|8.7| 1\

15] $\displaystyle\int_0^1 \frac{\theta+1}{\sqrt{\theta^2+2\theta}}\,d\theta$

$u = \theta^2 + 2\theta$

$du = 2(\theta+1)\,d\theta$

$\displaystyle\int \frac{1}{2\sqrt{u}} = \frac{1}{2}\int u^{-\frac{1}{2}} = \sqrt{u} = \sqrt{\theta^2+2\theta}$

$\displaystyle\lim_{c\to 0^+}\int_c^1 \frac{\theta+1}{\sqrt{\theta^2+2\theta}}\,d\theta = \lim_{c\to 0^+}\sqrt{\theta^2+2\theta}\,\Big|_c^1 = \lim_{c\to 0^+}\sqrt{3} - \sqrt{0^2+2(0)} = \sqrt{3}$ conv

47] $\displaystyle\int_1^\infty \frac{dx}{x^3+1}$ $\qquad \dfrac{1}{x^3+1} \leq \dfrac{1}{x^3}$

$\displaystyle\int_1^\infty \frac{1}{x^3}$ conv $p>1$

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Hence, $\displaystyle\int_1^\infty \frac{dx}{x^3+1}$ conv by DCT

63] $\displaystyle\int_{-\infty}^\infty \frac{dx}{\sqrt{x^4+1}}$ $\qquad \dfrac{1}{\sqrt{x^4+1}} \leq \dfrac{1}{x^2}$

$\displaystyle = 2\int_0^\infty \frac{dx}{\sqrt{x^4+1}} = \underbrace{2\int_0^1 \frac{dx}{\sqrt{x^4+1}}}_{\text{?}} + \underbrace{2\int_1^\infty \frac{dx}{\sqrt{x^4+1}}}_{\text{conv by DCT}}$

Hence, $\displaystyle\int_{-\infty}^\infty \frac{dx}{\sqrt{x^4+1}}$ conv

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1] $\int_0^4 \frac{dx}{\sqrt{4-x}}$

4] $\lim_{c\to4^-} \int_0^c \frac{dx}{\sqrt{4-x}}$  $u = 4-x$, $du = -dx$  $\int \frac{-du}{\sqrt{u}} = -2\sqrt{4-x}$

$\lim_{c\to4^-} -2\sqrt{4-x}\Big|_0^c = \lim_{c\to4^-} -2\sqrt{4-c} + 2\sqrt{4-0} = 4$ conv

10] $\int_{-\infty}^{2} \frac{2dx}{x^2+4}$

$\lim_{b\to-\infty} \int_b^2 \frac{2dx}{x^2+4}$

$\lim_{b\to-\infty} \cancel{2}\,\frac{1}{\cancel{2}} \tan^{-1}\frac{x}{2}\Big|_b^2 = \lim_{b\to-\infty} \left[\tan^{-1}1 - \tan^{-1}\frac{b}{2}\right] = \frac{\pi}{4} - -\frac{\pi}{2} = \frac{\pi}{4} + \frac{\pi}{2} = \frac{3\pi}{4}$

14] $\int_{-\infty}^{\infty} \frac{x\,dx}{(x^2+4)^{3/2}}$  $u = x^2+4$, $du = 2x\,dx$  $\int \frac{du}{2u^{3/2}} = \frac{1}{2}\int u^{-\frac{3}{2}}du = \frac{1}{2}\, u^{-\frac{1}{2}} \cdot -2 = \frac{-1}{\sqrt{x^2+4}}$

$= \int_{-\infty}^{0} \frac{x\,dx}{(x^2+4)^{\frac{3}{2}}} + \int_0^{\infty} \frac{x\,dx}{(x^2+4)^{\frac{3}{2}}}$

$= \lim_{b\to-\infty} \frac{-1}{\sqrt{x^2+4}}\Big|_b^0 + \lim_{b\to\infty} \frac{-1}{\sqrt{x^2+4}}\Big|_0^b = \lim_{b\to-\infty}\left[\cancel{-\frac{1}{2}} + \frac{1}{\sqrt{b^2+4}}\right] + \lim_{b\to\infty}\left[\frac{-1}{\sqrt{b^2+4}} + \cancel{\frac{1}{2}}\right] = 0$ conv

16] $\int_0^2 \frac{s+1}{\sqrt{4-s^2}}\,ds$  $u = 4-s^2$, $du = -2s\,ds$  $-\frac{1}{2}\int \frac{du}{\sqrt{u}} = -\frac{1}{\cancel{2}}\,\cancel{2}\sqrt{u} = -\sqrt{4-s^2}$

$= \int_0^2 \frac{s\,ds}{\sqrt{4-s^2}} + \int_0^2 \frac{1}{\sqrt{4-s^2}}\,ds$

$\lim_{c\to2^-} -\sqrt{4-s^2}\Big|_0^c + \lim_{c\to2^-} \sin^{-1}\frac{x}{2}\Big|_0^c = \lim_{c\to2^-}[0+2] + \lim_{c\to2^-}\left[\sin^{-1}1 - \sin^{-1}0\right]$

$= 2 + \frac{\pi}{2}$ conv

25] $\int_0^1 x\ln x\,dx$

$u = \ln x \qquad dv = x\,dx$

$du = \frac{1}{x} \qquad v = \frac{x^2}{2}$

$\lim_{c\to 0^+}\left[\frac{x^2}{2}\ln x - \frac{1}{2}\int x\,dx\right]$

$\lim_{c\to 0^+}\left[\frac{x^2}{2}\ln x - \frac{1}{4}x^2\right]\Big|_c^1 = \lim_{c\to 0^+}\left[\left(\frac{1}{2}\ln 1 - \frac{1}{4}\right) - (0)\right] = -\frac{1}{4}$ conv

32] $\int_0^2 \frac{dx}{\sqrt{|x-1|}}$

$x-1=0$

$x=1$

$-x+1 \quad x-1$

$0 \quad 1 \quad 2$

$= \int_0^1 \frac{dx}{\sqrt{-x+1}} + \int_1^2 \frac{dx}{\sqrt{x-1}}$

$= \lim_{c\to 1^-} -2\sqrt{-x+1}\Big|_0^c + \lim_{c\to 1^+} 2\sqrt{x-1}\Big|_c^2 = \lim_{c\to 1^-}[0+2] + \lim_{c\to 1^+}[2+0] = 4$ conv

37] $\int_0^\pi \frac{\sin\theta\,d\theta}{\sqrt{\pi-\theta}}$

$\frac{\sin\theta}{\sqrt{\pi-\theta}} \leq \frac{1}{\sqrt{\pi-\theta}}$

$\int_0^\pi \frac{1}{\sqrt{\pi-\theta}}\,d\theta$

$\lim_{c\to\pi^-} -2\sqrt{\pi-\theta}\Big|_0^c = \lim_{c\to\pi^-}[0+2\sqrt{\pi}] = 2\sqrt{\pi}$

Hence, $\int_0^\pi \frac{\sin\theta\,d\theta}{\sqrt{\pi-\theta}}$ conv by DCT

41] $\int_0^\pi \frac{dt}{\sqrt{t}+\sin t}$

$\frac{1}{\sqrt{t}+\sin t} \leq \frac{1}{\sqrt{t}}$

$\int_0^\pi \frac{1}{\sqrt{t}}\,dt$

$\lim_{c\to 0^+}\int_c^\pi \frac{1}{\sqrt{t}}\,dt = \lim_{c\to 0^+} 2\sqrt{t}\Big|_c^\pi = \lim_{c\to 0^+}[2\sqrt{\pi}-0] = 2\sqrt{\pi}$

Hence, $\int_0^\pi \frac{dt}{\sqrt{t}+\sin t}$ conv by DCT

50] $\displaystyle\int_0^\infty \frac{d\theta}{1+e^\theta}$ $\qquad \frac{1}{1+e^\theta} \leq \frac{1}{e^\theta}$



$\displaystyle\int_0^\infty \frac{d\theta}{e^\theta}$

$$\lim_{b\to\infty} \int_0^b e^{-\theta}\, d\theta = \lim_{b\to\infty} -\frac{1}{e^\theta}\Big|_0^b = \lim_{b\to\infty} [0+1] = 1$$

Hence, $\displaystyle\int_0^\infty \frac{d\theta}{1+e^\theta}$ conv by DCT

58] $\displaystyle\int_2^\infty \frac{1}{\ln x}\, dx$ $\qquad \frac{1}{x} < \frac{1}{\ln x}$

Hence, $\displaystyle\int_2^\infty \frac{1}{\ln x}$ div by DCT

$$\int_2^\infty \frac{1}{x}\, dx = \lim_{b\to\infty} \ln b - \ln 2 = \infty$$

62] $\displaystyle\int_1^\infty \frac{1}{e^x - 2^x}\, dx$

$$\lim_{b\to\infty} \frac{-1}{e^x}\Big|_1^b = \lim_{b\to\infty} \left[-\frac{1}{e^b} + \frac{1}{e}\right] = \frac{1}{e} \quad \text{conv}$$

62] $\displaystyle\int_1^\infty \frac{1}{e^x - 2^x}\, dx$

$$\lim_{x\to\infty} \frac{1}{e^x - 2^x} \cdot e^x = \lim_{x\to\infty} \frac{1}{1-\left(\frac{2}{e}\right)^x} = \frac{1}{1-0} = 1$$

Hence, $\displaystyle\int_1^\infty \frac{1}{e^x - 2^x}$ conv by LCT

1) $\int_0^{\infty} \frac{dx}{x^2+1}$

$\lim_{b \to \infty} \left[\tan^{-1} b - \tan^{-1} 0\right] = \frac{\pi}{2}$ conv

7) $\int_0^1 \frac{dx}{\sqrt{1-x^2}}$

$\lim_{c \to 1^-} \left[\sin^{-1} c - \sin^{-1} 0\right] = \frac{\pi}{2} - 0 = \frac{\pi}{2}$ conv

21) $\int_{-\infty}^{0} \theta e^{\theta} \, d\theta$

$u = \theta \qquad dv = e^{\theta} d\theta$

$du = d\theta \qquad v = e^{\theta}$

$= \theta e^{\theta} - e^{\theta}$

$\lim_{b \to -\infty} \left[-1 - (b e^{b} - e^{b})\right] = -1$

40) $\int_0^1 \frac{e^{-\sqrt{x}}}{\sqrt{x}} dx$

$u = \sqrt{x}$

$du = \frac{1}{2\sqrt{x}} dx$

$\lim_{c \to 0^+} \int 2 e^{-u} du$

$\lim_{c \to 0^+} -2 e^{-\sqrt{x}} \Big|_c^1 = -2 \lim_{c \to 0^+} \left[ \frac{1}{e} - 1 \right] = \frac{-2}{e} + 2$

56) $\int_{\pi}^{\infty} \frac{1+\sin x}{x^2}\,dx$

56]

$$\frac{1+\sin x}{x^2} \leq \frac{2}{x^2}$$

$$\int_{\pi}^{\infty} \frac{2}{x^2} = \lim_{b\to\infty} \frac{-2}{x}\Big|_{\pi}^{b}$$

$$= -2\lim_{b\to\infty} \left[\frac{1}{b} - \frac{1}{\pi}\right] = \frac{2}{\pi} \quad \text{conv}$$

Hence, $\int_{\pi}^{\infty} \frac{1+\sin x}{x^2}$ conv by DCT

65) a- $\int_{1}^{2} \frac{dx}{x(\ln x)^p}$ b- $\int_{2}^{\infty} \frac{dx}{x(\ln x)^p}$

$u = \ln x$
$du = \frac{dx}{x}$

$$\int \frac{du}{u^p}$$

$x=2 \to u=\ln 2$
$x=1 \to u=0$

$$P=1 \Rightarrow \lim_{a\to 0^+} \ln|u|\Big|_{a}^{\ln 2}$$

$$= \lim_{a\to 0^+} (\ln\ln 2 - \ln a) = \infty \quad P \geq 1 \text{ div}$$

$$P \neq 1 \Rightarrow \lim_{a\to 0^+} \frac{u^{-P+1}}{-P+1}\Big|_{a}^{\ln 2} = \lim_{a\to 0^+}\left(\frac{(\ln 2)^{1-P}}{1-P} - \frac{a^{1-P}}{1-P}\right) \quad P<1$$

$$= \frac{(\ln 2)^{1-P}}{1-P} \quad P<1 \text{ conv}$$

65) 6- $\int_2^\infty \frac{dx}{x(\ln x)^P}$ 7

$\underline{P=1}$

$\lim_{b\to\infty} \ln|u| \Big|_{\ln 2}^{b}$ $u=\ln x$

$x=\infty \to u=\infty$

$x=2 \to u=\ln 2$

$\lim_{b\to\infty} (\ln b - \ln\ln 2) = \infty$ div $P\leq 1$

$\underline{P\neq 1}$

$\lim_{b\to\infty} \frac{u^{-P+1}}{-P+1} \Big|_{\ln 2}^{b} = \lim_{b\to\infty} \frac{b^{-P+1}}{-P+1} - \frac{\ln 2^{-P+1}}{-P+1} = \frac{\ln 2^{1-P}}{P-1}$ conv $P>1$