first 4:

1) $\int_0^{\frac{\pi}{2}} \frac{\cos x}{\sqrt{\sin x}}\,dx$

$u = \sin x$
$du = \cos x\,dx$ $\int \frac{du}{\sqrt{u}} = 2\sqrt{\sin x}$

$\lim_{c\to 0^+} \left[2\sqrt{\sin\frac{\pi}{2}} - 2\sqrt{\sin c}\right]$
$= 2 - 0 = 2$ conv

d-

2) $\int_1^{\infty} \frac{\ln x}{x^3}\,dx$

$\ln x < x$

$\frac{\ln x}{x^3} < \frac{1}{x^2}$

conv by DCT

a-

3) $a_n = \frac{\ln(2n)}{\ln(3n)}$

a-

$\lim_{n\to\infty} \frac{\ln(2n)}{\ln(3n)} = \lim_{n\to\infty} \frac{2}{2n}\cdot\frac{3n}{3} = \frac{2}{3}\cdot\frac{3}{2} = 1$ conv

4) $\int_2^{\infty} \frac{dx}{\sqrt{x^2-1}}$

$x^2 - 1 < x^2$

$\sqrt{x^2-1} < \sqrt{x^2}$

$\frac{1}{x} < \frac{1}{\sqrt{x^2-1}}$

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c-

div by DCT

5) $\sum_{n=2}^{\infty} \frac{\sqrt{n}}{\ln n}$

$\lim_{n\to\infty} \frac{\sqrt{n}}{\ln n} = \lim_{n\to\infty} \frac{1}{2\sqrt{n}}\cdot n = \infty \neq 0$ div by n^{th} term test

a-

6) $\sum_{n=1}^{\infty} \frac{n}{e^n}$

$\lim_{n\to\infty} \frac{n^{\frac{1}{n}}}{e} = \frac{1}{e} < 1$ conv by Root test

c-

$$\sum_{n=2} \frac{(-1)^n}{\sqrt{n}-1}$$

$\lim_{n\to\infty} U_n = 0$ conv cond

$$\sum_{n=2}^{\infty} \frac{1}{\sqrt{n}-1} \qquad \sum_{n=2}^{\infty} \frac{1}{\sqrt{n}} \text{ div}$$

$$\lim_{n\to\infty} \frac{1}{\sqrt{n}-1} \cdot \sqrt{n} = 1$$

8) $\sum_{n=1}^{\infty} \frac{1}{n-\ln n}$

$n - \ln n < n$

$\frac{1}{n} < \frac{1}{n-\ln n}$

div by DCT

$\sum_{n=1}^{\infty} \frac{1}{n}$ div

or

$\lim_{n\to\infty} \frac{1}{n-\ln n} \cdot n = \frac{1}{1-\frac{1}{n}} = 1$ div by LCT

9) $1 - \frac{\pi^2}{2!} + \frac{\pi^4}{4!} - \frac{\pi^6}{6!} + \cdots\cdots$

$$\sum_{n=0}^{\infty} \frac{(-1)^n (\pi)^{2n}}{n!}$$

10) $\cos x \approx 1 - \frac{x^2}{2!}$, $|x| < 0.1$

$E < \frac{(0.1)^4}{4!}$

$E < \frac{10^{-4}}{4!}$

11) $a_n = (2^n + 3^n)^{\frac{1}{n}}$

$\sqrt{\lim_{n\to\infty}[(\frac{2}{3})^n + 1]3^n} = 3$ conv

12) $\sum_{n=1}^{\infty}\left(\frac{1}{2^n} + \frac{2}{3^n}\right)$

$= \left[\frac{1}{2} \cdot 2\right] + \left[\frac{2}{3} \cdot \frac{3}{2}\right] = 1 + 1 = 2$

13) $\sum_{n=1}^{\infty} \frac{4}{(2n-1)(2n+1)}$

$\frac{A}{2n-1} + \frac{B}{2n+1}$

$\sum_{n=1}^{\infty} \left[\frac{2}{2n-1} - \frac{2}{2n+1}\right]$

$\frac{2}{2n-1} - \frac{2}{2n+1}$

$S_n = 2 - \frac{2}{2n+1}$

14) $\sum_{n=1}^{\infty} \frac{(n!)^2}{(2n)!}$

$\lim_{n\to\infty} \frac{((n+1)!)^2}{(2n+2)!} \cdot \frac{(2n)!}{(n!)^2} = \lim_{n\to\infty} \frac{(n+1)^2}{(2n+2)(2n+1)} = \lim_{n\to\infty} \frac{n^2+2n+1}{4n^2+2n+4n+2} = \frac{1}{4} < 1$

conv ratio test

15) $\sum_{n=1}^{\infty} (\ln x)^n$

Geometric Series

$r \in (-1, 1)$

$-1 < \ln x < 1$

$e^{-1} < x < e$

16) $f(x) = 2^x$

$\sum_{n=0}^{\infty} \frac{(\ln 2)^n x^n}{n!}$

$f(x) = 2^x$

$f'(x) = 2^x \ln 2$

$f''(x) = 2^x (\ln 2)^2$

$f'''(x) = 2^x (\ln 2)^3$

$\vdots$

17) $f(x) = (1+x)^{-\frac{1}{3}}$

$1 + \sum_{k=1}^{\infty} \binom{-\frac{1}{3}}{k} x^k$

$1 + -\frac{1}{3}x + \frac{2}{9}x^2 - \cdots$

$$\sum_{n=1}^{\infty} \frac{n^{\frac{1}{n}}}{n}$$

$$\lim_{n\to\infty} \frac{n^{\frac{1}{n}}}{n} \cdot \frac{n}{1} = 1 \qquad \sum_{n=1}^{\infty} \frac{1}{n} \text{ div}$$

div by LCT

19) $\frac{x^2}{(1-x)^2}$,

$$\sum_{n=1}^{\infty} n\, x^{n+1}$$

$$f(x) = \frac{x^2}{(1-x)^2}$$

$$f'(x) = \frac{-2x^2+2x}{(1-x)^4}$$

$$f''(x) = \frac{(-4x+2)(1-x)^4 - -4(-2x^2+2x)(1-x)^3}{(1-x)^8}$$

$f(0) = 0$

$f'(0) = 0$

$f''(0) = 2$

Question 2: $\sum_{n=2}^{\infty} \frac{x^n}{n(\ln n)^2}$

$$\lim_{n\to\infty} \frac{x^{n+1}}{(n+1)(\ln(n+1))^2} \cdot \frac{n(\ln n)^2}{x^n} = |x| \lim_{n\to\infty} \left[\frac{n}{n+1} \cdot \frac{(\ln n)^2}{(\ln(n+1))^2}\right] = |x| < 1$$

$-1 < x < 1$

$x = -1 \to \sum_{n=2}^{\infty} (-1)^n \frac{1}{n(\ln n)^2} \to$ conv AST

$x = 1 \to \sum_{n=2}^{\infty} \frac{1}{n(\ln n)^2} \to$ conv IT

$u = \ln x$

$du = \frac{dx}{x}$

$$f(x) = \frac{1}{x(\ln x)^2}$$

$$\int \frac{du}{u^2} = \frac{-1}{\ln x}$$

$$\lim_{b\to\infty} \left[\frac{-1}{\ln b} + \frac{1}{\ln 2}\right] = \frac{1}{\ln 2}$$

$$\lim_{n\to\infty} U_n = \lim_{n\to\infty} \frac{1}{n(\ln n)^2} = 0$$

IC conv $\to -1 \leq x \leq 1$

Radius of conv $\to R = 1$

Question 3:

$$\frac{1}{1+t} = 1-t+t^2-t^3+\cdots \qquad -1<t<1$$

a) $$\frac{1}{1+t^2} = 1-t^2+t^4-t^6+t^8-t^{10}+\cdots$$

b) $$\int_0^x \frac{dt}{1+t^2} = \int_0^x (1-t^2+t^4-t^6+t^8-t^{10}+\cdots)\,dt$$

$$\tan^{-1}t\Big|_0^x = t-\frac{t^3}{3}+\frac{t^5}{5}-\frac{t^7}{7}+\frac{t^9}{9}-\frac{t^{11}}{11}+\cdots\Big|_0^x$$

$$\tan^{-1}x = x-\frac{x^3}{3}+\frac{x^5}{5}-\frac{x^7}{7}+\frac{x^9}{9}-\frac{x^{11}}{11}+\cdots$$

c) $$\int_0^1 \frac{\tan^{-1}x}{x}\,dx \qquad E<0.01$$

$$\frac{\tan^{-1}x}{x} = 1-\frac{x^2}{3}+\frac{x^4}{5}-\frac{x^6}{7}+\frac{x^8}{9}-\frac{x^{10}}{11}+\cdots$$

$$\int_0^1 \frac{\tan^{-1}x}{x} = \int_0^1 \left(1-\frac{x^2}{3}+\frac{x^4}{5}-\frac{x^6}{7}+\frac{x^8}{9}-\frac{x^{10}}{11}+\cdots\right)dx$$

$$= x-\frac{x^3}{9}+\frac{x^5}{25}-\frac{x^7}{49}+\frac{x^9}{81}-\frac{x^{11}}{121}+\cdots\Big|_0^1$$

$$= 1-\frac{1}{9}+\frac{1}{25}-\frac{1}{49}+\frac{1}{81}-\frac{1}{121}+\cdots$$

$$\int_0^1 \frac{\tan^{-1}x}{x} \approx 1-\frac{1}{9}+\frac{1}{25}-\frac{1}{49}+\frac{1}{81}$$

$$E<\frac{1}{121}$$