19) $\cosh x = \frac{e^{x}+e^{-x}}{2} = \frac{1}{2}\left[1+x+\frac{x^2}{2!}+\frac{x^3}{3!}+\cdots\cdots+1-x+\frac{x^2}{2!}-\frac{x^3}{3!}+\cdots\right]$ **10.8**

$$= 1+\frac{x^2}{2!}+\frac{x^4}{4!}+\frac{x^6}{6!}+\cdots$$

$$\sum_{n=0}^{\infty} \frac{x^{2n}}{(2n)!}$$

30) $f(x) = 2^x$, $a=1$

$f'(x) = 2^x \ln 2$

$f''(x) = 2^x (\ln 2)^2$

$f'''(x) = 2^x (\ln 2)^3$

$$\sum_{n=0}^{\infty} \frac{f^{n}(1)(x-1)^n}{n!} = f(1)+f'(1)(x-1)+\frac{f''(1)}{2!}(x-1)^2+\frac{f'''(1)}{3!}(x-1)^3+\cdots$$

$$= 2+2\ln 2\,(x-1)+\frac{2(\ln 2)^2(x-1)^2}{2!}+\frac{2(\ln 2)^3(x-1)^3}{3!}+\cdots$$

$$= \sum_{n=0}^{\infty} \frac{2(\ln 2)^n(x-1)^n}{n!}$$

3) $f(x) = \ln x$, $a=1$

$$\sum_{n=0}^{\infty} \frac{f^{n}(1)(x-1)^n}{n!} = 0+(x-1)-\frac{(x-1)^2}{2!}+\frac{2(x-1)^3}{3!}$$

$f'(x) = \frac{1}{x}$

$f''(x) = \frac{-1}{x^2}$

$f'''(x) = \frac{2}{x^3}$

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$P_0(x) = 0$

$P_1(x) = (x-1)$

$P_2(x) = (x-1)-\frac{1}{2}(x-1)^2$

$P_3(x) = (x-1)-\frac{1}{2}(x-1)^2+\frac{1}{3}(x-1)^3$

14) $f(x) = \frac{2+x}{1-x}$

$f'(x) = \frac{(1-x)-(2+x)\cdot -1}{(1-x)^2} = \frac{1-x+2+x}{(1-x)^2} = \frac{3}{(1-x)^2}$

$f''(x) = \frac{6}{(1-x)^3}$

$f'''(x) = \frac{18}{(1-x)^7}$

$$\sum_{n=0}^{\infty} \frac{f^{n}(0)}{n!}x^n = f(0)+f'(0)x+\frac{f''(0)}{2!}x^2+\frac{f'''(0)}{3!}x^3+\cdots$$

$$= 2+3x+\frac{6}{2!}x^2+\frac{18}{3!}x^3+\cdots$$

$$= 2+\sum_{n=1}^{\infty} 3x^n$$

$\sinh x = \frac{e^{x} - e^{-x}}{2}$

20]

$$= \frac{1}{2}\left[1+x+\frac{x^2}{2!}+\frac{x^3}{3!}+\frac{x^4}{4!}+\dots 1-x+\frac{x^2}{2!}-\frac{x^3}{3!}+\frac{x^4}{4!}-\dots\right]$$

$$= x + \frac{x^3}{3!} + \frac{x^5}{5!} + \frac{x^7}{7!} + \dots$$

$$= \sum_{n=0}^{\infty} \frac{x^{2n+1}}{(2n+1)!}$$

22] $f(x) = \frac{x^2}{x+1}$

$$f'(x) = \frac{2x(x+1) - x^2}{(x+1)^2} = \frac{2x^2+2x-x^2}{(x+1)^2} = \frac{x^2+2x}{(x+1)^2}$$

$$f''(x) = \frac{(2x+2)(x^2+2x+1) - (2x+2)(x^2+2x)}{(x+1)^4} = \frac{2}{(x+1)^3}$$

$$f'''(x) = \frac{-2 \cdot 3(x+1)^2}{(x+1)^6} = \frac{-6}{(x+1)^4}$$

$$\sum_{n=0}^{\infty} \frac{f^{n}(0)}{n!}x^n = f(0) + f'(0)x + \frac{f''(0)}{2!}x^2 + \frac{f'''(0)}{3!}x^3 + \dots$$

$$= 0 + 0 + \frac{2}{2!}x^2 - \frac{6}{3!}x^3 + \dots$$

$$= \sum_{n=2}^{\infty} (-1)^n x^n$$

27] $f(x) = \frac{1}{x^2}$, $a = 1$

$$f'(x) = \frac{-2}{x^3}$$

$$f''(x) = \frac{6}{x^4}$$

$$f'''(x) = \frac{-24}{x^5}$$

$$\sum_{n=0}^{\infty} \frac{f^{n}(1)}{n!}(x-1)^n = f(1) + f'(1)(x-1) + \frac{f''(1)}{2!}(x-1)^2 + \frac{f'''(1)}{3!}(x-1)^3 + \dots$$

$$= 1 - 2(x-1) + \frac{6}{2!}(x-1)^2 - \frac{24}{3!}(x-1)^3 + \dots$$

$$= \sum_{n=0}^{\infty} (-1)^n (x-1)^n (n+1)$$

32] $f(x) = \sqrt{x+1}$, $a = 0$

$f'(x) = \frac{1}{2\sqrt{x+1}}$

$f''(x) = \frac{-1}{4(x+1)^{\frac{3}{2}}}$

$f'''(x) = \frac{3}{8(x+1)^{\frac{5}{2}}}$

$$\sum_{n=0}^{\infty} \frac{f^{n}(0)}{n!} x^n = f(0) + f'(0)x + \frac{f''(0)}{2!}x^2 + \frac{f'''(0)}{3!}x^3 + \cdots$$

$$= 1 + \frac{1}{2}x - \frac{1}{4\,2!}x^2 + \frac{3}{8\,3!}x^3 + \cdots$$

$$= \cancel{\sum_{n=0}^{\infty} (-1)^n \left(\frac{1}{2}\right)^n x^n} \quad X$$

37] $f(x) = e^x$ $x = a$

$f'(x) = e^x$

$f''(x) = e^x$

$f'''(x) = e^x$

$\vdots$

$f^{n}(x) = e^x$

$$\sum_{n=0}^{\infty} \frac{f^{n}(a)}{n!}(x-a)^n = \cancel{f(a) + f'(a)x + \frac{f''(a)}{2!}x^2 + f'''(a)x}$$

$$= f(a) + f'(a)(x-a) + \frac{f''(a)}{2!}(x-a)^2 + \frac{f'''(a)}{3!}(x-a)^3 + \frac{f''''(a)}{4!}(x-a)^4 + \cdots$$

$$= e^a + e^a(x-a) + \frac{e^a}{2!}(x-a)^2 + \frac{e^a}{3!}(x-a)^3 + \frac{e^a}{4!}(x-a)^4$$

$$= e^a\left[1 + (x-a) + \frac{(x-a)^2}{2!} + \cdots\cdots\right].$$