10.4

10) $\displaystyle\sum_{n=1}^{\infty} \sqrt{\frac{n+1}{n^2+2}}$

$\displaystyle\sum_{n=1}^{\infty} \frac{1}{\sqrt{n}}$ div P-series

$$\lim_{n\to\infty} \frac{\sqrt{n+1}}{\sqrt{n^2+2}} \cdot \frac{n^{\frac{1}{2}}}{1}$$

$$\lim_{n\to\infty} \sqrt{\frac{n^2+n}{n^2+2}} = 1$$

Hence, $\displaystyle\sum_{n=1}^{\infty} \sqrt{\frac{n+1}{n^2+2}}$ div by LCT

22) $\displaystyle\sum_{n=1}^{\infty} \frac{n+1}{n^2\sqrt{n}}$

$\displaystyle\sum_{n=1}^{\infty} \frac{1}{n^{\frac{3}{2}}}$ conv $P>1$ P-series

$$\lim_{n\to\infty} \frac{n+1}{n^2\sqrt{n}} \cdot \frac{n^{\frac{3}{2}}}{1} = \lim_{n\to\infty} \frac{n^{\frac{5}{2}} + n^{\frac{3}{2}}}{n^{\frac{5}{2}}} = 1$$

Hence, $\displaystyle\sum_{n=1}^{\infty} \frac{n+1}{n^2\sqrt{n}}$ conv by LCT

Haytham Shehadeh

36) $\displaystyle\sum_{n=1}^{\infty} \frac{n+2^n}{n^2\, 2^n}$

$\displaystyle\sum_{n=1}^{\infty} \frac{1}{n^2}$ conv $P>1$ P-series

$$\lim_{n\to\infty} \frac{n+2^n}{n^2\, 2^n} \cdot \frac{n^2}{1} = \lim_{n\to\infty} \frac{n+2^n}{2^n} = \lim_{n\to\infty} \frac{1+2^n \ln 2}{2^n \ln 2} = \lim_{n\to\infty} \frac{2^n (\ln 2)^2}{2^n (\ln 2)^2} = 1$$

Hence, $\displaystyle\sum_{n=1}^{\infty} \frac{n+2^n}{n^2\, 2^n}$ conv by LCT

46) $\displaystyle\sum_{n=1}^{\infty} \tan\frac{1}{n}$

$u = \frac{1}{n}$

$\displaystyle\sum_{n=1}^{\infty} \frac{1}{n}$ Harmonic series div

$$\lim_{n\to\infty} \frac{\tan\frac{1}{n}}{\frac{1}{n}} = \lim_{u\to 0^+} \frac{\tan u}{u} = \lim_{u\to 0^+} \sec^2 u = 1$$

Hence, $\displaystyle\sum_{n=1}^{\infty} \tan\frac{1}{n}$ div by LCT

8] $\sum_{n=1}^{\infty} \frac{\sqrt{n}+1}{\sqrt{n^2+3}}$

$\sum_{n=1}^{\infty} \frac{1}{n^{\frac{1}{2}}}$ div $P<1$ P-series

$$\lim_{n\to\infty} \frac{\sqrt{n}+1}{\sqrt{n^2+3}} \cdot \frac{n^{\frac{1}{2}}}{1} = \lim_{n\to\infty} \frac{n+\sqrt{n}}{\sqrt{n^2+3}} = 1$$

Hence, $\sum_{n=1}^{\infty} \frac{\sqrt{n}+1}{\sqrt{n^2+3}}$ div by LCT

15] $\sum_{n=2}^{\infty} \frac{1}{\ln n}$

$\sum_{n=2}^{\infty} \frac{1}{n} \to$ div

$$\frac{1}{n} > \cancel{<} \frac{1}{\ln n}$$

$$\lim_{n\to\infty} \frac{1}{\ln n} \cdot \frac{n}{1} = \lim_{n\to\infty} \frac{n}{\ln n} = \lim_{n\to\infty} \frac{1}{\frac{1}{n}} = \lim_{n\to\infty} n = \infty$$

Hence, $\sum_{n=2}^{\infty} \frac{1}{\ln n}$ div by LCT

18] $\sum_{n=1}^{\infty} \frac{3}{n+\sqrt{n}}$

$$\frac{1}{n} < \frac{3}{n+\sqrt{n}}$$

$\sum_{n=1}^{\infty} \frac{1}{n}$ div

Hence, $\sum_{n=1}^{\infty} \frac{3}{n+\sqrt{n}}$ div by DCT

27] $\sum_{n=3}^{\infty} \frac{1}{\ln(\ln n)}$

$$\frac{1}{\ln(\ln n)} > \frac{1}{\ln n} > \frac{1}{n}$$

$\sum_{n=3}^{\infty} \frac{1}{n}$ div

Hence, $\sum_{n=3}^{\infty} \frac{1}{\ln(\ln n)}$

div by DCT

28] $\sum_{n=1}^{\infty} \frac{(\ln n)^n}{n^3}$

$\sum_{n=1}^{\infty} \frac{1}{n^2}$ conv

$$\lim_{n\to\infty} \frac{(\ln n)^n}{n^3} \cdot \frac{n^{\cancel{3}2}}{1} = \lim_{n\to\infty} \frac{(\ln n)^n}{n} = \lim_{n\to\infty} \frac{2\ln n\left(\frac{1}{n}\right)}{1}$$

$$= 2\lim_{n\to\infty} \frac{\ln n}{n} = 0$$

Hence, $\sum_{n=1}^{\infty} \frac{(\ln n)^n}{n^3}$ conv by LCT

32] $\sum_{n=2}^{\infty} \frac{\ln(n+1)}{n+1}$

$u = \ln(x+1)$

$du = \frac{1}{x+1}\,dx$

$\int u\,du = \frac{1}{2}u^2$

$$\int_2^{\infty} \frac{\ln(x+1)}{x+1} = \lim_{b\to\infty} \frac{1}{2}u^2 \Big|_{\ln 3}^{b}$$

$$= \lim_{b\to\infty} \frac{1}{2}\left(b^2 - (\ln 3)^2\right) = \infty$$

Hence, $\sum_{n=2}^{\infty} \frac{\ln(n+1)}{n+1}$ div by Integral test

40] $\sum_{n=1}^{\infty} \frac{2^n+3^n}{3^n+4^n}$

$\sum_{n=1}^{\infty} \left(\frac{3}{4}\right)^n$

$r = \frac{3}{4} \in (-1,1)$

Conv Geometric Series

$\lim_{n\to\infty} \frac{2^n+3^n}{3^n+4^n}$ ~~$= \lim_{n\to\infty} \frac{\left(\frac{2}{3}\right)^n+1}{\left(\frac{4}{3}\right)^n+1}$~~

$$\lim_{n\to\infty} \frac{\frac{2^n+3^n}{3^n+4^n}}{\left(\frac{3}{4}\right)^n} = \lim_{n\to\infty} \frac{8^n+12^n}{9^n+12^n} = \lim_{n\to\infty} \frac{1+\left(\frac{8}{12}\right)^n}{1+\left(\frac{9}{12}\right)^n} = 1$$

Hence, $\sum_{n=1}^{\infty} \frac{2^n+3^n}{3^n+4^n}$ conv by LCT

43] $\sum_{n=2}^{\infty} \frac{1}{n!}$

$\frac{1}{n!} \leq \frac{1}{n(n-1)}$ $\quad n \geq 2$

$\sum_{n=2}^{\infty} \left(\frac{1}{n-1} - \frac{1}{n}\right) \to$ conv by n^{th} Partial Sum test

$\frac{A}{n} + \frac{B}{n-1} = \frac{-1}{n} + \frac{1}{n-1}$

$S_n = 1 - \cancel{\frac{1}{2}} + \cancel{\frac{1}{2}} - \cancel{\frac{1}{3}} + \cancel{\frac{1}{3}} - \frac{1}{4} + \cdots$

$S_n = 1 - \frac{1}{n}$

Hence, $\sum_{n=2}^{\infty} \frac{1}{n!}$ conv by ~~[illegible]~~ DCT

$\lim_{n\to\infty} S_n = 1$

52] $\sum_{n=1}^{\infty} \frac{\sqrt[n]{n}}{n^2}$

$\sum_{n=1}^{\infty} \frac{1}{n^2}$ conv $P>1$ P-series

$\lim_{n\to\infty} \frac{\sqrt[n]{n}}{n^2} \cdot \frac{n^2}{1} = 1$

Hence, $\sum_{n=1}^{\infty} \frac{\sqrt[n]{n}}{n^2}$ conv by LCT