8.7

improper integrals

• Types of improper integrals

Type I

• one or both of integration limits are infinit

→ if $f(x)$ is continuous on $[a,\infty)$

$$\boxed{\int_a^\infty f(x)\,dx = \lim_{b\to\infty}\int_a^b f(x)\,dx}$$

→ if $f(x)$ is continuous on $(-\infty, a]$

$$\boxed{\int_{-\infty}^a f(x)\,dx = \lim_{b\to-\infty}\int_b^a f(x)\,dx}$$

→ if $f(x)$ is continuous on $(-\infty,\infty)$ Then

$$\int_{-\infty}^\infty f(x)\,dx = \int_{-\infty}^c f(x)\,dx + \int_c^\infty f(x)\,dx$$

when $-\infty < c < \infty$

Then $= \boxed{\lim_{a\to-\infty}\int_a^c f(x)\,dx + \lim_{b\to\infty}\int_c^b f(x)\,dx}$

• if The lim = finit Then The integral converges

• if The lim = ∞ or DNE Then The integral diverges

Type II

• Dicontinuety in the function

① → if $f(x)$ is cont on $[a,b)$ and discont at $x=b$

Then:

$$\boxed{\int_a^b f(x)\,dx = \lim_{c\to b^-}\int_a^c f(x)\,dx}$$

② → if $f(x)$ is cont on $(a,b]$ and discont at $x=a$

Then:

$$\boxed{\int_a^b f(x)\,dx = \lim_{c\to a^+}\int_c^b f(x)\,dx}$$

→ if $f(x)$ is cont on $[a,c)\cup(c,b]$ & discont at $x=c$

Then:-

$$\int_a^b f(x) = \underbrace{\int_a^c f(x)\,dx}_{①} + \underbrace{\int_c^b f(x)\,dx}_{②}$$

$$= \lim_{d\to c^-}\int_a^d f(x)\,dx + \lim_{e\to c^+}\int_e^b f(x)\,dx$$

Alaa Eltaimi

How To Know if a function diverges or converges:-

1- • P-integrals

(The rules works only when $\int_a^{\infty} f(x)\,dx$)

$\int_1^{\infty} \frac{1}{x^P}\,dx$ is a P-integral where P can be: $\pm1, \pm2, \pm3 \ldots$

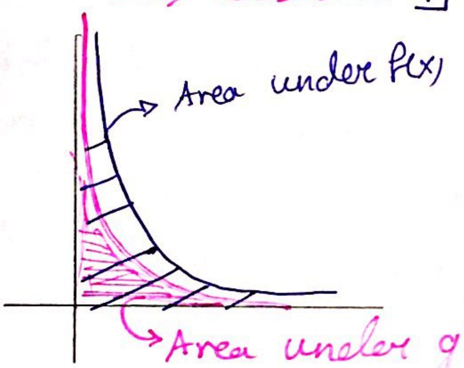
→ Cases:- → 1 ⇒ $P=1$ integral diverges

2 ⇒ $P>1$ integral converges To $\frac{1}{P-1}$

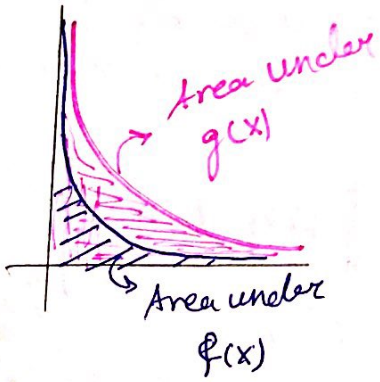
3 ⇒ $P<1$ integral diverges

2-• Direct Comparison Test

→ Cases:- [1]- If $f(x), g(x)$ is ≥ 0 on $[a,\infty)$ & $\int_a^{\infty} f(x)\,dx$ converges & $g(x) \leq f(x)$ where $a \leq x < \infty$ Then $\int_a^{\infty} g(x)\,dx$ also converges



[2]- If $f(x), g(x)$ is ≥ 0 on $[a,\infty)$ & $\int_a^{\infty} f(x)\,dx$ diverges and $g(x) \geq f(x)$ Where $a \leq x < \infty$ Then $\int_a^{\infty} g(x)\,dx$ also diverges



3-• limit comparison Test

cases:-

1 → If $f(x), g(x) \geq 0$ and $\lim_{x\to\infty} \frac{f(x)}{g(x)} = L$ $0 < L < \infty$

Then $\int_a^{\infty} f(x)\,dx$ and $\int_a^{\infty} g(x)\,dx$ both Converge or diverges

Haa Etaiwi

2-. If $\lim_{x \to \infty} \frac{f(x)}{g(x)} = \infty$ This means

→ $f(x) \ggg g(x)$ and If we Knew That $\int_0^\infty g(x)$ diverges Then $\int_0^\infty f(x)\,dx$ diverges

→ If we Knew That $\int_a^\infty f(x)$ converges Then $\int_0^\infty g(x)\,dx$ converg

3-. If $\lim_{x \to \infty} \frac{f(x)}{g(x)} = 0$ Then

→ If $\int_a^\infty g(x)\,dx$ conv Then $\int_a^\infty f(x)\,dx$ converges

→ If $\int_a^\infty f(x)\,dx$ diverges Then $\int_a^\infty g(x)\,dx$ diverges

Note : Comparison Tests can be used in integrals lik $\int_a^b$

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