|10.7|

4] $\sum_{n=1}^{\infty} \frac{(3x-2)^n}{n}$

$\lim_{n\to\infty} \frac{(3x-2)^{n+1}}{n+1} \cdot \frac{n}{(3x-2)^n} = \lim_{n\to\infty} \frac{|3x-2|\,n}{n+1} = |3x-2| < 1$

$-1 < 3x-2 < 1$

$1 < 3x < 3$

$\frac{1}{3} < x < 1 \to$ conv Abs

$\frac{1}{3} \le x < 1 \to$ conv

$x = \frac{1}{3} \to \sum_{n=1}^{\infty} (-1)^n \frac{1}{n} \to$ conv AST

$x = 1 \to \sum_{n=1}^{\infty} \frac{1}{n} \to$ div P-series

at $x = \frac{1}{3} \leftarrow$ conv cond


$R = \frac{1}{3}$

12] $\sum_{n=0}^{\infty} \frac{3^n x^n}{n!}$

$\lim_{n\to\infty} \frac{3^{n+1} x^{n+1}}{(n+1)!} \cdot \frac{n!}{3^n x^n} = \lim_{n\to\infty} \frac{3|x|}{n+1} = 0 < 1$ for all x

$R = \infty$

conv Abs for all x

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14] $\sum_{n=1}^{\infty} \frac{(x-1)^n}{3^n n^2}$

$\lim_{n\to\infty} \frac{(x-1)^{n+1}}{3^{n+1}(n+1)^2} \cdot \frac{3^n n^2}{(x-1)^n} = \lim_{n\to\infty} \frac{|x-1|\, n^2}{3\,(n+1)^2} = \frac{1}{3}|x-1| < 1$

$-1 < \frac{1}{3}(x-1) < 1$

$-3 < (x-1) < 3$

$-2 \le x \le 4 \to$ conv Abs

$R = 3$

$x = -2 \to \sum_{n=1}^{\infty} \frac{(-3)^n}{3^n n^2} = \sum_{n=1}^{\infty} (-1)^n \frac{1}{n^2} \to$ conv AST + Abs

$x = 4 \to \sum_{n=1}^{\infty} \frac{1}{n^2} \to$ conv

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$$\sum_{n=1}^{\infty} \left(1+\frac{1}{n}\right)^n x^n$$

23]

$$\lim_{n\to\infty} \frac{\left(1+\frac{1}{n+1}\right)^{n+1} x^{n+1}}{\left(1+\frac{1}{n}\right)^n x^n} = |x| \lim_{n\to\infty} \frac{\left(1+\frac{1}{n+1}\right)^{n+1}}{\left(1+\frac{1}{n}\right)^n} = \frac{|x|\, e}{e} = |x| < 1$$

$-1 < x < 1$ Abs conv

$R = 1$

$x=-1 \to \sum_{n=1}^{\infty} (-1)^n \left(1+\frac{1}{n}\right)^n$

$\lim_{n\to\infty} \left(1+\frac{1}{n}\right)^n = e$ div by $n^{th}$ term test

$x=1 \to \sum_{n=1}^{\infty} \left(1+\frac{1}{n}\right)^n \to$ div by $n^{th}$ term test

29]

$$\sum_{n=2}^{\infty} \frac{x^n}{n(\ln n)^2}$$

$$\lim_{n\to\infty} \frac{x^{n+1}}{(n+1)(\ln(n+1))^2} \cdot \frac{n(\ln n)^2}{x^n} = |x| \lim_{n\to\infty} \frac{n}{n+1} \cdot \frac{(\ln n)^2}{(\ln(n+1))^2} = |x| < 1$$

$-1 \leq x \leq 1 \to$ conv Abs

$R = 1$

$x=-1 \to \sum_{n=2}^{\infty} (-1)^n \frac{1}{n(\ln n)^2} \to$ conv

$$\sum_{n=2}^{\infty} \frac{1}{n(\ln n)^2}$$

$$\int_2^{\infty} \frac{1}{x(\ln x)^2} \qquad u = \ln x \quad du = \frac{dx}{x}$$

$$\lim_{b\to\infty} \left.\frac{-1}{\ln x}\right|_2^b = \lim_{b\to\infty} \left[\frac{-1}{\ln b} + \frac{1}{\ln 2}\right] = \frac{1}{\ln 2} \text{ conv}$$

$x=1 \to \sum_{n=2}^{\infty} \frac{1}{n(\ln n)^2} \to$ conv

32] $\sum_{n=1}^{\infty} \frac{(3x+1)^{n+1}}{2n+2}$

|10.7|

$$\lim_{n\to\infty} \frac{(3x+1)^{n+2}}{2n+4} \cdot \frac{2n+2}{(3x+1)^{n+1}} = |3x+1| \lim_{n\to\infty} \frac{2n+2}{2n+4} = |3x+1| < 1$$

$-1 < 3x+1 < 1$

$-2 < 3x < 0$

$-\frac{2}{3} < x < 0 \to$ conv Abs

$R = \frac{1}{3}$

$x = -\frac{2}{3} \to \sum_{n=1}^{\infty} (-1)^{n+1} \frac{1}{2n+2} \to$ conv AST

$\sum_{n=1}^{\infty} \frac{1}{2n+2}$ $\sum_{n=1}^{\infty} \frac{1}{n}$

$\lim_{n\to\infty} \frac{1}{2n+2} \cdot n = \frac{1}{2} > 0$ div

conv cond at $x = -\frac{2}{3}$

Interval of conv $-\frac{2}{3} \leq x < 0$

$x = 0 \to \sum_{n=1}^{\infty} \frac{1}{2n+2} \to$ div

40] $\sum_{n=1}^{\infty} \left(\frac{n}{n+1}\right)^{n^2} x^n$

$$\lim_{n\to\infty} \sqrt[n]{\frac{n}{n+1}} |x| = \frac{|x|}{e} < 1$$

$-1 < \frac{x}{e} < 1$

$-e < x < e$

$R = \frac{e - -e}{2} \Rightarrow R = e$

$\sum_{n=0}^{\infty} (\ln x)^n$

46] $\lim_{n\to\infty} \frac{(\ln x)^{n+1}}{(\ln x)^n}$

$-1 < \ln x < 1$

$\lim_{n\to\infty} |\ln x| = |\ln x| < 1$

$e^{-1} < x < e \to$ conv Abs

Sum $= \frac{1}{1 - \ln x}$

$x = e^{-1} \to \sum_{n=0}^{\infty} (\ln e^{-1})^n = \sum_{n=0}^{\infty} (-1)^n$ div

$x = e \to \sum_{n=0}^{\infty} (\ln e)^n = \sum_{n=0}^{\infty} 1^n$ div

49] $\sum_{n=0}^{\infty} \left(\frac{-1}{2}\right)^n (x-3)^n$

$\lim_{n\to\infty} \frac{\left(\frac{-1}{2}\right)^{n+1}(x-3)^{n+1}}{\left(\frac{-1}{2}\right)^n (x-3)^n} = \lim_{n\to\infty} \frac{1}{2}|x-3| = \frac{1}{2}|x-3| < 1$

$-1 < \frac{1}{2}(x-3) < 1$

$-2 < x-3 < 2$

$1 < x < 5 \rightarrow$ conv Abs

$R = 2$

$x=1 \rightarrow \sum_{n=0}^{\infty} \left(\frac{-1}{2}\right)^n (-2)^n = \sum_{n=0}^{\infty} 1^n$ div

$x=5 \rightarrow \sum_{n=0}^{\infty} \left(\frac{-1}{2}\right)^n 2^n = \sum_{n=0}^{\infty} (-1)^n$ div

Sum $= \frac{1}{1+\frac{x-3}{2}} = \frac{2}{x-1}$

11] $\sum_{n=0}^{\infty} \frac{(-1)^n x^n}{n!}$

$\lim_{n\to\infty} \frac{x^{n+1}}{(n+1)!} \cdot \frac{n!}{x^n} = \lim_{n\to\infty} \frac{|x|}{n+1} = 0 < 1$ for all x conv Abs $R = \infty$

24] $\sum_{n=1}^{\infty} (\ln n) x^n$

$\lim_{n\to\infty} \frac{\ln(n+1)\, x^{n+1}}{\ln(n)\, x^n} = |x| \lim_{n\to\infty} \frac{\ln(n+1)}{\ln n} = |x| < 1$

$-1 < x < 1 \rightarrow$ conv Abs

$R = 1$

$x=-1 \rightarrow \sum_{n=1}^{\infty} (\ln n)(-1)^n \rightarrow$ div by $n^{th}$ term test

$x=1 \rightarrow \sum_{n=1}^{\infty} (\ln n) \rightarrow$ div by $n^{th}$ term test

41] $\sum_{n=0}^{\infty} 3^n x^n$

$\lim_{n\to\infty} \frac{3^{n+1} x^{n+1}}{3^n x^n} = |x| \lim_{n\to\infty} 3 = 3|x| < 1$

$-1 < 3x < 1$

$-\frac{1}{3} < x < \frac{1}{3} \rightarrow$ conv Abs

$R = \frac{\frac{1}{3} - -\frac{1}{3}}{2} \Rightarrow R = \frac{1}{3}$

$x=-\frac{1}{3} \rightarrow \sum_{n=0}^{\infty} \left(\frac{-1}{3}\right)^n 3^n = \sum_{n=0}^{\infty} (-1)^n$ div

$x=\frac{1}{3} \rightarrow \sum_{n=0}^{\infty} \left(\frac{1}{3}\right)^n 3^n = \sum_{n=0}^{\infty} 1^n$ div

$\sum_{n=0}^{\infty} (3x)^n$

Sum $= \frac{1}{1-3x}$ $\quad -\frac{1}{3} < x < \frac{1}{3}$