$|10.5\}$ 1

6] $\sum_{n=2}^{\infty} \frac{3^{n+2}}{\ln n}$

$$\lim_{n\to\infty} \frac{3^{n+3}}{\ln(n+1)} \cdot \frac{\ln n}{3^{n+2}} = 3\lim_{n\to\infty} \frac{\ln n}{\ln(n+1)} = 3\lim_{n\to\infty} \frac{1}{n}\cdot \frac{n+1}{} = 3>1 \quad \text{div by Ratio Test}$$

15] $\sum_{n=1}^{\infty} \left(1-\frac{1}{n}\right)^{n^2}$

$$\lim_{n\to\infty} \left(1-\frac{1}{n}\right)^{n^2\cdot\frac{1}{n}} = \lim_{n\to\infty} \left(1-\frac{1}{n}\right)^{n} = e^{-1} = \frac{1}{e} < 1 \quad \text{conv by Root test}$$

20] $\sum_{n=1}^{\infty} \frac{n!}{10^n}$

$$\lim_{n\to\infty} \frac{(n+1)!}{10^{n+1}} \cdot \frac{10^n}{n!} = \lim_{n\to\infty} \frac{(n+1)}{10} = \infty > 1 \quad \text{div by Ratio test}$$

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30] $\sum_{n=1}^{\infty} \left(\frac{1}{n}-\frac{1}{n^2}\right)^n$

$$\lim_{n\to\infty} \left(\frac{1}{n}-\frac{1}{n^2}\right)^{n\cdot\frac{1}{n}} = \lim_{n\to\infty} \frac{1}{n}-\frac{1}{n^2} = 0<1 \quad \text{conv by Root test}$$

7] $\sum_{n=1}^{\infty} \frac{n^2(n+2)!}{n!\,3^{2n}}$

$$\lim_{n\to\infty} \frac{(n+1)^2\,\cancel{(n+3)!}}{(n+1)!\,\cancel{3}^{2n+2}} \cdot \frac{n!\,\cancel{3}^{2n}}{n^2\,\cancel{(n+2)!}} = \lim_{n\to\infty} \frac{(n+1)^2(n+3)}{3^2(n+1)n^2} = \lim_{n\to\infty} \frac{(n^2+2n+1)(n+3)}{3^2n^3+3^2n^2}$$

$$= \lim_{n\to\infty} \frac{n^3+2n^2+n+3n^2+6n+3}{3^2n^3+3^2n^2} = \lim_{n\to\infty} \frac{n^3+5n^2+7n+3}{3^2n^3+3^2n^2} = \frac{1}{9} < 1 \quad \text{conv by Ratio test}$$

12] $\sum_{n=1}^{\infty} \left(\ln\left(e^2+\frac{1}{n}\right)\right)^{n+1}$

$$\ln \lim_{n\to\infty} \left(e^2+\frac{1}{n}\right)^{\frac{n+1}{n}} = \ln \lim_{n\to\infty} \left(e^2+\frac{1}{n}\right)^{1+\frac{1}{n}} = \ln e^2 = 2>1 \quad \text{div by Root test}$$

16] $\sum_{n=2}^{\infty} \frac{1}{n^{1+n}}$

$\lim_{n\to\infty} \frac{\sqrt[n]{1}}{\sqrt[n]{n^{1+n}}} = \lim_{n\to\infty} \frac{1^{\frac{1}{n}}}{n\, n^{\frac{1}{n}}} = 0 < 1$ conv by Root test

20] $\sum_{n=1}^{\infty} \frac{n!}{10^n}$

$\lim_{n\to\infty} \frac{(n+1)!}{10^{n+1}} \cdot \frac{10^n}{n!} = \lim_{n\to\infty} \frac{n+1}{10} = \infty > 1$ div by Ratio test

28] $\sum_{n=1}^{\infty} \frac{(\ln n)^n}{n^n}$

$\lim_{n\to\infty} \frac{a_{n+1}}{a_n} = \lim_{n\to\infty} \frac{\ln n}{n} = 0 < 1$ conv by Root test

38] $\sum_{n=1}^{\infty} \frac{n!}{n^n}$

$\lim_{n\to\infty} \frac{(n+1)!}{(n+1)^{n+1}} \cdot \frac{n^n}{n!} = \lim_{n\to\infty} \frac{\cancel{(n+1)}\, n^n}{(n+1)^n \cancel{(n+1)}} = \lim_{n\to\infty} \left(\frac{n}{n+1}\right)^n = \lim_{n\to\infty} \left(\frac{n+1 \; -1}{n+1}\right)^n$

$\lim_{n\to\infty} \left(1 - \frac{1}{n+1}\right)^n = \lim_{n\to\infty} \left(1 - \frac{1}{u}\right)^{u-1} = \lim_{n\to\infty} \left(1 - \frac{1}{u}\right)^u \left(1 - \frac{1}{u}\right)^{-1} = e^{-1} = \frac{1}{e} < 1$ $\quad u = n+1$

conv by Ratio Test

43] $\sum_{n=1}^{\infty} \frac{(n!)^2}{(2n)!}$

$\lim_{n\to\infty} \frac{((n+1)!)^2}{(2n+2)!} \cdot \frac{(2n)!}{(n!)^2} = \lim_{n\to\infty} \frac{(n+1)^2}{(2n+2)(2n+1)} = \lim_{n\to\infty} \frac{n^2 + 2n + 1}{4n^2 + 2n + 4n + 2} = \frac{1}{4} < 1$

conv by Ratio test

46] $a_1 = 1 \quad a_{n+1} = \frac{1 + \tan^{-1} n}{n} a_n$

$\lim_{n\to\infty} \frac{1 + \tan^{-1} n}{n} = 0 < 1$ conv by Ratio test

60] $\sum_{n=1}^{\infty} \frac{n^n}{(2^n)^2}$

$\lim_{n\to\infty} \frac{n}{4} = \infty > 1$

$\lim_{n\to\infty} \sqrt[n]{a_n}$ div by Root test