Power series

Ex : $\sum_{n=0}^{\infty} x^n = 1 + x + x^2 + \cdots$

- General form of power series with center a :

$$\sum_{n=0}^{\infty} C_n (x-a)^n$$

*a is the center

- Convergance of this series has 3 cases :- R: Radius of convergence

[1] exist R such that $\sum C_n (x-a)^n$ converges abs for

$|x-a| < R \Rightarrow [\underset{+a}{-R} < \underset{+a}{x-a} < \underset{+a}{R}] \Rightarrow [a-R < x < a+R]$

and diverges for $|x-a| > R \Rightarrow [\, x-a > R$ or $x-a < -R \,]$

The series may or may not conv at End points

End Points are :- $x = a+R$, $x = a-R$

or [2] The series converges abs for all x : $R = \infty$, $-\infty < x < \infty$

or [3] The series converges only at $x = a$

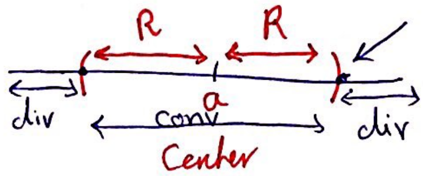
Note: It's always Conv at $x = a$

- How to solve Problems like this?

1- you find the series of Abs. val / Then you test if it converges or not

2- If it Conv then The series Conv. abs for an interval such that

End Points



3- you Check the End Points by putting Them in the series instead of x and finding wheather the series Converges or div at them

4- You summurize the intervals of convergance & divergence

Power series (Completation of 10.7)

Operations on power series

Multiplication

- When you multiply two series the Radius of Convergance stays the same

Differentiation

- If you have a series (such as $\sum x^n$) converges for a specefic interval (here at $-1 < x < 1$) Then

$$\sum x^n = 1 + x + \cdots = \frac{1}{1-x} \rightsquigarrow f(x) \quad \text{at } -1 < x < 1$$

and if you differentiate it you get a new series and a new function and the Radius of conv stays the same

$$f'(x) = \frac{-1}{(1-x)^2} = 1 + 2x + 3x^2 + \cdots \quad \text{at } -1 < x < 1$$

integration

The only difference here is that you have to find C (The integration constant) :-

Ex $\int \frac{1}{1-x} = \int 1 + x + x^2 \cdots$

$$= -\ln|1-x| = x + \frac{x^2}{2} + \frac{x^3}{3} \cdots + \frac{x^{n+1}}{n+1} + C$$

you find a value of x to find C

here we use $x=0$

$0 = 0 + C$ So $\Rightarrow -\ln|1-x| = x + \frac{x^2}{2} + \frac{x^3}{3} \cdots$ at $-1 < x < 1$.

$C = 0$

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