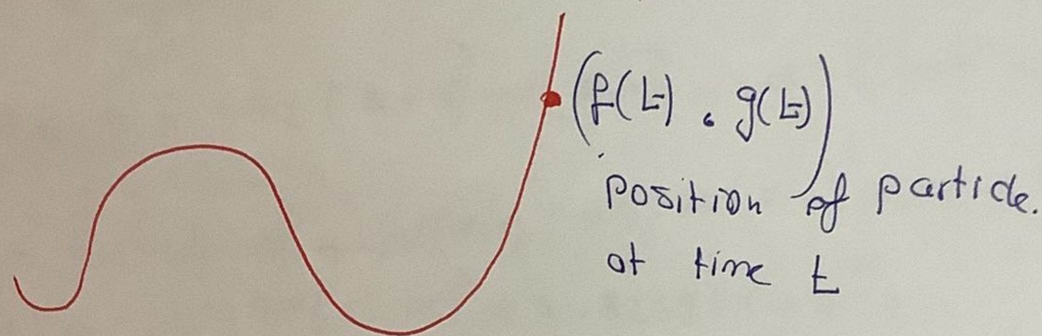
Ch 11 : Parametric Equations and Polar coordinates ①

11.1 Parametrization of Plane Curves

Parametric Equations



Definition:

If x and y are given as function

$$x = f(t), \quad y = g(t)$$

over an interval I of t-values, then the set of points $(x, y) = (f(t), g(t))$ defined by these equations is $\boxed{\text{a parametric curve}}$ and the equations are $\boxed{\text{parametric equations}}$ of the curve

t = parameter of the curve

I = parameter interval

$a \leq t \leq b \rightarrow$ $(f(a), g(a))$ initial point
$(f(b), g(b))$ terminal point

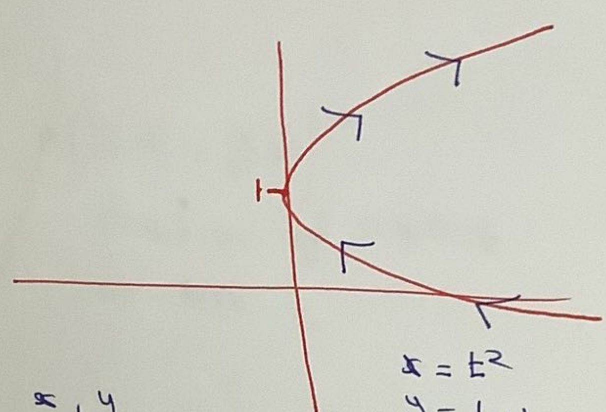
Example: sketch the curve defined by the parametric equations 3

$x = t^2$, $y = t+1$, $-\infty < t < \infty$

Solution

$x = t^2 = (y-1)^2$

so $x = (y-1)^2$



$x = t^2$
$y = t+1$
$-\infty < t < \infty$

or make a table

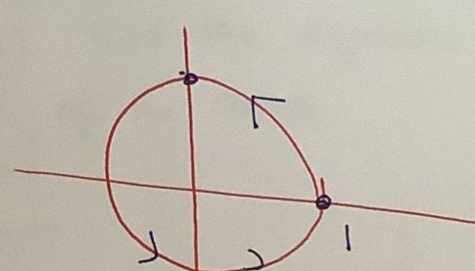
t	x	y

Ex| Graph
$x = \cos t$, $y = \sin t$ $0 \le t \le 2\pi$

$\cos^2 t + \sin^2 t = 1$

$x^2 + y^2 = 1$

unit circle



$t = 0 \longrightarrow x = 1 , y = 0$

$t = \frac{\pi}{2} \longrightarrow x = 0 , y = 1$

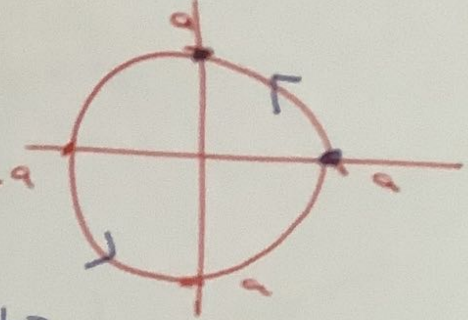
$t = 0 \longrightarrow x = 1 , y = 0$

Ex Graph $x = a\cos t \quad y = a\sin t \quad 0 \le t \le 2\pi$

Solution:

$$\cos^2 t + \sin^2 t = 1$$

$$\frac{x^2}{a^2} + \frac{y^2}{a^2} = 1$$



$$x^2 + y^2 = a^2$$

Circle $(0,0)$ $r = a$

$t = 0 \qquad x = a \qquad y = 0$

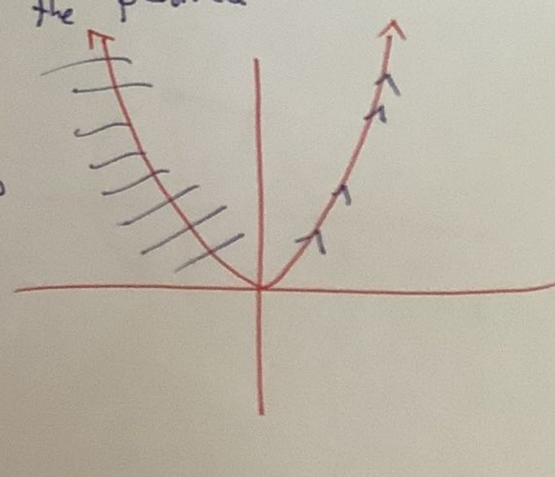
$t = \frac{\pi}{2} \qquad x = 0 \qquad y = a$

$t = 2\pi \qquad x = \cdot$

Example $x = \sqrt{t}$, $y = t$, $t \ge 0$

This is the position of a particle moving in the xy-plane.

Identify the path traced by the particle



Solution:

$$x = \sqrt{t} = \sqrt{y} \quad , t \geq 0$$

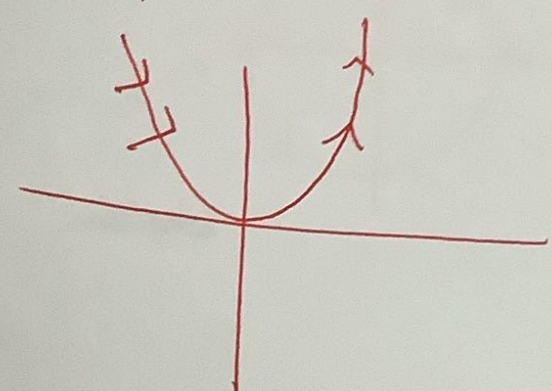
$$\boxed{x^2 = y}$$

$t = 0 \longrightarrow x = 0,\ y = 0$

$t = 1 \longrightarrow x = 1,\ y = 1$

Ex A parametrization of the graph of $f(x) = x^2$ is given by

$x = t \qquad y = f(t) = t^2, \qquad -\infty < t < \infty$



$x = t$

$y = t^2$

$y = x^2, \qquad -\infty < t < \infty$

Ex Find a parametrization of the line through the point (a, b) having slope m

Solution:

$y = m(x - a) + b$ (slope - point form)

$t = x - a \longrightarrow x = t + a$

$y = mt + b$

So the parametric equations are

$x = a + t \quad , \quad y = b + mt, \qquad -\infty < t < \infty$

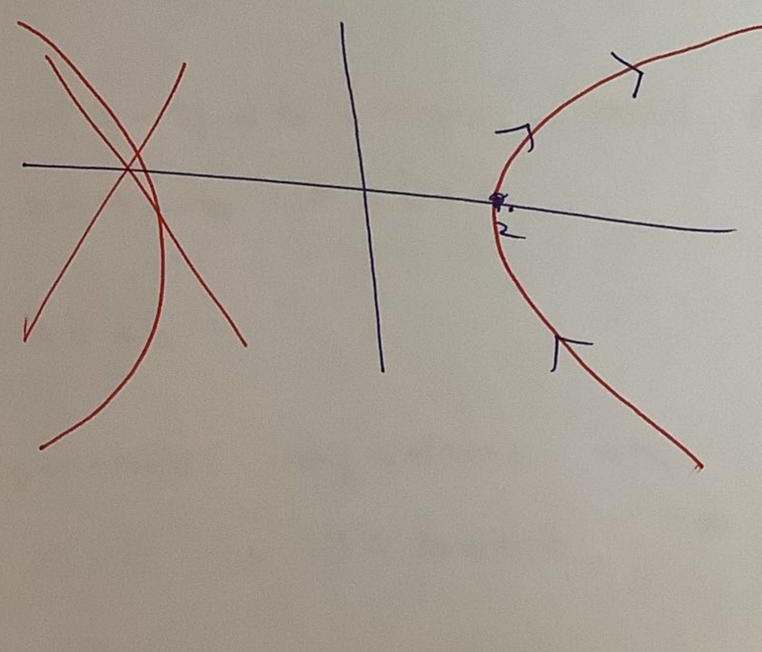
Ex) Sketch and Identify the path traced by the point $P(x, y)$ if

$$x = t + \frac{1}{t}, \quad y = t - \frac{1}{t}, \quad t > 0$$

$$x - y = \frac{2}{t}, \quad x + y = 2t$$

$$(x-y)(x+y) = 4$$

$$x^2 - y^2 = 4$$



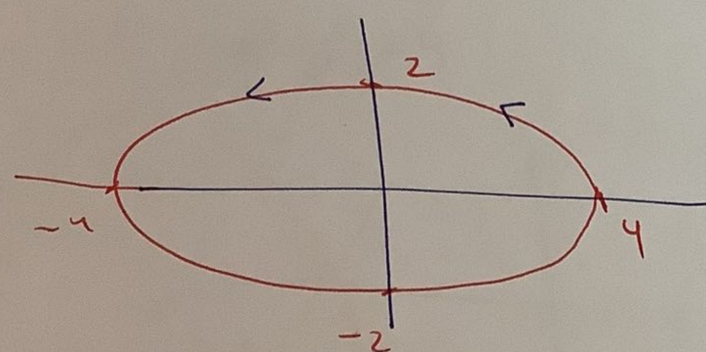
Exercise 7

7. $x = 4\cos t$, $y = 2\sin t$, $0 \le t \le 2\pi$

$\cos^2 t + \sin^2 t = 1$

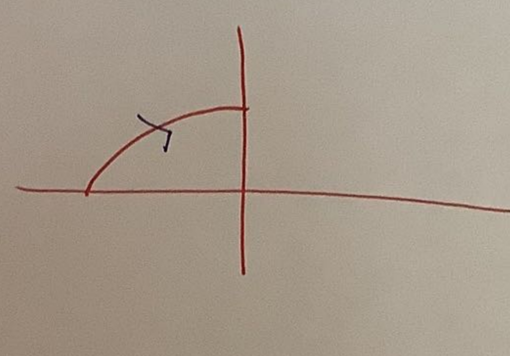
$\left(\frac{x}{4}\right)^2 + \left(\frac{y}{2}\right)^2 = 1$

$\frac{x^2}{16} + \frac{y^2}{4} = 1$ ellipse = قطع ناقص



13. $x = t$, $y = \sqrt{1 - t^2}$, $-1 \le t \le 0$

$y = \sqrt{1 - x^2}$, $-1 \le x \le 0$



(16) $x = -\sec t \qquad y = \tan t \qquad -\frac{\pi}{2} < t < \frac{\pi}{2}$ 8

$\cancel{\sec^2 t + \tan^2 t =}$.

$1 + \tan^2 t = \sec^2 t$

$1 + y^2 = x^2$

$y^2 = x^2 - 1$

$\cancel{y^2 - x^2 = 1}$

$x^2 - y^2 = 1$