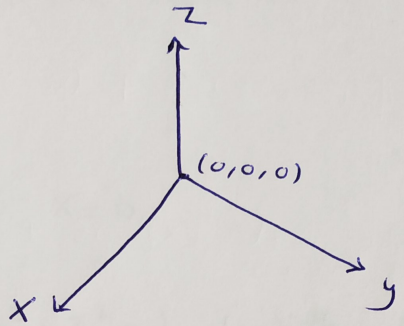
12.1 Three-Dimensional Coordinate Systems.



* The distance between $P_1(x_1, y_1, z_1)$ and $P_2(x_2, y_2, z_2)$ is $|P_1P_2| = \sqrt{(x_2-x_1)^2 + (y_2-y_1)^2 + (z_2-z_1)^2}$

* The standard equation of the sphere of radius a and center (x_0, y_0, z_0) is

$$(x - x_0)^2 + (y - y_0)^2 + (z - z_0)^2 = a^2$$

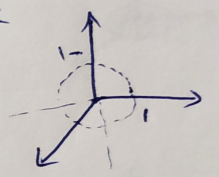
Questions: 8, 12, 14, 20, 26, 30, 36, 43

56, 64, 65.

Give a geometric description of the set of points in space whose coordinates satisfy the following:-

[8] $y^2 + z^2 = 1$, $x = 0$

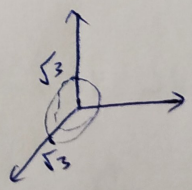
The circle $y^2 + z^2 = 1$ (with center $(0,0,0)$ and radius of 1) in the zy-plane



[12] $x^2 + (y-1)^2 + z^2 = 4$, $y = 0$

$x^2 + (0-1)^2 + z^2 = 4 \rightarrow x^2 + z^2 = 3$

The circle $x^2 + z^2 = 3$ (with center $(0,0,0)$ and radius of $\sqrt{3}$) in the xz-plane



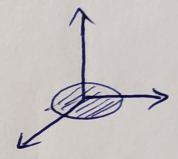
[14] $x^2 + y^2 + z^2 = 4$, $y = x$

a circle (with center $(0,0,0)$ and radius of 2) formed by the intersection of the sphere $x^2 + y^2 + z^2 = 4$ and the plane $y = x$

20 Describe the sets of points in space whose coordinates satisfy the following:-

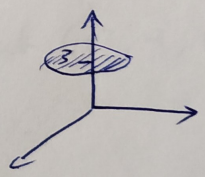
a) $x^2+y^2 \leq 1$, $z=0$

The disk (circumference and interior of the circle) $x^2+y^2 \leq 1$ in the xy-plane.



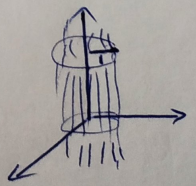
b) $x^2+y^2 \leq 1$, $z=3$

The disk (circumference and interior of $x^2+y^2=1$) in the plane $z=3$.



c) $x^2+y^2 \leq 1$, no restriction on z

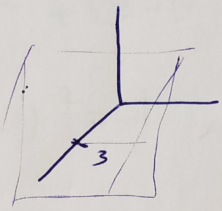
a solid cylindrical column of radius 1 whose axis is the z-axis



26 The plane through the point $(3, -1, 2)$ Perpendicular to the :-

a) x-axis

$x = 3$



b) y-axis

$y = -1$

c) z-axis :- $z = 2$

30 The circle of radius 1 centered at $(-3, 4,$ and lying in a plane parallel to the :-

a) xy-plane :- $z = 1$

$$(x+3)^2 + (y-4)^2 + (z-1)^2 = 1$$

$$(x+3)^2 + (y-4)^2 + (1-1)^2 = 1$$

$$(x+3)^2 + (y-4)^2 = 1 \quad , \quad z = 1$$

b) yz-plane $\qquad x = -3$

$$(x+3)^2 + (y-4)^2 + (z-1)^2 = 1$$

$$(-3+3)^2 + (y-4)^2 + (z-1)^2 = 1$$

$$(y-4)^2 + (z-1)^2 = 1, \quad x = -3$$

c) xz-plane :- $\qquad y = 4$

$$(x+3)^2 + (y-4)^2 + (z-1)^2 = 1$$

$$(x+3)^2 + (z-1)^2 = 1, \quad y = 4$$

36 Write inequality to describe :

The solid cube in the first octant bounded by the coordinate planes and the planes $x = 2$, $y = 2$ and $z = 2$

$$0 \le x \le 2, \quad 0 \le y \le 2, \quad 0 \le z \le 2.$$

43) Find the distance between points

$P_1(1,4,5)$, $P_2(4,-2,7)$

$$|P_1P_2| = \sqrt{(4-1)^2+(-2-4)^2+(5-7)^2}$$

$$= \sqrt{9+36+4} = 7$$

56) Find the center and the radius of the sphere

$$x^2+y^2+z^2-6y+8z=0$$

$$x^2+(y^2-6y)+(z^2+8z)=0$$

$$x^2+\left(y^2-6y+\left(\frac{-6}{2}\right)^2\right)+\left(z^2+8z+\left(\frac{8}{2}\right)^2\right)=0+\left(\frac{-6}{2}\right)^2+\left(\frac{8}{2}\right)^2$$

$$x^2+(y^2-6y+9)+(z^2+8z+16)=9+16$$

$$x^2+(y-3)^2+(z+4)^2=25$$

Center : $(0,+3,-4)$, radius : 5

64 Find the equation for the set of points equidistant from the point $P_1(0,0,2)$ and the xy-plane

we need to find the point $P_2(x,y,z)$

we take a point $P_3(x,y,0)$ on the xy-plane

$$|P_1P_2| = |P_2P_3|$$

$$\sqrt{(x-0)^2+(y-0)^2+(z-2)^2} = \sqrt{(x-x)^2+(y-y)^2+(z-0)^2}$$

$$\sqrt{x^2+y^2+z^2-4z+4} = \sqrt{z^2}$$

$$x^2+y^2+z^2-4z+4 = z^2$$

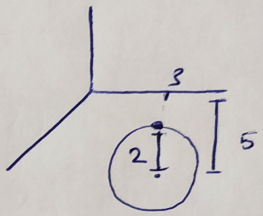
$$x^2+y^2-4z+4=0$$

$$\frac{x^2}{4}+\frac{y^2}{4}+1=z$$

65 Find the point on the sphere $x^2+(y-3)^2+(z+5)^2=4$ nearest

a) the xy - plane

center $(0,3,-5)$, radius 2



the nearest point is the point at the top of the sphere

$(0, 3, -3)$

b) the point $(0,7,-5)$

$(0, 5, -5)$