|10.9|

22) $f(x) = \frac{2}{(1-x)^3}$

$\sum_{n=0}^{\infty} \frac{f^{n}(0)x^n}{n!} = 2 + 6x + \frac{24}{2!}x^2 + \frac{120}{3!}x^3 + \dots$

$= 2 + 6x + 12x^2 + 20x^3 + \dots$

$= \sum_{n=0}^{\infty} (n+2)(n+1)x^n$

$f'(x) = 6(1-x)^{-4}$

$f''(x) = 24(1-x)^{-5}$

$f'''(x) = 120(1-x)^{-6}$

$\frac{2}{(1-x)^3} = \frac{d^2}{dx^2}\left[\frac{1}{1-x}\right] = \frac{d^2}{dx^2}\sum_{n=0}^{\infty} x^n$

$= \sum_{n=2}^{\infty} n(n-1)x^{n-2}$

40) $\sqrt{1+x} = 1 + \frac{x}{2}$ $\quad |x| < 0.01$

$\sqrt{1+x} = 1 + \frac{x}{2} - \frac{x^2}{8} + \frac{x^3}{16} - \dots$

$|e| < \left|\frac{-x^2}{8}\right| < \frac{(0.01)^2}{8} = 1.25 \times 10^{-5}$

10) $\frac{1}{2-x}$

$\frac{1}{1-x} = \sum_{n=0}^{\infty} x^n$

$\frac{1}{2}\left[\frac{1}{1-\frac{x}{2}}\right] = \frac{1}{2}\sum_{n=0}^{\infty}\left(\frac{1}{2}x\right)^n$

$= \sum_{n=0}^{\infty} \frac{x^n}{2^{n+1}}$

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12) $x^2 \sin x$

$\sin x = \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n+1}}{(2n+1)!}$

$x^2 \sin x = x^2 \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n+1}}{(2n+1)!}$

$= \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n+3}}{(2n+1)!}$

18] $\sin^2 x$

$$\sin^2 x = \frac{1}{2}\left[1 - \cos 2x\right]$$

$$= \frac{1}{2} - \frac{1}{2}\cos 2x = \frac{1}{2} - \frac{1}{2}\left[1 - \frac{(2x)^2}{2!} + \frac{(2x)^4}{4!} - \frac{(2x)^6}{6!} + \dots\right]$$

$$= \frac{(2x)^2}{2\,2!} - \frac{(2x)^4}{2\,4!} + \frac{(2x)^6}{2\,6!} - \dots$$

$$= \sum_{n=1}^{\infty} \frac{(-1)^{n+1}(2x)^{2n}}{2.(2n)!}$$

28] $\ln(1+x) - \ln(1-x)$

$$\frac{1}{1-x} = \sum_{n=0}^{\infty} x^n = 1+x+x^2+x^3+\dots$$

$$\frac{1}{1+x} = \sum_{n=0}^{\infty} (-1)^n x^n = 1-x+x^2-x^3+\dots$$

$$-\ln(1-x) = x + \frac{x^2}{2} + \frac{x^3}{3} + \frac{x^4}{4} + \dots$$

$$\int \frac{1}{1+x} - \int \frac{1}{1-x}$$

$$= \int 1+x+x^2+x^3+\dots - \int 1-x+x^2\cancel{+}-x^3+\dots$$

$$= \cancel{x} + \cancel{\frac{x^2}{2}} + \cancel{\frac{x^3}{3}} + \cancel{\frac{x^4}{4}} + \dots - \cancel{x} + \frac{x^2}{2} - \cancel{\frac{x^3}{3}} + \cancel{\frac{x^4}{4}} - \dots$$

$$= x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \dots + x + \frac{x^2}{2} + \frac{x^3}{3} + \frac{x^4}{4}$$

$$= 2x + \frac{2}{3}x^3 + \frac{2}{5}x^5 + \dots$$

$$= \sum_{n=0}^{\infty} \frac{2x^{2n+1}}{(2n+1)}$$

35] $P_3(x) = x - \frac{x^3}{6}$

$\sin x$ at $x = 0.1$

$f(x) = \sin x = P_3(x) + R_3(x)$

$f(x) = \sin x = x - \frac{x^3}{3!} + R_3(x)$

$|E| < \frac{(0.1)^4}{4!}$

$\approx 4.167 \times 10^{-6}$

$\text{Error} = |R_3(x)|$

$R_3(x) = \frac{f^{4}(c)}{4!} x^4$

37] $\sin x = x - \frac{x^3}{6}$

$E < 5 \times 10^{-4}$

$\sin x = \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n+1}}{(2n+1)!}$

$\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!}$

$\frac{|x^5|}{5!} < 5 \times 10^{-4}$

$|x| < \sqrt[5]{0.06} \approx 0.5698$

$\sin x = P(x) + R(x)$

41] $e^x = 1 + x + \frac{x^2}{2}$

$|x| < 0.1$

$f(x) = P_2(x) + R_2(x)$

$|R_2(x)| < \frac{e^c |x|^3}{3!}$

$f^{n+1}(x) = f^{3}(x)$

$< \frac{3^{0.1} (0.1)^3}{3!} = 1.87 \times 10^{-4}$

$f^3(x) < e^x \leq e^{0.1} < 3^{0.1}$