1) $\sum_{n=1}^{\infty} n\tan^{-1}(\frac{2}{\pi})$

first 1:

$n\tan^{-1}(\frac{2}{\pi}) \leq n\frac{\pi}{2}$

$\lim_{n\to\infty} n\tan^{-1}(\frac{2}{\pi}) = \infty \cdot \frac{\pi}{2} = \infty \neq 0$

div n^{th} term tesc

2) $\sum_{n=1}^{\infty} (-1)^{n-1} \frac{(2n-1)}{(2n-1)!}$

$\sum_{n=0}^{\infty} \frac{1}{(2n)!}$

$\lim_{n\to\infty} \frac{1}{(2n+2)!} \cdot (2n)! = \lim_{n\to\infty} \frac{\cancel{(2n)!}}{(2n+1)(2n)\cancel{!}} = 0 < 1$

conv

$\lim_{n\to\infty} \frac{1}{(2n)!} = 0$

$\therefore$ conv Abs

3) $\sum \frac{(-1)^n \pi^{2n+1}}{(2n+1)!}$

$\sin(\pi) = 0$

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4) $\sum_{n=1}^{\infty} \frac{(2x+3)^n}{\sqrt{n}}$

$\lim_{n\to\infty} \frac{(2x+3)^{n+1}}{\sqrt{n+1}} \cdot \frac{\sqrt{n}}{(2x+3)^n} = |2x+3|\sqrt{\lim_{n\to\infty} \frac{n}{n+1}} = |2x+3| < 1$

$-1 < 2x+3 < 1$

$-3 \quad -3 \quad -3$

$-4 < 2x < -2$

$-2 \leq x < -1$

conv Abs

$x=-2 \to \sum (-1)^n \frac{1}{\sqrt{n}} \to$ conv

$x=-1 \to \sum \frac{1}{\sqrt{n}} \to$ div

IC conv $= -2 \leq x < -1$

$$\sum_{n=1}^{\infty} \left(\frac{\sqrt{x-1}-1}{2}\right)^n$$

$$\lim_{n\to\infty} \frac{\sqrt{x-1}-1}{2} = \frac{\sqrt{x-1}-1}{2}$$

$x=2 \to \sum 0 = 0$

$x=10 \to \sum 1 = 1$

$$-1 < \frac{\sqrt{x-1}-1}{2} < 1$$

$$-2 < \sqrt{x-1}-1 < 2$$

$$-1 < \sqrt{x-1} < 3$$

$$1 < x-1 < 9$$

$$2 < x < 10$$

conv Abs

6) $\lim_{n\to\infty} n^{\ln(n^2)}$

$$e^{\lim_{n\to\infty} \ln n \ln n^2} = e^{\infty} = \infty$$

7) $a_n = (-1)^n \cos(n\pi)$

$$\lim_{n\to\infty} (-1)^n(-1)^n = \lim_{n\to\infty} (-1)^{2n} = 1 \quad \text{conv}$$

8) $\sum_{n=1}^{\infty} \frac{(-1)^n 2^{n+1}}{5^n}$

$$\sum_{n=1}^{\infty} \frac{(-1)^n 2^n 2}{5^n}$$

$$2\sum_{n=1}^{\infty} (-1)^n \left(\frac{2}{5}\right)^n$$

$$\text{Sum} = 2\left[-\frac{2}{\cancel{5}} \cdot \frac{\cancel{5}}{7}\right] = -\frac{4}{7}$$

9) $a_n = e^{\left(1+\frac{2}{n}\right)^n}$

$$e^{\lim_{n\to\infty}\left(1+\frac{2}{n}\right)^n} = e^{e^2} \quad \text{conv}$$

10) $\sum_{n=0}^{\infty} r^{-n} = \frac{5}{4}$ $\quad r?$

$\sum_{n=0}^{\infty} \left(\frac{1}{r}\right)^n$

$$\text{Sum} = \frac{1}{1-\frac{1}{r}} = \frac{5}{4}$$

$$1-\frac{1}{r} = \frac{4}{5}$$

$$\frac{1}{r} = \frac{1}{5}$$

$\boxed{r=5}$

11) $e^{-x} = 1-x+\frac{x^2}{2} \quad 0 < x < 0.4$

$$e^{-x} = \sum (-1)^n \frac{x^n}{n!}$$

$$e^{-x} \leq 1-x+\frac{x^2}{2}$$

$$L - P \leq |a_{n+1}|$$

12) $f(x) = e^x \quad a=9$

$$e^x = \sum_{n=0}^{\infty} \frac{e^9(x-9)^n}{n!}$$

13) $a_n > 0 \quad b_n > 0 \quad n > N$

$$\lim_{n\to\infty} \frac{a_n}{b_n} = \infty$$

$\sum b_n$ div $\rightarrow \sum a_n$ div

$$\sum_{n=1} \frac{(-1)^n}{\ln(n+1)} = \frac{-1}{\ln 2} + \frac{1}{\ln 3} - \frac{1}{\ln 4}$$

$$\lim_{n\to\infty} U_n = \lim_{n\to\infty} \frac{1}{\ln(n+1)} = 0 \quad \text{conv cond}$$

$$\sum_{n=1}^{\infty} \frac{1}{\ln(n+1)} \qquad \frac{1}{\ln(n+1)} < \frac{1}{\ln n} \qquad \underset{\text{div}}{\frac{1}{n}} < \underset{\text{div}}{\frac{1}{\ln n}}$$

$$\lim_{n\to\infty} \frac{1}{\ln(n+1)} \cdot \ln n = 1$$

15) $\sum_{n=1}^{\infty} \frac{n}{(\ln n+10)^n}$ $\qquad \lim_{n\to\infty} \frac{n^{\frac{1}{n}}}{(\ln n+10)} = \frac{1}{\infty} = 0 < 1$ conv Root test

16) $a_n = \ln\left(1+\frac{1}{n}\right)^n$

$$\ln\left(\lim_{n\to\infty}\left(1+\frac{1}{n}\right)^n\right) = \ln e^1 = 1 \quad \text{conv}$$

17) $f(x) = x^3 e^x$

$$= x^3\left(1 + x + \frac{x^2}{2} + \frac{x^3}{3!} + \frac{x^4}{4!} + \cdots\right) \qquad \sum_{n=0}^{\infty} \frac{x^{n+3}}{n!}$$

$$= x^3 + x^4 + \frac{x^5}{2} + \frac{x^6}{3!} + \frac{x^7}{4!} + \cdots$$

18) $\lim_{n\to\infty} \frac{8+(-1)^n}{n}$

$$\frac{7}{n} < \frac{8+(-1)^n}{n} < \frac{9}{n}$$

$$\lim_{n\to\infty} \frac{7}{n} = 0 \qquad \lim_{n\to\infty} \frac{9}{n} = 0$$

$$\therefore \lim_{n\to\infty} \frac{8+(-1)^n}{n} = 0 \quad \text{by Sandwish Th}$$

19) $\sum_{n=1}^{\infty} \frac{5}{n(n+1)}$ $\qquad f(x) = \frac{5}{n(n+1)} = \frac{A}{n} + \frac{B}{n+1}$

$\sum_{n=1}^{\infty} \left(\frac{5}{n} - \frac{5}{n+1}\right) = \left(5 - \cancel{\frac{5}{2}}\right) + \left(\cancel{\frac{5}{2}} - \cancel{\frac{5}{3}}\right) + \left(\cancel{\frac{5}{3}} - \frac{5}{4}\right) + \dots = \frac{5}{n} - \frac{5}{n+1}$

$S_n = 5 - \frac{5}{n+1} = \frac{5n+5-5}{n+1} = \frac{5n}{n+1}$

$\lim_{n\to\infty} S_n = 5 - 0 = 5$ conv

20) If $\sum a_n$ and $\sum b_n$ conv $\to$ $\sum (a_n b_n)$ conv $\quad$ False?

21) $a_n = \frac{2n}{n+1}$ $\qquad \lim_{n\to\infty} \frac{2n}{n+1} = \frac{2}{1} = 2$ conv

$a_1 = \frac{2}{2} = 1$, $a_2 = \frac{4}{3}$, $a_3 = \frac{6}{4}$ $\qquad$ nondecreasing $=$ increasing

22) $\frac{1}{2}, \frac{-1}{6}, \frac{1}{12}, \frac{-1}{20}$ $\qquad \frac{(-1)^{n+1}}{n^2+n}$ $\qquad n = 1, 2, 3, 4, \dots\dots \infty$

23) $a_1 = 3$, $a_{n+1} = \frac{a_n}{3}$ $\qquad a_2 = 1$

$a_3 = \frac{1}{3}$

$\lim_{n\to\infty} \frac{a_{n+1}}{a_n} = \lim_{n\to\infty} \frac{1}{3} = \frac{1}{3} < 1$ conv $\qquad a_4 = \frac{1}{9}$

$\vdots$

conv to 0 $\qquad 0$

24) $5.\bar{4} = 5.44444\dots = 5 + \frac{4}{10} + \frac{4}{100} + \frac{4}{1000} + \dots$

$5 + \frac{a}{1-r} = 5 + \left[\frac{4}{10} \cdot \frac{10}{9}\right]$

$= 5 + \frac{4}{9} = \frac{49}{9}$

$$\sum_{n=1} \frac{\sqrt{n+1}}{n^2+1}$$

$$n^2 - n^{\frac{1}{2}} = n^{\frac{3}{2}}$$

$$\lim_{n\to\infty} \frac{\sqrt{n+1}}{n^2+1} \cdot n^{\frac{3}{2}} = \lim_{n\to\infty} \frac{\sqrt{n+1}\, n^{\frac{3}{2}}}{n^2+1} = 1$$

$$b_n = \frac{1}{n^{\frac{3}{2}}}$$

$\sum_{n=1}^{\infty} \frac{1}{n^{\frac{3}{2}}}$ conv P-series

26) $\sum_{n=1}^{\infty} \left(\frac{\ln n}{n}\right)^n$

$\lim_{n\to\infty} \frac{\ln n}{n} = 0 < 1$ conv by Root test

27) $\sum_{n=1}^{\infty} \frac{(-1)^n}{n!} = -1 + \frac{1}{2} - \frac{1}{6} + \frac{1}{24} - \frac{1}{120} + \cdots\cdots$

$E > 0$, $E < \frac{1}{24}$

28) $\sum_{n=0}^{\infty} n!\,(x-2)^n$

$$\lim_{n\to\infty} \frac{(n+1)!\,(x-2)^{n+1}}{n!\,(x-2)^n} = |x-2| \lim_{n\to\infty} (n+1) = \infty > 1$$

$\xleftarrow{\ \ \text{div}\ \ }\ 2\ \xrightarrow{\ \ }$

radius conv $\to 0$

29) $\sum_{n=1}^{\infty} \frac{4^n x^{2n}}{n}$

$$\lim_{n\to\infty} \frac{4^{n+1} x^{2n+2}}{n+1} \cdot \frac{n}{4^n x^{2n}} = \lim_{n\to\infty} \frac{n x^2 4}{n+1} = |4x^2| \lim_{n\to\infty} \frac{n}{n+1} = |4x^2| < 1$$

$-1 < 4x^2 < 1$

$-\frac{1}{4} < x^2 < \frac{1}{4}$

$-\frac{1}{2} \leq x < \frac{1}{2}$

$x = -\frac{1}{2} \to \sum_{n=1}^{\infty} \frac{4^n}{n} \left(-\frac{1}{2}\right)^{2n} \to$ conv

$x = \frac{1}{2} \to \sum_{n=1}^{\infty} \frac{4^n \left(\frac{1}{2}\right)^{2n}}{n} = \sum_{n=1}^{\infty} \frac{1}{n} \to$ div

$x = -\frac{1}{2}$

$$\lim_{n\to\infty} \frac{4^{n+1}\left(-\frac{1}{2}\right)^{2n+2}}{n+1} \cdot \frac{n}{4^n\left(-\frac{1}{2}\right)^{2n}}$$

$$\lim_{n\to\infty} \frac{4n}{4n+4} = 1$$

conv by Ratio Test

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30) $\sum_{n=1}^{\infty} \frac{(-2)^{n-1}}{4^n}$

$$\sum_{n=1}^{\infty} \frac{(-2)^n(-2)^{-1}}{4^n} = -\frac{1}{2}\sum_{n=1}^{\infty}\left(-\frac{1}{2}\right)^n = -\frac{1}{2}\left[-\frac{1}{2}\cdot\frac{2}{3}\right] = \frac{1}{6}$$

31) $\sum_{n=1}^{\infty} \frac{n^n}{n!}$

$$\lim_{n\to\infty} \frac{(n+1)^{n+1}}{(n+1)!}\cdot\frac{n!}{n^n} = \lim_{n\to\infty}\frac{(n+1)^{n+1}}{(n+1)n^n} = \lim_{n\to\infty}\left(\frac{n+1}{n}\right)^n = \lim_{n\to\infty}\left(1+\frac{1}{n}\right)^n = e^1 > 1 \quad \text{div}$$

32) $\sum_{n=0}^{\infty} \frac{2^{3n}}{3^{2n}}$

$$\sum_{n=0}^{\infty}\left(\frac{8}{9}\right)^n \qquad \text{Sum} = 1\cdot\frac{9}{1} = 9$$

33) $\sum_{n=1}^{\infty} \frac{1}{n^{\ln 2}}$ $\qquad \ln 2 < 1$

div by p-series

Question 2:

$$\sum_{n=1}^{\infty} \frac{(-1)^{n+1}(x-2)^{n+1}}{n4^n}$$

$$\lim \frac{(x-2)^{n+2}}{(n+1)4^{n+1}}\cdot\frac{n4^n}{(x-2)^{n+1}} = \frac{1}{4}|x-2|\lim_{n\to\infty}\frac{n}{n+1} = \frac{1}{4}|x-2| < 1$$

$$-1 < \frac{1}{4}(x-2) < 1$$

$$-4 < x-2 < 4$$

$$\boxed{-2 < x < 6} \quad \text{conv Abs}$$

$x=-2 \to \sum \frac{(-1)^{n+1}(-4)^{n+1}}{n4^n} \to \sum \frac{4}{n}$ div

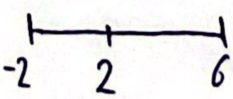
$x=6 \to \sum \frac{(-1)^{n+1}(4)^{n+1}}{n4^n} \to \sum (-1)^{n+1}\frac{4}{n}$ conv

IC conv $\to -2 < x \leq 6$

conv cond at $x=6$

center $= 2$

Radius $= 4$



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