12.3 : The Dot Product

- Using the Dot product in finding the Angle between two vectors :

if $\vec{v}\langle v_1, v_2, v_3\rangle$ and $\vec{u}\langle u_1, u_2, u_3\rangle$ Then θ between them :-

$$\theta = \cos^{-1}\left(\frac{u_1v_1 + u_2v_2 + u_3v_3}{|u|\,|v|}\right)$$

The dot product *

- Using dot product to know wheather a two vectors are (orthogonal) peripendicular or not

→ if $u.v=0$ ~ Then u and v are peripendicular

proprieties

- $u.v = v.u$
- $u.(v+w) = u.v + u.w$
- $0.u = 0$
- $u.u = |u|^2$

- How to find θ :- *

Example :

Question 5 page 680 (outline)

$v = 5\hat{j} - 3\hat{k}$, $u = \hat{i} + \hat{j} + \hat{k}$

a) find $v.u = (0\times1) + (5\times1) + (-3\times1)$

$= 0 + 5 + -3 = 2$

$|v| = \sqrt{25+9} = \sqrt{34}$

$|u| = \sqrt{1+1+1} = \sqrt{3}$

b) find $\cos\theta = \dfrac{u.v}{|u||v|} = \dfrac{2}{\sqrt{34}\sqrt{3}} = \dfrac{2}{\sqrt{3\times34}} \approx 0.2$

The Rest of the Question (c, d) is in the second page

Alaa Etaimi

Projection

- The vector projection of u onto v is the vector

$$\mathrm{Proj}_v u = \left(\frac{u \cdot v}{|v|^2}\right) v$$

- The scalar component of u in the direction of v is the scalar

$$|u| \cos\theta = \frac{u \cdot v}{|v|} = u \cdot \frac{v}{|v|}$$

How to find projection ?

Continue → c- The scalar component of u in the direction of v

$$|u| = \sqrt{3}$$

$$\cos\theta = \frac{2}{\sqrt{3}\sqrt{34}}$$

So $|u| \cos\theta = \frac{2}{\sqrt{34}}$

d- $\mathrm{Proj}_v u = \left(\frac{u \cdot v}{|v|^2}\right) v$

$$= \left(\frac{2}{(\sqrt{34})^2}\right)(5\hat{j} - 3\hat{k})$$

$$= \frac{1}{17}(5\hat{j} - 3\hat{k})$$

- Example : Question 33 page 681 (outline)

$P(2,1)$ $\quad \vec{V} = \hat{i} + 2\hat{j}$

- find the equation of the line that passes through p and is peripenclicular to $\vec{V}$

Slope of $\vec{V} = \frac{2}{1} = 2$

so slope of the line $= -\frac{1}{2}$

equation :- $y - y_0 = m(x - x_0) \Rightarrow y - 1 = -\frac{1}{2}(x-2)$

$$\boxed{y = -\frac{x}{2} + 2}$$