

11.4 Graphing in Polar Coordinates

* Symmetry Tests for Polar Graphs.

a) Symmetry about x-axis:-

if (r, θ) on the graph, then $(r, -\theta)$ or $(+r, \pi - \theta)$ on the graph

b) Symmetry about the y-axis:-

if (r, θ) on the graph, then $(r, \pi - \theta)$ or $(-r, -\theta)$ on the graph

c) Symmetry about the origin:

if (r, θ) on the graph, then $(-r, \theta)$ or $(r, \theta + \pi)$ lies on the graph

* slope:-

the slope of the curve $r = f(\theta)$ is

$$\frac{dy}{dx} \Big|_{(r, \theta)} = \frac{f'(\theta) \sin \theta + f(\theta) \cos \theta}{f'(\theta) \cos \theta - f(\theta) \sin \theta}$$

$$\frac{dy}{dx} \Big|_{(r, \theta_0)} = \tan \theta_0$$

Identify the symmetric of the curves. Then sketch the curves.

$$\boxed{1} \quad r = 1 + \cos \theta$$

$$1 + \cos(-\theta) = 1 + \cos \theta = r$$

$$\uparrow$$

$$(r, -\theta)$$

$$\uparrow$$

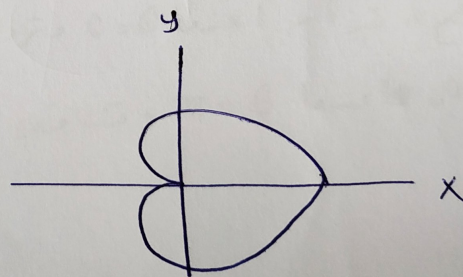
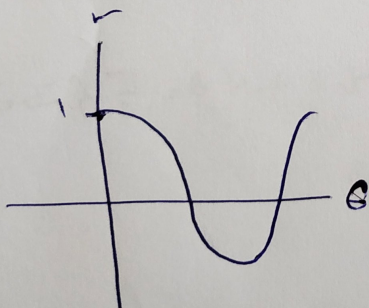
$$(r, \theta)$$

symmetric about the x-axis

$$r \neq 1 + \cos(-\theta)$$

$$r \neq 1 + \cos(\pi - \theta)$$

not symmetric about the y-axis



فقط r تبدأ من 2 وتتناقص حتى 0 ثم تبدأ تتزايد حتى 2

$$\boxed{6} \quad r = 1 + 2 \sin \theta$$

$$1 + 2 \sin \theta = 1 - 2 \sin \theta \neq r$$

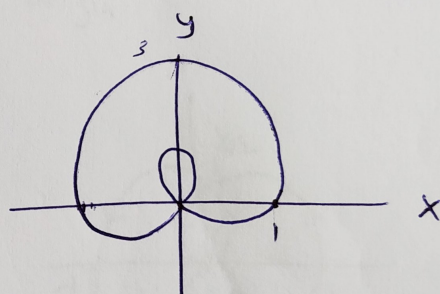
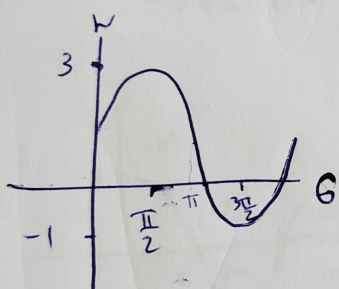
$$1 + 2 \sin(\pi - \theta) = 1 + 2 \sin \theta \neq -r$$

} not symmetric
about the
x-axis

$$1 + 2 \sin(\pi - \theta) = 1 + 2 \sin \theta = r$$

symmetric about the y-axis.

not symmetric about the origin



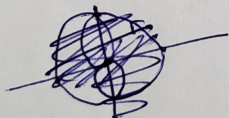
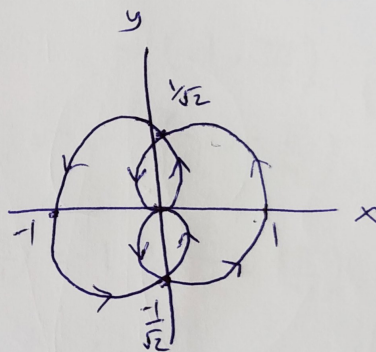
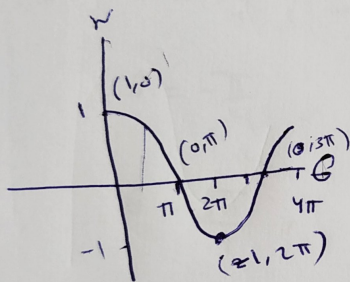
قیمت r تباراً ۱ عندما $\theta = 0$ و تزداد حتى تصل ۳ عندما $\theta = \frac{\pi}{2}$ ثم تنقص
حتى -1 عندما $\theta = \pi$ و تنقص حتى تصل ۰

8 $w = \cos\left(\frac{\theta}{2}\right)$

$\cos\left(-\frac{\theta}{2}\right) = \cos\left(\frac{\theta}{2}\right) = w \rightarrow$ symmetric about the x-axis

$\cos\left(\pi - \frac{\theta}{2}\right) = \cos\left(\frac{2\pi - \theta}{2}\right) \rightarrow$ symmetric about the y-axis.
 $= -\cos\left(\frac{\theta}{2}\right) = -w$

So symmetric about the origin



14) What symmetric do curve has?

$w^2 = 4 \sin 2\theta$

$4 \sin(2(-\theta)) = -4 \sin 2\theta \neq w^2$

$4 \sin(2(\pi - \theta)) = 4 \sin(2\pi - 2\theta) = 4 \left(\sin 2\pi \cos(-2\theta) + \cos(2\pi) \sin(-2\theta) \right)$

$= -4 \sin 2\theta \neq w^2$

not symmetric about the x or y axis

$$(-r)^2 = r^2 = 4 \sin 2\theta$$

the graph is symmetric about the origin

so it is not symmetric about the y-axis

19 Find the slope of the curve

$$r = \sin 2\theta, \text{ at } \theta = \pm \frac{\pi}{4}, \pm \frac{3\pi}{4}$$

$$\text{at } \theta = \frac{\pi}{4}, r = \sin 2\left(\frac{\pi}{4}\right) = 1 \rightarrow \left(1, \frac{\pi}{4}\right)$$

$$\theta = -\frac{\pi}{4}, r = \sin 2\left(-\frac{\pi}{4}\right) = -1 \rightarrow \left(-1, -\frac{\pi}{4}\right)$$

$$\theta = \frac{3\pi}{4}, r = \sin 2\left(\frac{3\pi}{4}\right) = -1 \rightarrow \left(-1, \frac{3\pi}{4}\right)$$

$$\theta = -\frac{3\pi}{4}, r = 1 \rightarrow \left(1, -\frac{3\pi}{4}\right)$$

$$r = \sin 2\theta = f(\theta)$$

$$f'(\theta) = 2 \cos 2\theta$$

$$\text{slope} = \frac{f'(\theta) \sin \theta + f(\theta) \cos \theta}{f'(\theta) \cos \theta - f(\theta) \sin \theta}$$

$$= \frac{2 \cos 2\theta \sin \theta + \sin 2\theta \cos \theta}{2 \cos 2\theta \cos \theta - \sin 2\theta \sin \theta}$$

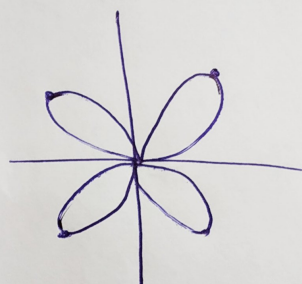
$$\text{slope at } \theta = \frac{\pi}{4} : \frac{2 \cos(\frac{\pi}{2}) \sin(\frac{\pi}{4}) + \sin(\frac{\pi}{2}) \cos(\frac{\pi}{4})}{2 \cos(\frac{\pi}{2}) \cos(\frac{\pi}{4}) - \sin(\frac{\pi}{2}) \sin(\frac{\pi}{4})}$$

$$= -1$$

$$\text{slope at } \theta = \frac{-\pi}{4} : 1$$

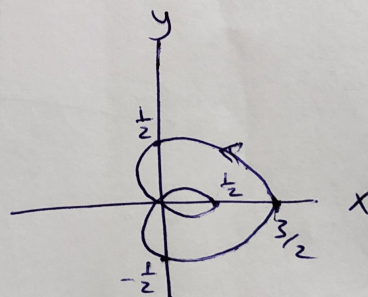
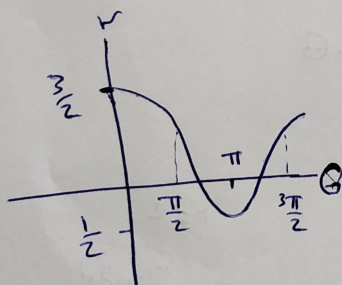
$$\text{slope at } \theta = \frac{3\pi}{4} : 1$$

$$\text{slope at } \theta = \frac{-3\pi}{4} : -1$$



21 Graph

a) $r = \frac{1}{2} + \cos \theta$



b) $r = \frac{1}{2} + \sin \theta$

