$\boxed{11.2}$

9) $x = 2t^2 + 3 \quad y = t^4 \ , \ t = -1$

$t = -1 \rightarrow x = 5$

$y = 1$

$\left.\frac{d^2y}{dx^2}\right|_{t=-1} \ ?$

$$\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}} = \frac{4t^3}{4t} = t^2 = 1$$

$y - 1 = x - 5$

$y = x - 4$

$$\frac{d^2y}{dx^2} = \frac{\frac{dy'}{dt}}{\frac{dx}{dt}} = \frac{2t}{4t} = \frac{1}{2}$$

15) $x^3 + 2t^2 = 9 \ , \ 2y^3 - 3t^2 = 4 \ , \ t = 2$

$3x^2x' + 4t = 0 \quad | \quad 6y^2y' - 6t = 0$

$x' = \frac{-4t}{3x^2} \quad | \quad y' = \frac{t}{y^2}$

$t = 2 \rightarrow x^3 + 8 = 9$

$x = 1$

$t = 2 \rightarrow 2y^3 - 12 = 4$

$y = 2$

$$\left.\frac{dy}{dx}\right|_{t=2} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}} = \frac{\frac{t}{y^2}}{\frac{-4t}{3x^2}} = \frac{\cancel{2}^{1}}{\cancel{2}4} \cdot \frac{3}{-8} = \frac{-3}{16}$$

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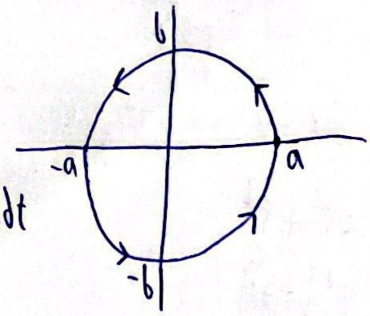
23) $x = a\cos t \quad y = b\sin t \quad 0 \leq t \leq 2\pi$

$\cos t = \frac{x}{a} \quad \sin t = \frac{y}{b}$

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$



$dx = -a\sin t\,dt$

$t_0 = 0 \rightarrow (a, 0)$

$t_1 = 2\pi \rightarrow (a, 0)$

$$A = 2\int_0^{\pi} y\,dx = 2\left|\int_0^{\pi} -ab\sin^2 t\,dt\right| = 2ab\int_0^{\pi}\left(\frac{1-\cos 2t}{2}\right)dt = ab\int_0^{\pi}(1-\cos 2t)\,dt$$

$$= ab\left[t - \frac{\sin 2t}{2}\right]\Big|_0^{\pi} = \pi ab$$

$x = \sec t$, $y = \tan t$, $t = \frac{\pi}{6}$

7)

$$\left.\frac{dy}{dx}\right|_{t=\frac{\pi}{6}} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}} = \frac{\sec^2 t}{\sec t \tan t} = \frac{\sec t}{\tan t} = \csc t = \csc\frac{\pi}{6} = \frac{1}{\sin\frac{\pi}{6}} = 2$$

$$\left.\frac{d^2y}{dx^2}\right|_{t=\frac{\pi}{6}} = \frac{\frac{dy'}{dt}}{\frac{dx}{dt}} = \frac{-\csc t \cot t}{\sec t \tan t} = -\cot^3 t \Big|_{\frac{\pi}{6}} = -3\sqrt{3}$$

$t = \frac{\pi}{6} \to x = \frac{2}{\sqrt{3}}$

$y = \frac{1}{\sqrt{3}}$

$y - \frac{1}{\sqrt{3}} = 2\left(x - \frac{2}{\sqrt{3}}\right)$

$y = 2x - \frac{4}{\sqrt{3}} + \frac{1}{\sqrt{3}}$

$y = 2x - \frac{3}{\sqrt{3}}$

$y = 2x - \sqrt{3}$

14) $x = t + e^t$, $y = 1 - e^t$, $t = 0$

$$\left.\frac{dy}{dx}\right|_{t=0} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}} = \frac{-e^t}{1+e^t} = \frac{-1}{2}$$

$$\left.\frac{d^2y}{dx^2}\right|_{t=0} = \frac{\frac{dy'}{dt}}{\frac{dx}{dt}} = \frac{-e^t}{(1+e^t)^2} \cdot \frac{1}{(1+e^t)} = \frac{-e^t}{(1+e^t)^3}$$

$$= \frac{-1}{8}$$

$t = 0 \to x = 1$

$y = 0$

$y = \frac{-1}{2}(x-1)$

$y = \frac{-x}{2} + \frac{1}{2}$

20) $t = \ln(x-t)$, $y = te^t$, $t = 0$

$1 = \frac{x' - 1}{(x-t)}$

$x - t = x' - 1$

$x' = x - t + 1$

$y' = te^t + e^t$

$y' = e^t[t+1]$

$t = 0 \to 0 = \ln x$

$e^0 = e^{\ln x}$

$x = 1$

$$\left.\frac{dy}{dx}\right|_{t=0} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}} = \frac{e^t[t+1]}{x - t + 1} = \frac{1[0+1]}{1-0+1} = \frac{1}{2}$$

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22] $x = t - t^2 \quad , \quad y = 1 + e^{-t}$

$dy = -e^{-t}\,dt$

$t - t^2 = 0$

$t(1-t) = 0$

$t = 0, 1$

$$\int_0^1 x\,dy = \left| \int_0^1 (t - t^2) - e^{-t}\,dt \right|$$

$$= \left| \int_0^1 (t - t^2)\, e^{-t}\,dt \right|$$

$$= \left[+e^{-t}(t - t^2) + e^{-t}(1 - 2t) - 2e^{-t} \right]\Big|_0^1$$

$$= \left[\frac{1}{e}(0) + \frac{1}{e}(-1) - \frac{2}{e} \right]$$

$$-\left[0 + 1 - 2 \right]$$

$$= \frac{-1}{e} - \frac{2}{e} + 1 = \left| \frac{-3}{e} + 1 \right|$$

$$\begin{array}{ccc} t - t^2 & & -e^{-t} \\ & + & \\ 1 - 2t & & +e^{-t} \\ & - & \\ -2 & & -e^{-t} \\ & + & \\ 0 & & +e^{-t} \end{array}$$

25] $x = \cos t \quad , \quad y = t + \sin t \qquad 0 \le t \le \pi$

$\frac{dx}{dt} = -\sin t \rightarrow \left(\frac{dx}{dt}\right)^2 = \sin^2 t$

$\frac{dy}{dt} = 1 + \cos t \rightarrow \left(\frac{dy}{dt}\right) = (1 + \cos t)^2$

$= 1 + 2\cos t + \cos^2 t$

$$L = \int_0^\pi \sqrt{\left(\frac{dy}{dt}\right)^2 + \left(\frac{dx}{dt}\right)^2}\,dt$$

$$= \int_0^\pi \sqrt{2 + 2\cos t}\,dt = \sqrt{2} \int_0^\pi \sqrt{(1 + \cos t) \cdot \frac{1 - \cos t}{1 - \cos t}}\,dt$$

$$= \sqrt{2} \int_0^\pi \sqrt{\frac{1 - \cos^2 t}{1 - \cos t}}\,dt = \sqrt{2} \int_0^\pi \frac{\sin t}{\sqrt{1 - \cos t}}\,dt$$

$u = 1 - \cos t$

$du = \sin t\,dt$

$$= \sqrt{2} \int \frac{du}{\sqrt{u}} = 2\sqrt{2} \left[\sqrt{1 - \cos t} \right]\Big|_0^\pi = 2\sqrt{2}\left[\sqrt{2} - 0\right] = 2(2) = 4$$

27\] $x = \frac{t^2}{2}$, $y = \frac{(2t+1)^{\frac{3}{2}}}{3}$, $0 \le t \le 4$

$\frac{dx}{dt} = t \Rightarrow \left(\frac{dx}{dt}\right)^2 = t^2$

$\frac{dy}{dt} = \frac{1}{2}(2t+1)^{\frac{1}{2}} \cdot 2 = \sqrt{2t+1} \Rightarrow \left(\frac{dy}{dt}\right)^2 = 2t+1$

$L = \int_0^4 \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}\,dt = \int_0^4 \sqrt{t^2+2t+1}\,dt = \int_0^4 \sqrt{(t+1)^2}\,dt = \int_0^4 (t+1)\,dt$

$= \left[\frac{t^2}{2} + t\right]\Big|_0^4 = 12 - 0 = 12$