11.1 Parametrizations of Plane Curves

Definition: If x and y are given as functions

$$x = f(t), \quad y = g(t)$$

over an interval I of t-values, then the set of points $(x, y) = (f(t), g(t))$ defined by these equations is a parametric curve. The equations are parametric equations for the curve

- The variable t is a parameter for the curve.
- I is the parameter interval.
- If I is a closed interval $[a, b]$, $(f(a), g(a))$ is the initial point and $(f(b), g(b))$ is the terminal point

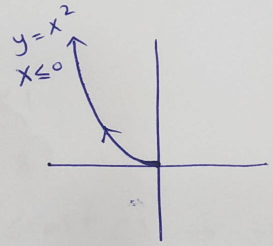
Questions: 2, 6, 10, 14, 15, 18, 20, 22, 26

Pages: 634 - 635

Identify the particle's path by finding a Cartesian equation ~~for~~. Graph the Cartesian equation. Indicate the portion of the graph traced by the particle and the direction of motion.

$\boxed{2}$ $x = -\sqrt{t}$, $y = t$, $t \geq 0$

$x = -\sqrt{y} \rightarrow y = x^2$ but $y \geq 0$, $x \leq 0$



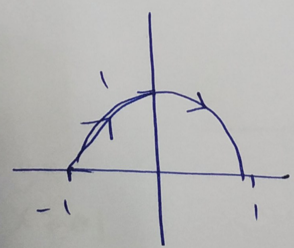
$\boxed{6}$ $x = \cos(\pi - t)$, $y = \sin(\pi - t)$, $0 \leq t \leq \pi$

$x^2 + y^2 = \cos^2(\pi - t) + \sin^2(\pi - t) = 1$

$y \geq 0$

$\sin(\pi - t) \geq 0$

for $0 \leq t \leq \pi$



$\boxed{10}$ $x = 1 + \sin t$, $y = \cos t - 2$, $0 \le t \le \pi$

$$\sin^2 t = (x-1)^2 \qquad \cos^2 t = (y+2)^2$$

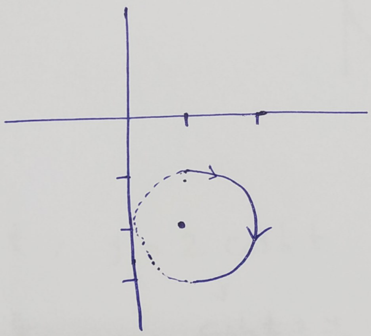
$$\sin^2 t + \cos^2 t = (x-1)^2 + (y+2)^2 = 1$$

a circle with center $(1,-2)$ and a radius of 1

Note that for $0 \le t \le \pi$

$x \geq 0$

$y < 0$



$0 \le \sin t \le 1$

$1 \le x \le 2$

and $-1 \le \cos t \le 1 \rightarrow -3 \le y \le -1$

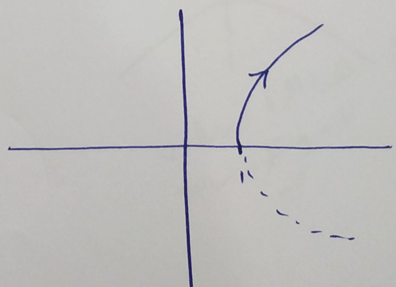
$\boxed{14}$ $x = \sqrt{t+1}$, $y = \sqrt{t}$, $t \geq 0$

$$y^2 = t, \quad y \geq 0$$

$$x = \sqrt{y^2 + 1}$$

$$x^2 = y^2 + 1$$

$$x^2 - y^2 = 1$$



a hyperbola with $y \geq 0$

$x \geq 1$

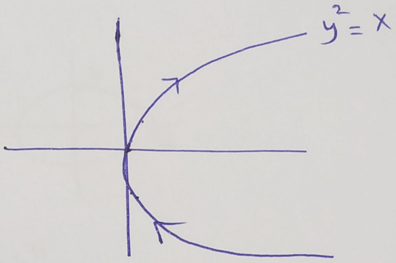
$\boxed{15}$ $x = \sec^2 t - 1$, $y = \tan t$, $\frac{-\pi}{2} < t < \frac{\pi}{2}$

$\downarrow$

$y^2 = \tan^2 t$

$x = \sec^2 t - 1 = \tan^2 t = y^2$

$x = y^2$



$\boxed{18}$ $x = 2 \sinh t$, $y = 2 \cosh t$, $-\infty < t < \infty$

$\downarrow$ $\sinh t = \frac{x}{2}$ $\downarrow$ $\cosh t = \frac{y}{2}$, note that

~~[illegible]~~ $y > 0$

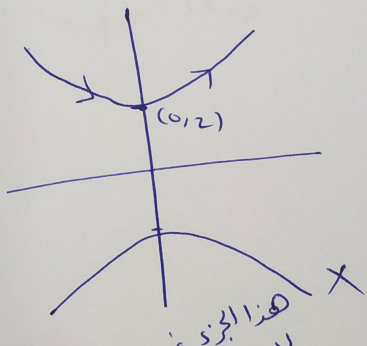
$\cosh^2 t - \sinh^2 t = 1$

$\left(\frac{y}{2}\right)^2 - \left(\frac{x}{2}\right)^2 = 1$

$\frac{y^2}{4} - \frac{x^2}{4} = 1$

$y^2 - x^2 = 1$

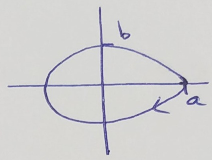
hyperbola



هذا الجزء غير مطلوب
لانه y تكون سالبة
فيه

$\boxed{20}$ Find parametric equations and parameter interval for the motion of a particle that starts at $(a, 0)$ and traces the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$

a) once clockwise

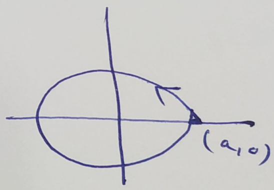


$x = a \sin t$

$y = b \cos t \qquad \frac{\pi}{2} \le t \le \frac{5\pi}{2}$

$$\left.\begin{array}{l} a \sin t = a \rightarrow \sin t = 1 \\ b \cos t = 0 \rightarrow \text{cost } t = 1 \end{array}\right\} t = \frac{\pi}{2}$$

b) once counterclockwise



$x = a \cos t \quad , \quad y = b \sin t$

$$\left.\begin{array}{l} a \cos t = a \rightarrow \text{cost } t = 1 \\ b \sin t = 0 \rightarrow \sin t = 0 \end{array}\right\} t = 0$$

$0 \le t \le 2\pi$

c) twice clockwise :-

$x = a \sin t \quad , \quad y = b \cos t \qquad \frac{\pi}{2} \leq t \leq \frac{9\pi}{2}$

d) twice counterclockwise

$x = a \cos t \quad , \quad y = b \sin t \qquad 0 \leq t \leq 4\pi$

$\boxed{22}$ Find a parametrization of the curve :-

the line segment with endpoints $(-1, 3)$ and $(3, -2)$

using $(-1, 3)$

$x = 3 + at$

$y = -2 + bt$

$\rightarrow \; -1 = 3 + at$

$(-1, 3)$

$$\left.\begin{aligned} x &= -1 + at \\ y &= 3 + bt \end{aligned}\right\} \quad (-1, 3) \text{ at } t = 0$$

$(3, -2)$, $t = 1$

$3 = -1 + a(1) \rightarrow a = 4$

$-2 = 3 + b(1) \rightarrow b = -5$

$x = -1 + 4t$

$y = 3 + -5t$

$\boxed{26}$ Find a parametrization for the ray with initial point $(-1, 2)$ that passes through the point $(0, 0)$

$$\left.\begin{aligned} x &= -1 + at \\ y &= 2 + bt \end{aligned}\right\} \quad t = 0$$

for $(0, 0)$ assume $t = 2$

$0 = -1 + a(2) \rightarrow a = \frac{1}{2}$

$0 = 2 + 2(b) \rightarrow b = -1$

$$\begin{cases} x = -1 + \frac{1}{2}t \\ y = 2 - t \end{cases}, \quad t \geq 0$$