1) $f(x) = \frac{1}{1-2x}$

$f'(x) = \frac{2}{(1-2x)^2}$

$f''(x) = \frac{2\times4(1-2x)}{(1-2x)^4} = \frac{8}{(1-2x)^3}$

$f'''(x) = \frac{8\times6(1-2x)^2}{(1-2x)^6} = \frac{48}{(1-2x)^4}$

$\sum \frac{f^{n}(c)\,x^n}{n!} = f(0) + f'(0)x + \frac{f''(c)x^2}{2} + \frac{f'''(c)x^3}{3!}$

$= 1 + 2x + \frac{8x^2}{2} + \frac{48x^3}{3!} + \cdots$

$= 1 + 2x + 4x^2 + 8x^3 + \cdots$

first 3:

$\sum_{n=0}^{\infty} (2x)^n,$

$r < 1$

$2x < 1$

$x < \frac{1}{2}$

2) $\sum_{n=1}^{\infty} \frac{2^{-n} + n}{n^3 + n^2}$

$\lim_{n\to\infty} \frac{2^{-n} + n}{n^3 + n^2} \cdot \frac{n^2}{1} = 1$

$\sum_{n=1}^{\infty} \frac{1}{n^2}$ conv

3) $\sum_{n=1}^{\infty} \frac{e^{2n}}{n^n}$

$\lim_{n\to\infty} \frac{e^2}{n} = 0 < 1$ conv by Root test

4) $\frac{1}{1+x} = 1 - x + x^2$

$E < 64\times10^{-6}$

$x^3 < 64\times10^{-6}$

$|x| < 0.04$

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5) $\sum_{n=2}^{\infty} \frac{1}{n\ln n}$

$\lim_{b\to\infty} \int_2^b \frac{dx}{x\ln x}$

$u = \ln x$

$du = \frac{dx}{x}$

$\int \frac{du}{u} = \ln u$

$x = b \to \ln b$

$x = 2 \to \ln 2$

$\lim_{b\to\infty} \left[\ln(\ln b) - \ln(\ln 2)\right]$

$= \infty$ div by Integral test

$\ln 3 + \frac{(\ln 3)^2}{2!} + \frac{(\ln 3)^3}{3!} + \dots\dots =$

$\sum_{n=0}^{\infty} \frac{(\ln 3)^n}{n!}$

$e^{\ln 3} = 3$

$e^x = \sum_{n=0}^{\infty} \frac{x^n}{n!}$

7) $\sqrt{1+x^3} = (1+x^3)^{\frac{1}{2}}$

$(1+x)^m$

$= 1 + \sum_{k=1}^{\infty} \binom{m}{k} x^k$

$= 1 + \frac{1}{2}x^3 + \frac{-1}{8}x^6 + \frac{x^9}{16} \dots\dots$

8) $\sum_{n=1}^{\infty} \frac{x^n}{\sqrt{n}}$

$x = -1 \rightarrow \sum (-1)^n \frac{1}{\sqrt{n}} \rightarrow$ conv

IC conv $= [-1, 1)$

$x = 1 \rightarrow \sum \frac{1}{\sqrt{n}} \rightarrow$ div

9) $\sum_{n=0}^{\infty} \left(\frac{1}{2^n} + \frac{(-1)^n}{3^n} \right)$

$\sum_{n=0}^{\infty} \left(\frac{1}{2}\right)^n + \sum_{n=0}^{\infty} \left(\frac{-1}{3}\right)^n = [1 \cdot 2] + [1 \cdot \frac{3}{4}] = \frac{4 \times 2}{4 \times 1} + \frac{3}{4} = \frac{11}{4}$

10) $e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!}$

$|x| < 0.1$

$E < \frac{(0.1)^3}{6} e^{0.1}$

$E < \frac{e^{0.1}}{6000}$

11) $\sum_{n=1}^{\infty} \frac{3}{n^2} - \frac{3}{(n+1)^2} = \left(\frac{3}{1} - \cancel{\frac{3}{4}}\right) + \left(\cancel{\frac{3}{4}} - \cancel{\frac{3}{9}}\right) + \left(\cancel{\frac{3}{9}} - \frac{3}{16}\right) + \cdots$

$S_n = 3 - \frac{3}{(n+1)^2}$

$\lim_{n\to\infty} S_n = 3$

12) $\sum_{n=1}^{\infty} \left(\frac{n}{n+1}\right)^{n^2} x^n$

$u = n+1$

$|x| \lim_{n\to\infty} \left(\frac{n+1-1}{n+1}\right)^n = |x| \lim_{u\to\infty} \left(1 - \frac{1}{u}\right)^u \left(1 - \frac{1}{u}\right)^{-1}$

$= \frac{|x|}{e} < 1$

$-1 < \frac{x}{e} < 1$

$-e < x < e$

$-e \quad 0 \quad e$

$R = e$

13) $a_n = \left(3 + \frac{3}{n}\right)^n$

$\lim \left(1 + \frac{1}{n}\right)^n = e^1$

$\lim_{n\to\infty} \left(3 + \frac{3}{n}\right)^n = e^3$ conv

14) $\sum_{n=1}^{\infty} \frac{n \ln n}{2^n}$

$\lim_{n\to\infty} \frac{(n+1)\ln(n+1)}{2^{n+1}} \cdot \frac{2^n}{n(\ln n)} = \lim_{n\to\infty} \frac{n+1}{2n} \cdot \frac{\ln(n+1)}{\ln(n)} = \frac{1}{2} \cdot 1 = \frac{1}{2} < 1$ conv

15) $\sum_{n=2}^{\infty} \frac{-2}{n(n+1)}$

$\frac{-2}{n(n+1)} = \frac{A}{n} + \frac{B}{n+1}$

$= \frac{-2}{n} + \frac{2}{n+1}$

$\sum_{n=2}^{\infty} \left(\frac{-2}{n} + \frac{2}{n+1}\right) = \left(-1 + \cancel{\frac{2}{3}}\right) + \left(\cancel{-\frac{2}{3}} + \frac{1}{2}\right)$

$\lim_{n\to\infty} S_n = \lim_{n\to\infty} \left[-1 + \frac{2}{n+1}\right] = -1$

$$\sum_{n=1} \frac{(-2)^n}{3^n+1}$$

$$b_n = \frac{(-2)^n}{(3)^n} = \left(-\frac{2}{3}\right)^n$$

$$\sum_{n=1}^{\infty} \left(\frac{-2}{3}\right)^n \text{ conv}$$

$$r = \frac{-2}{3} \in (-1, 1)$$

$$\lim_{n\to\infty} \frac{(-2)^n}{3^n+1} \cdot \frac{3^n}{-2^n} = \lim_{n\to\infty} \frac{3^n}{3^n+1}$$

$$= \lim_{n\to\infty} \frac{3^n \ln 3}{3^n \ln 3} = 1$$

$$\therefore \sum_{n=1}^{\infty} \frac{(-2)^n}{3^n+1} \text{ conv Abs}$$

$$\sum_{n=1}^{\infty} \frac{1}{3^n+1} \qquad \frac{1}{3^n+1} < \frac{1}{3^n}$$

17) $\sum_{n=1}^{\infty} \frac{e^n}{e^n+1}$

$$\lim_{n\to\infty} \frac{e^n}{e^n} = 1 \neq 0 \text{ div by } n^{th} \text{ term test}$$

18) $f(x) = \ln(1+x^2)$

$$f(x) = \ln x = \sum_{n=1}^{\infty} (-1)^{n-1} \frac{x^n}{n} = x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4}$$

$$\sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{n} x^{2n}$$

19) $\sum_{n=1}^{\infty} \frac{\sqrt{n}}{n^2+1}$

$$\lim_{n\to\infty} \frac{\sqrt{n}}{n^2+1} \cdot x^{\frac{3}{2}} = 1$$

$$\sum_{n=1}^{\infty} \frac{1}{x^{\frac{3}{2}}} \text{ conv}$$

conv LCT

20) $\sum_{n=1}^{\infty} (-1)^n \frac{n^2+1}{n^2+5}$

$$\lim_{n\to\infty} U_n = \lim_{n\to\infty} \frac{n^2+1}{n^2+5} = 1 \neq 0 \quad \text{div by } n^{th} \text{ term test}$$

Question 2 :

$$\sum_{n=1}^{\infty} (-1)^{n+1} \frac{1+n}{n^2}$$

$\lim_{n\to\infty} U_n = \lim_{n\to\infty} \frac{1+n}{n^2} = 0$ conv cond

$$\sum_{n=1}^{\infty} \frac{1+n}{n^2}$$

$\lim_{n\to\infty} \frac{1+n}{n^2} \cdot \frac{n}{1} = 1$

$\sum_{n=1}^{\infty} \frac{1}{n}$ div

Question 3 :

$$\ln(1+x) = \sum_{n=1}^{\infty} \frac{(-1)^{n+1} x^n}{n}$$

$f(x) = \frac{1}{(1+x)^2}$

Question 4: $\sum_{n=1}^{\infty} (-1)^{n+1} \frac{(x+2)^n}{n\,2^n}$

$$\lim_{n\to\infty} \frac{(x+2)^{n+1}}{(n+1)\,2^{n+1}} \cdot \frac{n\,2^n}{(x+2)^n} = \frac{|x+2|}{2} \lim_{n\to\infty} \frac{n}{n+1} = \frac{|x+2|}{2} < 1$$

$$-1 < \frac{x+2}{2} < 1$$

$$-2 < x+2 < 2$$

$$-4 < x < 0 \quad \text{conv Abs}$$

$x=-4 \to \sum_{n=1}^{\infty} (-1)^{n+1} \frac{-2^n}{n\,2^n}$

$= \sum_{n=1}^{\infty} \frac{(-1)^n(-1)\,-2^n}{n\,2^n} = \sum_{n=1}^{\infty} (-1)^{2n+1} \frac{1}{n} \to$ ~~conv~~ $\sum_{n=1}^{\infty} -\frac{1}{n} \to$ div

$x=0 \to \sum_{n=1}^{\infty} (-1)^{n+1} \frac{1}{n} \to$ conv

IC conv $\to -4 < x \le 0$ conv cond $\to x = 0$

Radius conv $\to \frac{0+4}{2} = 2$

center $= \frac{0-4}{2} = -2$