1) $\lim_{n\to\infty} \frac{4+(-1)^n}{n}$ $\qquad \frac{3}{n} < \frac{4+(-1)^n}{n} < \frac{5}{n}$ first 2:

$\lim_{n\to\infty} a_n = 0$

2) $\lim_{n\to\infty} \frac{2n+1}{6n+1} = \frac{1}{3}$ $\qquad a_n = \frac{2n+1}{6n+1}$

$a_1 = \frac{3}{7}$ , $a_2 = \frac{5}{13}$ , $a_3 = \frac{7}{19}$ , $a_4 = \frac{9}{25}$

3) $a_n = (-1)^n\left(1-\frac{6}{n}\right)$

$\lim_{n\to\infty} (-1)^n\left(1-\frac{6}{n}\right) =$ DNE div

Haytham Shehadeh

4) $\sum_{n=1}^{\infty} \frac{(\ln n)^n}{n^n}$

$\lim_{n\to\infty} \frac{\ln n}{n} = 0 < 1$ conv

5) $\sum b_n$ conv $\to \sum a_n$ conv

6) $\sum_{n=0}^{\infty}\left(\frac{1}{4^n} - \frac{(-1)^n}{4^{n+1}}\right) = \left(1-\frac{1}{4}\right)+\left(\frac{1}{4}+\frac{1}{16}\right)+\left(\frac{1}{16}-\frac{1}{64}\right)+\left(\frac{1}{64}+\frac{1}{4^4}\right)+\cdots$

$\sum_{n=0}^{\infty}\left(\frac{1}{4}\right)^n - \frac{1}{4}\sum_{n=0}^{\infty}\left(\frac{-1}{4}\right)^n$

$= \left(1 \cdot \frac{4}{3}\right) - \frac{1}{4}\left(1 \cdot \frac{4}{5}\right) = \frac{5\times 4}{5\times 3} - \frac{1}{5}\,{}^{\times 3}_{\times 3} = \frac{17}{15}$

$$\sum_{n=1} \frac{n}{(\ln n+10)^n}$$

$$\lim_{n\to\infty} \frac{n^{\frac{1}{n}}}{\ln n+10} = 0 < 1$$ conv by Root test

8) $$\sum_{n=1}^{\infty} \frac{5}{n(n+1)}$$

$$\frac{5}{n(n+1)} = \frac{A}{n} + \frac{B}{n+1}$$

$$= \frac{5}{n} - \frac{5}{n+1}$$

$$= \sum_{n=1}^{\infty}\left(\frac{5}{n} - \frac{5}{n+1}\right)$$

$$S_n = \left(5 - \cancel{\frac{5}{2}}\right) + \cancel{\left(\frac{5}{2}\right.} - \cancel{\frac{5}{3}}\Big) + \cancel{\Big(\frac{5}{3}} - \frac{5}{4}\Big)$$

$$S_n = 5 - \frac{5}{n+1} = \frac{5n}{n+1}$$

$$\lim_{n\to\infty} S_n = \lim_{n\to\infty}\left[5 - \frac{5}{n+1}\right] = 5$$

9) $$\sum_{n=1}^{\infty} \frac{(x+6)^n}{\sqrt{n}}$$

$$\lim_{n\to\infty} \frac{(x+6)^{n+1}}{\sqrt{n+1}} \cdot \frac{\sqrt{n}}{(x+6)^n} = |x+6|\sqrt{\lim_{n\to\infty}\frac{n}{n+1}} = |x+6| < 1$$

$$-1 < x+6 < 1$$

$$-7 < x < -5$$

$x=-7 \rightarrow \sum (-1)^n \frac{1}{\sqrt{n}} \rightarrow$ conv

$x=-5 \rightarrow \sum \frac{1}{\sqrt{n}} \rightarrow$ div

conv $x=-7$
cond

10) $$\sum_{n=1}^{\infty} \frac{(-1)^n}{n^{\frac{5}{4}}+3}$$

$$\sum_{n=1}^{\infty} \frac{1}{n^{\frac{5}{4}}+3}$$

$$\frac{1}{n^{\frac{5}{4}}+3} < \frac{1}{n^{\frac{5}{4}}}$$

$\downarrow$ conv $\qquad \downarrow$ conv P-series

conv Abs

11) $a_1 = 4$ , $a_{n+1} = \sqrt[n]{n}\, a_n$

$\lim_{n\to\infty} \sqrt[n]{n} = 1$

12) $\sum_{n=1}^{\infty} \frac{5n}{n^2+1}$

$\lim_{n\to\infty} \frac{5n}{n^2+1} \cdot \frac{n}{1} = 5$ · LCT $\sum_{n=1}^{\infty} \frac{1}{n}$ div

13) $\sum_{n=1}^{\infty} b_n$ conv $\to \sum_{n=1}^{\infty} a_n$ conv

14) $\sum_{n=1}^{\infty} (-1)^{n-1} \frac{(2n-1)}{(2n-1)!}$

$\sum_{n=1}^{\infty} \frac{1}{2n!}$ $\lim_{n\to\infty} \frac{1}{(2n+2)!} \cdot 2n! = \lim_{n\to\infty} \frac{1}{(2n+2)(2n+1)} = 0 < 1$ conv Abs

15) $a_1 = 3$ , $a_{n+1} = \frac{n}{n+1} a_n$

$\lim_{n\to\infty} \frac{a_{n+1}}{a_n} = \lim_{n\to\infty} \frac{n}{n+1} = 1$

conv to 0

$a_2 = \frac{2}{3} \cdot 3 = 2$ , $a_5 = \frac{5}{6} \cdot \frac{6}{5} = 1$

$a_3 = \frac{3}{4} \cdot 2 = \frac{3}{2}$ , $a_7 = \frac{7}{8} \cdot 1 = \frac{7}{8}$

$a_4 = \frac{4}{5} \cdot \frac{3}{2} = \frac{6}{5}$ , $a_8 = \frac{8}{9} \cdot \frac{7}{8} = \frac{7}{9}$

16) $\sum_{n=1}^{\infty} e^{-n} = \sum_{n=1}^{\infty} \left(\frac{1}{e}\right)^n$ conv $r = \frac{1}{e} \in (-1, 1)$

Sum $= \frac{\frac{1}{e}}{1 - \frac{1}{e}} = \frac{1}{e} \frac{e}{e-1} = \frac{1}{e-1}$

$$\sum_{n=1} \frac{(x+4)^n}{n3^n}$$

$$\lim_{n\to\infty} \frac{(x+4)^{n+1}}{(n+1)\,3^{n+1}} \cdot \frac{n3^n}{(x+4)^n} = \lim_{n\to\infty} \frac{(x+4)\,n}{3\,(n+1)} = \frac{|x+4|}{3} \lim_{n\to\infty} \frac{n}{n+1} = \frac{|x+4|}{3} < 1$$

$x=-7 \to \sum_{n=1}^{\infty} (-1)^n \frac{1}{n} \to$ conv

$x=-1 \to \sum_{n=1}^{\infty} \frac{1}{n} \to$ div

$-1 < \frac{x+4}{3} < 1$

$-3 < x+4 < 3$

$-7 < x < -1 \to$ conv Abs

IC conv $\to [-7,-1)$

18) div by $n^{th}$ term test

19) $1.\bar{7} = 1 + \frac{7}{10} + \frac{7}{100} + \frac{7}{1000} + \cdots$

$\text{Sum} = 1 + \left[\frac{7}{10} \cdot \frac{10}{9}\right] = 1 + \frac{7}{9} = \frac{16}{9}$

20) $\sum_{n=1}^{\infty} \frac{(-1)^n}{2^{n+1}} = \frac{-1}{4} + \frac{1}{8} - \frac{1}{16} + \frac{1}{32}$

$0 \leq E \leq \frac{1}{32}$

Bonus $\left(2^{2^n}\right)^2 = 2^{2^{n+1}}$

**Question 2:** a- $\sum_{n=1}^{\infty} \frac{(-1)^n 3n^2}{n^3+1}$

$U_n = \frac{3n^2}{n^3+1} > 0$, $\downarrow$ for Large n

$\lim_{n\to\infty} U_n = 0$ conv

$\sum_{n=1}^{\infty} \frac{3n^2}{n^3+1}$ div

$\sum_{n=1}^{\infty} \frac{1}{n}$ div

$\lim_{n\to\infty} \frac{3n^2}{n^3+1} \cdot \frac{n}{1} = 3$

$\therefore \sum_{n=1}^{\infty} \frac{(-1)^n 3n^2}{n^3+1}$ conv cond

b- $\sum_{n=2}^{\infty} \frac{x^n}{n(\ln n)^2}$

$\lim_{n\to\infty} \frac{x^{n+1}}{(n+1)(\ln(n+1))^2} \cdot \frac{n(\ln n)^2}{x^n} = |x| \lim_{n\to\infty} \left[\frac{n}{n+1} \cdot \frac{(\ln n)^2}{(\ln(n+1))^2}\right] = |x| < 1$

$-1 < x < 1 \to$ conv ABS

$x=-1 \to \sum_{n=2}^{\infty} (-1)^n \frac{1}{n(\ln n)^2} \to$ conv

$\therefore -1 \leq x \leq 1 \to$ conv ABS

$x=1 \to \sum_{n=2}^{\infty} \frac{1}{n(\ln n)^2} \to$ conv by Integral test

$f(x) = \frac{1}{x(\ln x)^2}$ $+, \downarrow$

$\lim_{b\to\infty} \int_2^b \frac{dn}{n(\ln n)^2}$

$u = \ln n$
$du = \frac{1}{n} dn$

$\int \frac{du}{u^2} = \frac{-1}{\ln n}$

$\lim_{b\to\infty} \left[\frac{-1}{\ln n}\right]_2^b = \lim_{b\to\infty} \left[\frac{-1}{\ln b} + \frac{1}{\ln 2}\right] = \frac{1}{\ln 2}$

so $\sum_{n=2}^{\infty} \frac{x^n}{n(\ln n)^2}$ conv ABS $-1 \leq x \leq 1$

tion 3: $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}(x-2)^{n+1}}{n4^n}$

$\lim_{n\to\infty} \frac{(x-2)^{n+2}}{(n+1)4^{n+1}} \cdot \frac{n4^n}{(x-2)^{n+1}} = \frac{|x-2|}{4} \lim_{n\to\infty} \frac{n}{n+1} = \frac{|x-2|}{4} < 1$

$-1 < \frac{|x-2|}{4} < 1$

$-4 < x-2 < 4$

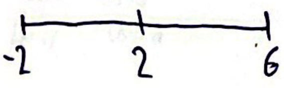
$\boxed{-2 < x < 6}$ conv Abs

$x=-2 \to \sum_{n=1}^{\infty} \frac{(-1)^{n+1}(-4)^{n+1}}{n4^n}$

$= \sum \frac{(-1)^n(-1)(-4)^n(-4)}{n4^n}$

$= 4\sum \frac{1}{n} \to$ div

$x=6 \to \sum_{n=1}^{\infty} \frac{(-1)^n(-1)(4)^n(4)}{n4^n} = -4\sum_{n=1}^{\infty} (-1)^n \frac{1}{n} \to$ conv cond



IC conv $\to -2 < x \leq 6$

Radius conv $\to R=4$

$x=6 \to$ conv cond

Question 4: $\sum_{n=0}^{\infty} (\log_2 x)^n$

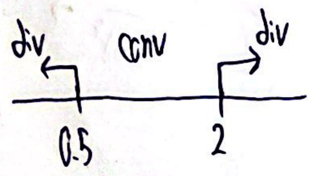
$r = \log_2 x$

conv $r \in (-1,1)$

$-1 < \log_2 x < 1$

$2^{-1} < x < 2$

conv $\boxed{\frac{1}{2} < x < 2}$




div $x \geq 2$ or $x \leq \frac{1}{2}$

Radius of conv $R =$

Scanned with CamScanner

Bonus: $\sum_{n=1}^{\infty} \frac{n}{2^{n-1}}$ (Hint $\frac{1}{1-x} = \sum_{n=0}^{\infty} x^n$, for $-1 < x < 1$)

$$\frac{1}{1-x} = \sum_{n=0}^{\infty} x^n$$ differentiale

$$\frac{1}{(1-x)^2} = \sum_{n=0}^{\infty} n x^{n-1}$$

$$x = \frac{1}{2} \rightarrow \sum_{n=0}^{\infty} n \left(\frac{1}{2}\right)^{n-1} = \sum_{n=0}^{\infty} \frac{n}{2^{n-1}} = \frac{1}{(1-\frac{1}{2})^2} = \frac{1}{\frac{1}{4}} = 4$$

so $\sum_{n=1}^{\infty} \frac{n}{2^{n-1}} = 4$