10.10 : The Binomial series

Def :- for $-1 < x < 1$

$$(1+x)^m = 1 + mx + \frac{m(m-1)x^2}{2!} + \cdots \frac{m(m-1)(m-2)\cdots(m-k+1)}{k!}$$

$$= 1 + \sum_{k=1}^{\infty} \binom{m}{k} x^k$$

$$\binom{m}{k} = \frac{m!}{k!\,(m-k)!}$$

Applications of Taylor series

- You can use it to estimate unknown integrals

s.a :-

$$\int \sin(x^2)\,dx$$

$$\sin(x) = x - \frac{x^3}{3!} + \cdots$$

$$\sin(x^2) = x^2 - \frac{x^6}{3!} \cdots$$

$$\int \sin(x^2) = \frac{x^3}{3} - \frac{x^7}{7 \cdot 3!} \cdots$$

- You can use it to Evaluate limits s.a :-

$$\lim_{x \to 0} \frac{e^x - e^{-x}}{x}$$

$$\lim_{x \to 0} \frac{\left(1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \cdots\right) - \left(1 - x + \frac{x^2}{2!} - \frac{x^3}{3!} + \cdots\right)}{x}$$

$$= \lim_{x \to 0} \frac{2x + \frac{x^3}{3} + \cdots}{x}$$

$$= \lim_{x \to 0} \frac{x\left(2 + \frac{x^2}{3} + \cdots\right)}{x} = 2$$

where $\frac{x^2}{3} \to 0$

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- You can use it in finding the sum of the series

$$\frac{\pi}{3} - \frac{\pi^3}{3^3 \cdot 3!} + \frac{\pi^5}{3^5 \cdot 5!} + \text{———}$$

$$\hookrightarrow \left(\frac{\pi}{3}\right) - \left(\frac{\pi}{3}\right)^3 \frac{1}{3!} + \left(\frac{\pi}{3}\right)^5 \frac{1}{5!} + \text{———}$$

If we consider $\frac{\pi}{3} = x$

$$\hookrightarrow x - \frac{x^3}{3!} + \frac{x^5}{5!} \text{———}$$

This is the T.S of $\sin x$

So $= \sin x = \sin \frac{\pi}{3} = \frac{\sqrt{3}}{2}$

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