Chapter II :- Parametric Curves

Def:- A parametric Curve is given as $x = f(t), y = g(t)$
$t \in I$

- Point $(x, y) = (f(t), g(t))$

How to Solve Exercises

- To find Cartesian Equations : Ex 4:- $x = 3 - 3t$, $y = 2t$, $0 \le t \le 1$

$x = 3 - 3t$ $\qquad$ $y = 2t$

→ you have to find a relation between x and y with no t in it.

$x = 3 - 3t$, $\frac{y}{2} = t$ $\Rightarrow$ $\boxed{x = 3 - \frac{3y}{2}}$ Now you have to draw this

$x - 3 = -\frac{3y}{2}$

$2x - 6 = -3y$

$\frac{2x - 6}{-3} = y \Rightarrow y = \frac{6 - 2x}{3}$

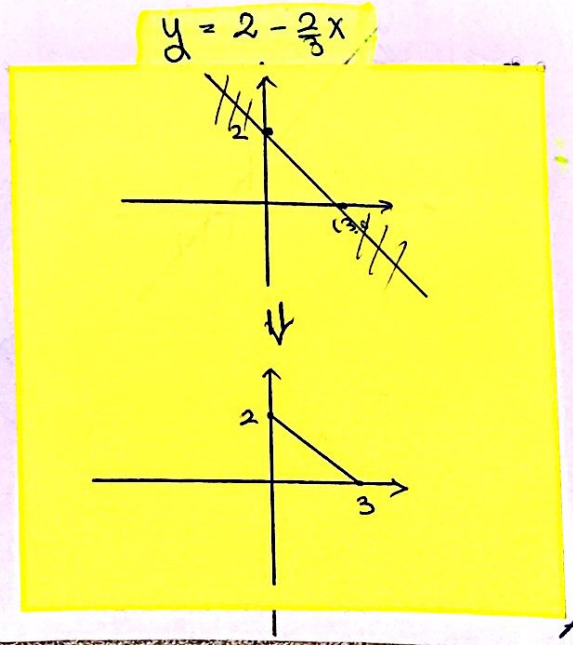
But $0 \le t \le 1$

Starting point at $t = 0$

at $t = 0$ $\quad x = 3$, $y = 0$

at $t = 1$ $\quad x = 0$, $y = 2$ ← Terminating Point

$y = 2 - \frac{2}{3}x$



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• Finding Parametric Equations

$x = f(t)$, $y = g(t)$ → are parametric equations

Ex 22 :- find a parametrization for the curve : line segment with endpoints $(-1, 3)$ and $(3, -2)$

1- • Since it's a line you can use line equation :-

$$y - b = m(x - a)$$

First :- set $\boxed{x = t + a}$

Now $y - b = m(t + a - a)$

Then :- $\boxed{y = mt + b}$

2- • find slope :- $m = \frac{-2-3}{3-(-1)} = \frac{-5}{4}$

using the two points given

$$x = t + a$$
$$y = -\frac{5}{4}t + b \qquad -\infty < t < \infty$$

3- • substituting point $(-1, 3)$ to know starting Point

$x = t - 1$

$y = -\frac{5}{4}t + 3$ ⇝ at $t = 0$ is the starting point

4- • Now to find the ending point let $(x, y) = (3, -2)$

$3 = t - 1 \Rightarrow t = 4$

$-2 = -\frac{5}{4}t + 3$

$-5 = -\frac{5}{4}t$

$\Rightarrow t = 4$ ⟹ at $t = 4$ is the ending point

5- So the parametrization is :- $x = t - 1$, $y = -\frac{5}{4}t + 3$ at $0 \le t \le 4$

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