < \frac{1}{2} \end{cases}$$