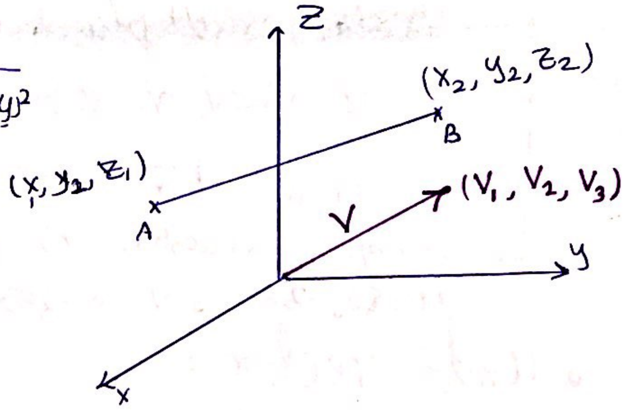
12.2 vectors



- $\overrightarrow{AB} \equiv V$ is a vector
- length $= |\overrightarrow{AB}| = \sqrt{(x_2-x_1)^2+(z_2-z_1)^2+(y_2-y_1)^2}$
  - A: initial point
  - B: terminal point

- standard position of $\vec{V}$
  - initial point at $(0,0,0)$
  - terminal point at (V_1, V_2, V_3)

- How to write vectors :-
  - component form $\langle V_1, V_2, V_3 \rangle$
  - length of the vector $\sqrt{(V_1)^2+(V_2)^2+(V_3)^2}$

- Midpoint M of $P_1(x_1, y_1, z_1)$ and $P_2(x_2, y_2, z_2)$ is :- $M = \left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}, \frac{z_1+z_2}{2}\right)$

- proprieties

1- Equality of vectors :-

$\vec{V} = \vec{W} \iff$ same length and Direction

2- Zero vector :- has no direction

$\vec{0} \Rightarrow |\vec{0}| = 0$

$\Rightarrow \langle 0, 0, 0 \rangle$

3-
- $\vec{u} + \vec{V} = \vec{V} + \vec{u}$
- $\vec{u} + 0 = \vec{u}$
- $a(\vec{u} + \vec{V}) = a\vec{u} + a\vec{V}$
- $(a+b)\vec{w} = a\vec{w} + b\vec{w}$

- where a and b are Constants

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- Operations on vectors

→ **Addition**

if $\vec{v}\langle v_1, v_2, v_3\rangle$, $\vec{u}\langle u_1, u_2, u_3\rangle$ Then

$\vec{V} \pm \vec{u} = \langle v_1 \pm u_1, v_2 \pm u_2, v_3 \pm u_3\rangle$

→ **Scalar Multiplication**

if $\vec{v}\langle v_1, v_2, v_3\rangle$, t is a Constant (scalar)

Then $t\vec{v} = \langle tv_1, tv_2, tv_3\rangle$

Example: Question 5 page 716:

$u=\langle 3,-2\rangle$, $v=\langle -2,5\rangle \Rightarrow 2u-3v = \langle 2\times 3 - 3\times -2), 2\times -2 - (-2\times 5\rangle$

$= \langle 12, 6\rangle$

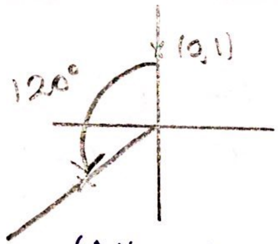
- Unit vector :-
  - Def: a vector that has length of 1
  - if $\vec{u}$ is a vector Then it's unit vector is $\dfrac{\vec{u}}{|\vec{u}|}$

has the same direction as $\vec{u}$

Example :- Question 15 page 716 (outline)

$(0,1) \xrightarrow[\text{Counterclockwise}]{120^\circ}$

$|V| = 1$



- The new vector is :-

$\left(\dfrac{-|V|\cos 30}{|V|}, \dfrac{-|V|\sin 30}{|V|}\right)$ or $(|V|\cos 210, |V|\sin 210)$

The unit vector is $= \left(\dfrac{-\frac{\sqrt{3}}{2}}{1}, \dfrac{\frac{1}{2}}{1}\right) = \left(-\frac{\sqrt{3}}{2}, \frac{1}{2}\right)$

Note:- You can express A vector $\vec{V}$ by it's length & direction using unit vector

$\vec{V} = \dfrac{\vec{V}}{|\vec{V}|}\,|\vec{V}|$

direction length

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