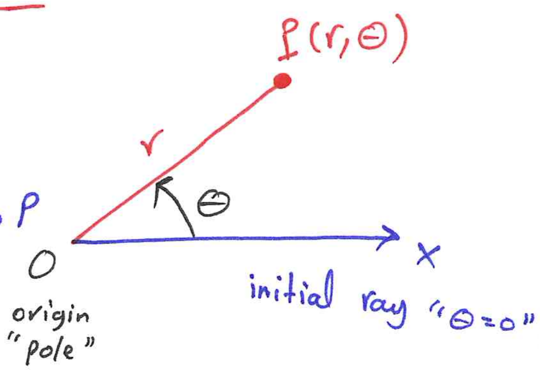
11.3 Polar Coordinates

$P(r, \theta)$ is Polar coordinate



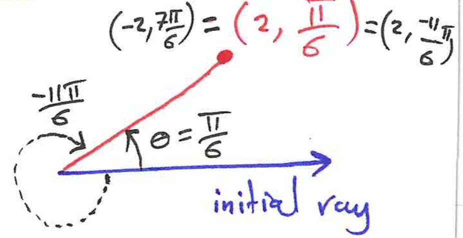
- r is the directed distance from O to P
  "r can be negative"
- θ is the directed angle from the initial ray to OP.

Polar coordinates are not unique:

$$P(r, \theta) = P(r, \theta + 2\pi m) \quad , \quad m = 0, \pm 1, \pm 2, \cdots$$

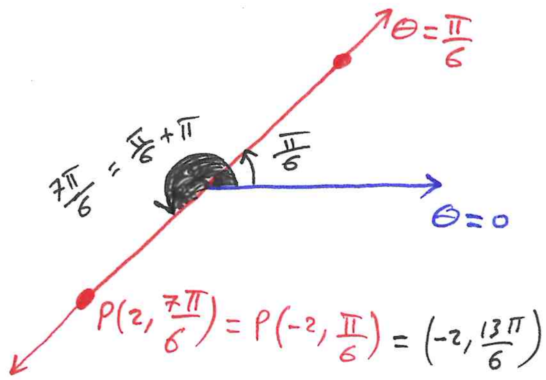
$$= P(-r, (\theta \pm \pi) + 2\pi m) = P(-r, \theta \pm (2m+1)\pi)$$

Exp Polar coordinate $P(2, \frac{\pi}{6})$



$$P(2, \tfrac{\pi}{6}) = (2, \tfrac{\pi}{6} - 2\pi) = (2, -\tfrac{11\pi}{6})$$

$$= (-2, \tfrac{\pi}{6} + \pi) = (-2, \tfrac{7\pi}{6})$$



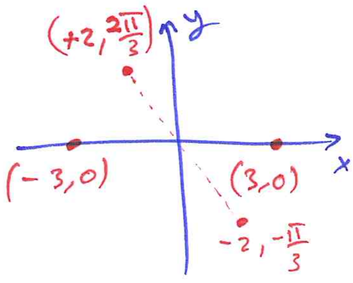
$$P(-2, \tfrac{\pi}{6}) = (-2, \tfrac{\pi}{6} + 2\pi) = (-2, \tfrac{13\pi}{6})$$

$$= (2, \tfrac{\pi}{6} + \pi) = (2, \tfrac{7\pi}{6})$$

Exp The following polar coordinates pair are the same:

$P(r, \theta) = P(3, 0) = P(-3, \pi)$, $(-3, 0) = (-3, 2\pi)$,

$(2, \frac{2\pi}{3}) = (-2, -\frac{\pi}{3})$ . . .



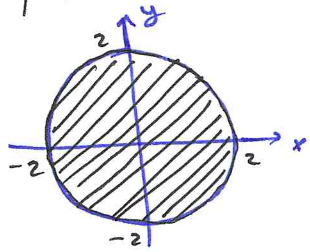
Polar Equations and Graphs:

$r = a$ circle of radius $|a|$ centered at O

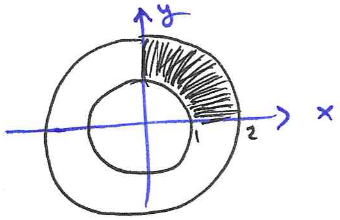
$\theta = \theta_0$ Line through O making an angle θ_0 with the initial ray.

Exp Graph the set of points whose polar coordinates satisfy:

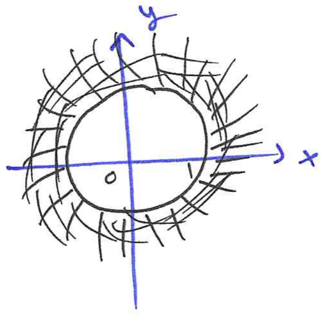
(a) $0 \le r \le 2$



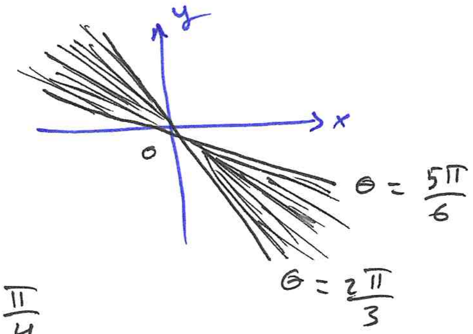
(b) $1 \le r \le 2$ and $0 \le \theta \le \frac{\pi}{2}$



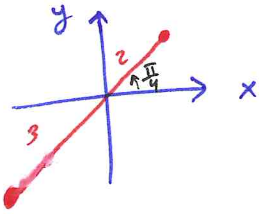
(c) $r \ge 1$



(d) $\frac{2\pi}{3} \le \theta \le \frac{5\pi}{6}$



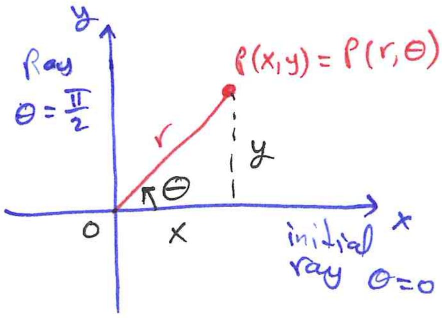
(e) $-3 \le r \le 2$ and $\theta = \frac{\pi}{4}$



Equations Relating Polar and Cartesian Coordinates:

$x = r\cos\theta$, $y = r\sin\theta$

$r^2 = x^2 + y^2$, $\tan\theta = \frac{y}{x}$



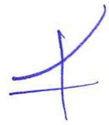
Exp Replace the polar equation with Cartisian equations:

a) $r\cos\theta = 2 \Rightarrow x = 2$ Vertical line passes (2,0)

b) $r = -3\sec\theta \Rightarrow r = \frac{-3}{\cos\theta} \Rightarrow r\cos\theta = -3 \Leftrightarrow x = -3$

c) $r^2 = 1 \Rightarrow x^2 + y^2 = 1$ circle center (0,0) and $r = 1$

d) $r = \csc\theta \, e^{r\cos\theta} \Rightarrow r\sin\theta = e^{r\cos\theta} \Rightarrow y = e^x$



Exp Replace the cartisian equation with equivalent polar eq.

a) $x = y \Rightarrow r\cos\theta = r\sin\theta \Rightarrow \cos\theta = \sin\theta \Rightarrow \theta = \frac{\pi}{4}$

b) $x^2 + y^2 = 4 \Rightarrow r^2 = 4 \Leftrightarrow r = \pm 2$

c) $(x-5)^2 + y^2 = 25$

$x^2 - 10x + 25 + y^2 = 25 \Leftrightarrow x^2 + y^2 = 10x$

$\Leftrightarrow r^2 = 10 r\cos\theta \Leftrightarrow r = 10\cos\theta$
