8) $\sum_{n=1}^{\infty} (-1)^n \frac{10^n}{(n+1)!}$ **|10.6|**

$\sum_{n=1}^{\infty} U_n = \sum_{n=1}^{\infty} \frac{10^n}{(n+1)!}$

$\lim_{n\to\infty} \frac{10^{n+1}}{(n+2)!} \cdot \frac{(n+1)!}{10^n} = 10 \lim_{n\to\infty} \frac{1}{n+2} = 0 < 1$

so $\sum_{n=1}^{\infty} U_n$ conv $\to \sum_{n=1}^{\infty} (-1)^n \frac{10^n}{(n+1)!}$ conv

20) $\sum_{n=1}^{\infty} (-1)^{n+1} \frac{n!}{2^n}$ $U_n = \frac{n!}{2^n}$ $\Rightarrow \lim_{n\to\infty} \frac{n!}{2^n} = \infty$

div by n^{th} term test

$\times \lim_{n\to\infty} \frac{U_{n+1}}{U_n} = \lim_{n\to\infty} \frac{(n+1)!}{2^{n+1}} \cdot \frac{2^n}{n!}$

$= \frac{1}{2} \lim_{n\to\infty} (n+1) = \infty > 1$ div by n^{th} term test

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13) $\sum_{n=1}^{\infty} (-1)^{n+1} \frac{\sqrt{n}+1}{n+1}$

$U_n = \frac{\sqrt{n}+1}{n+1} > 0$, $U_n > U_{n+1}$, $\lim_{n\to\infty} U_n = 0$

conv by AST

23) $\sum_{n=1}^{\infty} (-1)^{n+1} \frac{1+n}{n^2}$

$\sum_{n=1}^{\infty} \frac{1+n}{n^2} = \underbrace{\sum_{n=1}^{\infty} \frac{1}{n^2}}_{\text{conv}} + \underbrace{\sum_{n=1}^{\infty} \frac{1}{n}}_{\text{div}} \to \sum_{n=1}^{\infty} \frac{1+n}{n^2}$ div

$U_n = \frac{1+n}{n^2} > 0$, $U_n > U_{n+1}$, $\lim_{n\to\infty} U_n = 0$ conv AST

so $\sum_{n=1}^{\infty} (-1)^{n+1} \frac{1+n}{n^2}$ conv cond

38) $\sum_{n=1}^{\infty} (-1)^n \frac{\ln n}{n-\ln n}$

$\sum_{n=1}^{\infty} \frac{\ln n}{n-\ln n} = \sum_{n=1}^{\infty} \frac{\ln n}{n} - \sum_{n=1}^{\infty} 1 \to \text{div}$

$n-\ln n < n$

$\frac{1}{n-\ln n} > \frac{1}{n}$

$\sum_{n=1}^{\infty} \frac{1}{n}$ div

$U_n = \frac{\ln n}{n-\ln n} > 0$

$f(x) = \frac{\ln x}{x-\ln x}$

$f'(x) = \frac{\frac{x-\ln x}{x} - \ln x\left[1-\frac{1}{x}\right]}{(x-\ln x)^2}$

$f'(x) = \frac{1 - \frac{1}{x}\ln x - \ln x + \frac{1}{x}\ln x}{x(x-\ln x)^2} = \frac{(1-\ln x)}{x(x-\ln x)^2} < 0$

$\lim_{n\to\infty} \frac{\ln n}{n-\ln n} = \lim_{n\to\infty} \frac{\frac{1}{n}}{1-\frac{1}{n}} = 0$

$\sum_{n=1}^{\infty} (-1)^n \frac{\ln n}{n-\ln n}$ conv by AST

39) $\sum_{n=1}^{\infty} (-1)^n \frac{(2n)!}{2^n n! n}$

$U_n = \frac{(2n)!}{2^n n! n}$

$\lim \frac{(2n+2)!}{2^{n+1}(n+1)!(n+1)} \cdot \frac{2^n n! n}{(2n)!} = \lim_{n\to\infty} \frac{(2n+2)(2n+1)n}{2(n+1)(n+1)} = \infty$ div by n^{th} term test

42) $\sum_{n=1}^{\infty} (-1)^n (\sqrt{n^2+n} - n)$

$\lim_{n\to\infty} \sqrt{n^2+n} - n \cdot \frac{\sqrt{n^2+n}+n}{\sqrt{n^2+n}+n} = \lim_{n\to\infty} \frac{n^2+n-n^2}{\sqrt{n^2+n}+n} = \lim_{n\to\infty} \frac{n}{\sqrt{n^2\left[1+\frac{1}{n}\right]}+1} = \lim_{n\to\infty} \frac{\cancel{n}}{\cancel{n}\sqrt{1+\frac{1}{n}}+1}$

$= \lim_{n\to\infty} \frac{1}{\sqrt{1+\frac{1}{n}}+1} = \frac{1}{2} \neq 0$

div by n^{th} term test

14] $\sum_{n=1}^{\infty} (-1)^{n+1} \frac{3\sqrt{n+1}}{\sqrt{n}+1}$

$U_n = \frac{3\sqrt{n+1}}{\sqrt{n}+1} > 0$

$\lim_{n\to\infty} U_n = 3 \neq 0$ div by n^{th} term test

19] $\sum_{n=1}^{\infty} (-1)^{n+1} \frac{n}{n^3+1}$

$\sum_{n=1}^{\infty} \frac{n}{n^3+1}$ conv $\qquad \frac{n}{n^3+1} < \frac{1}{n^2}$

so $\sum_{n=1}^{\infty} (-1)^{n+1} \frac{n}{n^3+1}$ conv Abs

29] $\sum_{n=1}^{\infty} (-1)^{n} \frac{\tan^{-1} n}{n^2+1}$

$\sum_{n=1}^{\infty} \frac{\tan^{-1} n}{n^2+1}$ $\qquad u = \tan^{-1} x$, $du = \frac{dx}{1+x^2}$ $\qquad \lim_{b\to\infty} \int u\,du = \lim_{b\to\infty} \left.\frac{(\tan^{-1} x)^2}{2}\right|_1^b = \lim_{b\to\infty} \left[\frac{(\tan^{-1} b)^2}{2} - \frac{(\tan^{-1} 1)^2}{2} \right]$

$= \frac{3\pi^2}{32}$

so $\sum_{n=1}^{\infty} (-1)^{n} \frac{\tan^{-1} n}{n^2+1}$ conv Abs

50) $\sum_{n=1}^{\infty} (-1)^{n+1} \frac{1}{10^n}$

$|E| < U_{n+1} = |a_{n+1}|$

$|E| < a_5 = \left| (-1)^{5+1} \frac{1}{10^5} \right|$

$|E| < 0.00001$

54) $\sum_{n=1}^{\infty} (-1)^{n+1} \frac{n}{n^2+1}$

$|E| < 0.001$

$\frac{n+1}{(n+1)^2+1} < 0.001$

$(n+1)1000 < (n+1)^2+1$

$1000n + 1000 < n^2+2n+2$

$0 < n^2 - 998n - 998$

$n > \frac{998 \mp \sqrt{(-998)^2 - 4(1)(-998)}}{2(1)} \approx 998.99899$

$n \geq 999$