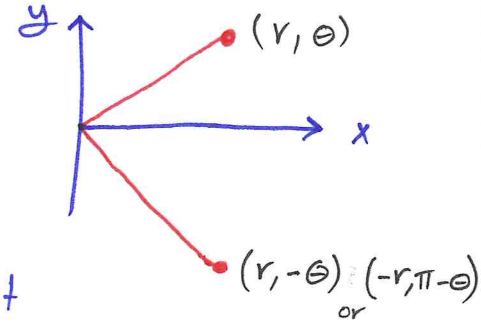
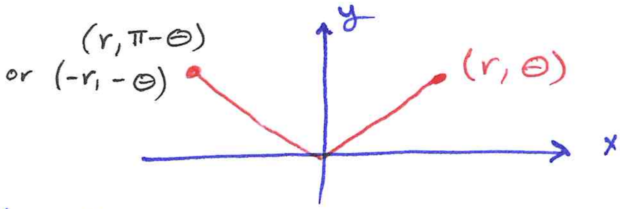
$\boxed{11.4}$ Graphing in Polar Coordinates

Symmetry Tests for Polar Graphs:

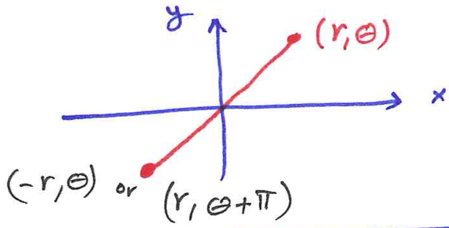
$\boxed{1}$ Symmetry about x-axis: If the point (r, θ) lies on the graph, then $(r, -\theta)$ or $(-r, \pi - \theta)$ lies on the graph.



$\boxed{2}$ Symmetry about y-axis: If the point (r, θ) lies on the graph, then the point $(r, \pi - \theta)$ or $(-r, -\theta)$ lies on the graph



$\boxed{3}$ Symmetry about the origin: If the point (r, θ) lies on the graph, then the point $(-r, \theta)$ or $(r, \theta + \pi)$ lies on the graph.



Slope Let $r = f(\theta)$. Recall the parametric equations:

$$x = r\cos\theta = f(\theta)\cos\theta, \quad y = r\sin\theta = f(\theta)\sin\theta \qquad r' = f'(\theta)$$

slope of the curve $r = f(\theta)$ at (r, θ) is

$$\left.\frac{dy}{dx}\right|_{(r,\theta)} = \frac{r'\sin\theta + r\cos\theta}{r'\cos\theta - r\sin\theta}$$

Proof $\dfrac{dy}{dx} = \dfrac{dy/d\theta}{dx/d\theta} = \dfrac{f'\sin\theta + f\cos\theta}{f'\cos\theta - f\sin\theta}$

Note that when the curve $r = f(\theta)$ passes through the origin at $\theta_0 \Rightarrow \left.\dfrac{dy}{dx}\right|_{(0,\theta_0)} = \tan\theta_0$

Exp Find the slope of $r = \cos 2\theta$ at $\theta = 0, \frac{\pi}{2}$

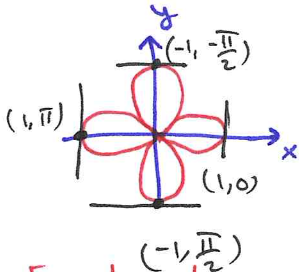
$r' = -2\sin 2\theta$

- when $\theta = 0 \Rightarrow r = 1 \Rightarrow (r,\theta) = (1,0)$

slope is $\left.\dfrac{dy}{dx}\right|_{(1,0)} = \left.\dfrac{r'\sin\theta + r\cos\theta}{r'\cos\theta - r\sin\theta}\right|_{(1,0)} = \dfrac{-2\sin(0)\sin(0) + (1)\cos(0)}{-2\sin(0)\cos(0) - (1)\sin(0)} = \dfrac{1}{0}$ undefined

- when $\theta = \frac{\pi}{2} \Rightarrow r = -1 \Rightarrow (r,\theta) = (-1, \frac{\pi}{2})$

the slope is $\left.\dfrac{dy}{dx}\right|_{(-1,\frac{\pi}{2})} = \dfrac{-2\sin(\pi)\sin(\frac{\pi}{2}) + (-1)\cos(\frac{\pi}{2})}{-2\sin(\pi)\cos(\frac{\pi}{2}) - (-1)\sin(\frac{\pi}{2})} = 0$



Four-leaved rose

Exp sketch the grave of the following curves, Identify the symmetry

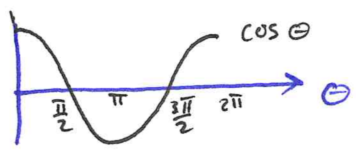
1) $r = 1 - \cos\theta$

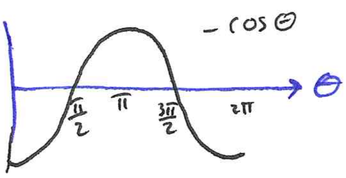
- (r,θ) on the graph $\Rightarrow r = 1 - \cos\theta$
  $\Rightarrow r = 1 - \cos(-\theta)$
  $\Rightarrow (r, -\theta)$ on the graph

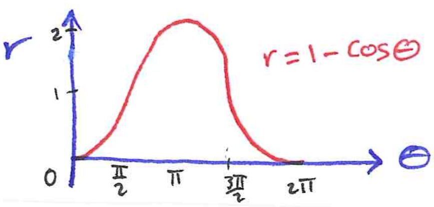
$\Rightarrow$ the curve is symmetric about x-axis.

- $1 - \cos(-\theta) = 1 - \cos\theta \neq -r$
  $1 - \cos(\pi - \theta) = 1 + \cos\theta \neq r$
  $\Rightarrow$ the curve is not symmetric about y-axis

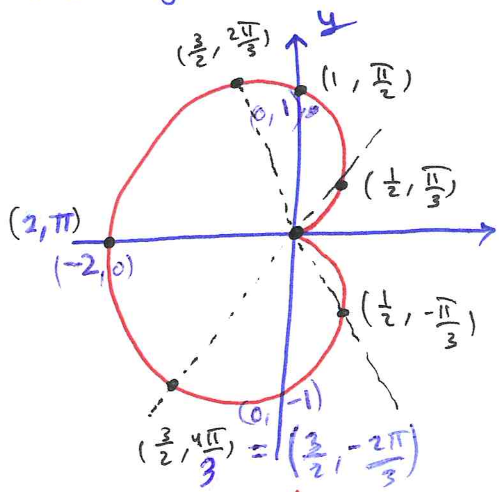
- $1 - \cos\theta \neq -r$
  $1 - \cos(\theta + \pi) = 1 + \cos\theta \neq r$
  $\Rightarrow$ the curve is not symmetric about the origin.







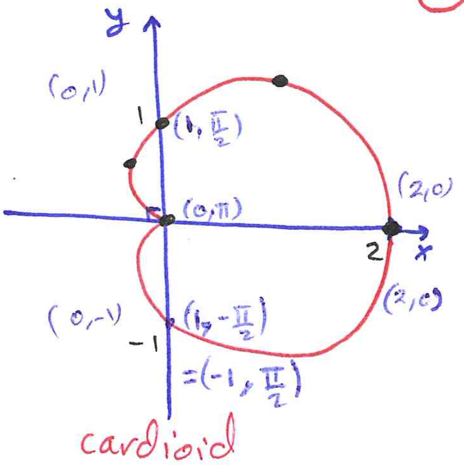
θ	$r = 1 - \cos\theta$
0	0
$\frac{\pi}{3}$	$\frac{1}{2}$
$\frac{\pi}{2}$	1
$\frac{2\pi}{3}$	$\frac{3}{2}$
π	2



cardioid

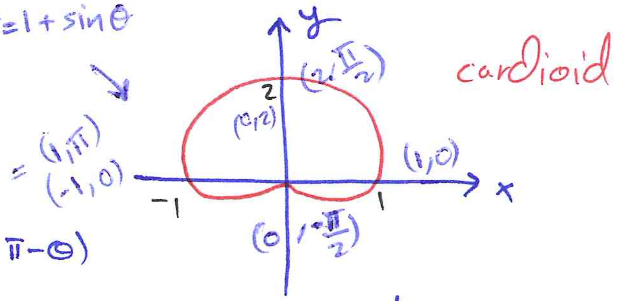
$\boxed{2}$ $r = 1 + \cos\theta$ ✓

- (r, θ) on the graph $\Rightarrow$
  $r = 1 + \cos\theta \Rightarrow r = 1 + \cos(-\theta) \Rightarrow$
  $(r, -\theta)$ on the graph $\Rightarrow$ symmetric about x-axis



$\boxed{3}$ $r = -1 + \sin\theta$ similar to $r = 1 + \sin\theta$ ✓

- (r, θ) on the graph $\Rightarrow$
  $r = -1 + \sin\theta \Rightarrow r = -1 + \sin(\pi - \theta)$
  $\Rightarrow (r, \pi - \theta)$ on the graph $\Rightarrow$ the curve is symmetric about y-axis

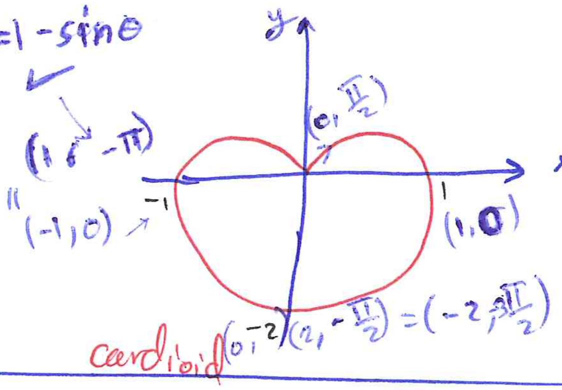


- $\left.\begin{array}{l} -1 + \sin(-\theta) = -1 - \sin\theta \neq r \\ -1 + \sin(\pi - \theta) = -1 + \sin\theta \neq -r \end{array}\right\} \Rightarrow$ not symmetric about x-axis.

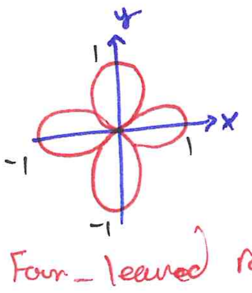
- $\left.\begin{array}{l} -1 + \sin\theta \neq -r \\ -1 + \sin(\theta + \pi) = -1 - \sin\theta \neq r \end{array}\right\} \Rightarrow$ not symmetric about origin.

$\boxed{4}$ $r = -1 - \sin\theta$ similar to $r = 1 - \sin\theta$ ✓

symmetric about y-axis

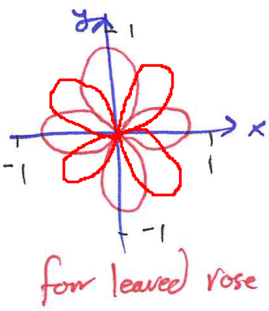


$\boxed{5}$ $r = \cos 2\theta$



symmetric about x-axis and y-axis and origin

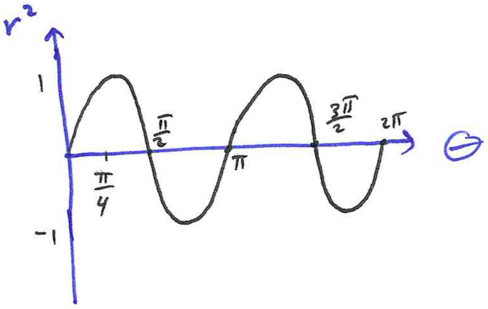
$\boxed{6}$ $r = \sin 2\theta$

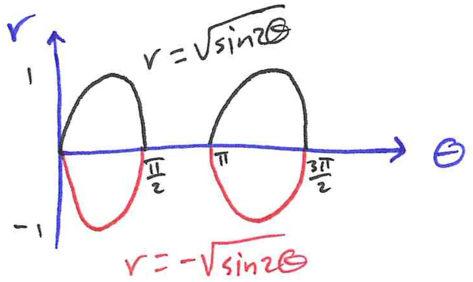


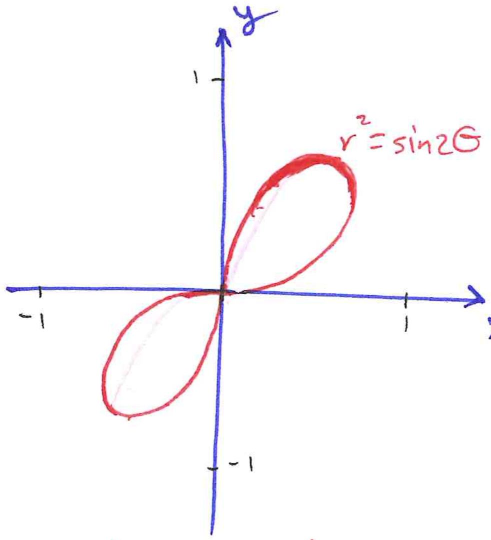
symmetric about x-axis, y-axis and origin.

7

[7] $r^2 = \sin 2\theta$

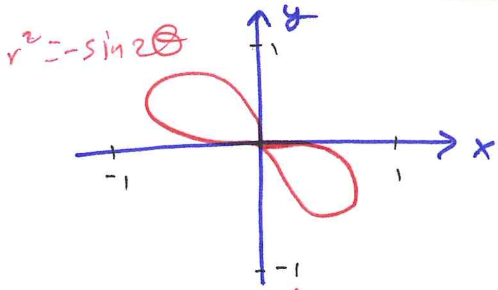






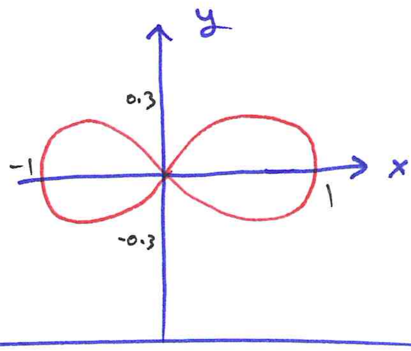
lemniscate

[7] $r^2 = -\sin 2\theta$



Lemniscate

[8] $r^2 = \cos 2\theta$



[9] $r^2 = -\cos 2\theta$