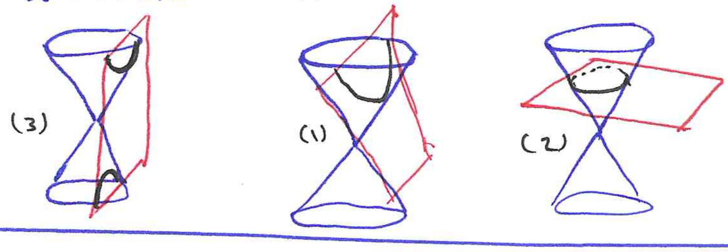
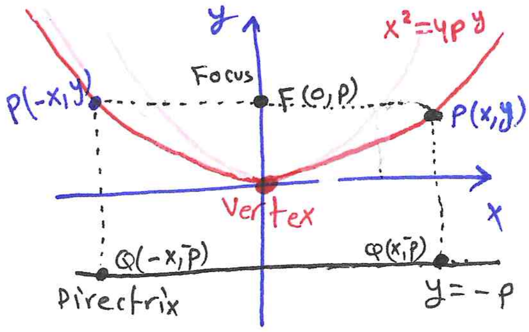
11.6 Conic Sections

* Conic Sections are Parabolas (1), Ellipses (2), Hyperbolas (3) curves in which a plane cuts a double cone.



Parabolas: [1] The standard form of the parabola is

$$\boxed{x^2 = 4py\ ,\ p>0}$$



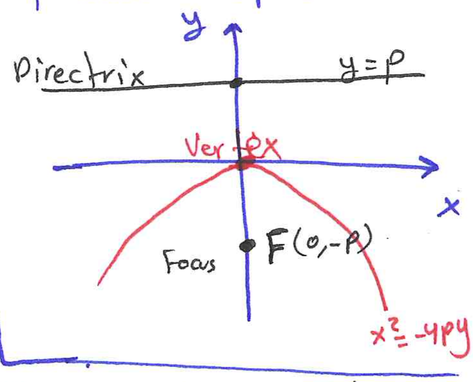
- the Parabola is the set of points s.t $PF = PQ \iff$

$$\sqrt{(x-0)^2+(y-p)^2} = \sqrt{(x-x)^2+(y--p)^2}$$

$$\sqrt{x^2+(y-p)^2} = \sqrt{(y+p)^2} \iff x^2+(y-p)^2 = (y+p)^2 \iff \boxed{x^2=4py}$$

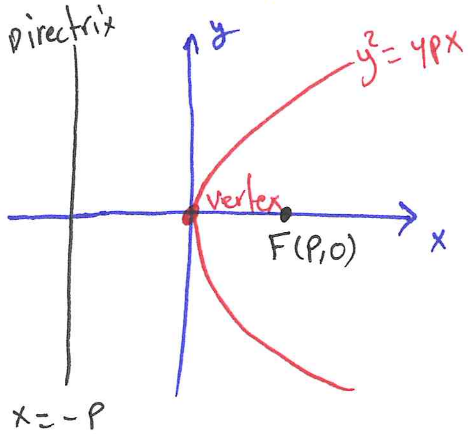
- p is positive number called the parabola's focal length.

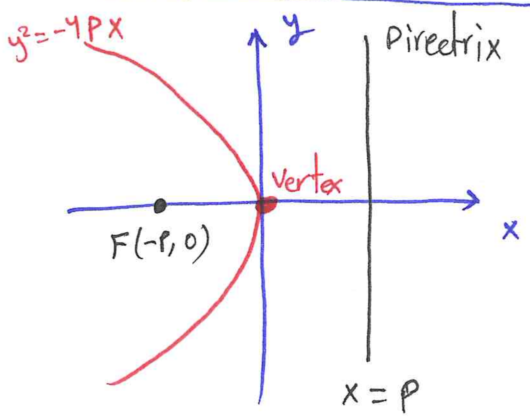
[2] when $x^2=-4py$, $p>0 \Rightarrow$ the parabola opens down:



[3] when $y^2=4px$, $p>0 \Rightarrow$ the parabola opens right

[4] when $y^2=-4px$, $p>0 \Rightarrow$ the parabola opens left



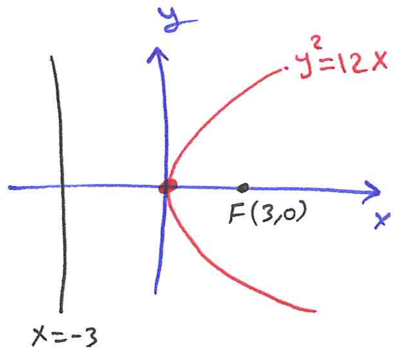


Exp Find the focus and directrix for each of the following parabolas. sketch each one.

1. $y^2 = 12x$

   $4p = 12 \Leftrightarrow p = 3$, vertex is $(0,0)$

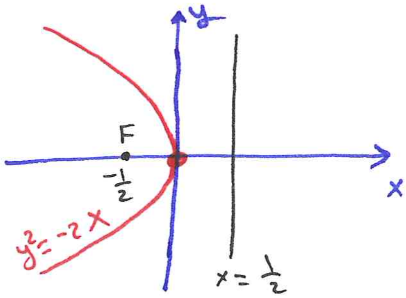
   Focus is $(3,0)$, Directrix is $x = -3$



2. $y^2 = -2x$

   $4p = 2 \Leftrightarrow p = \frac{1}{2}$, vertex is $(0,0)$

   Focus is $(-\frac{1}{2},0)$, Directrix is $x = \frac{1}{2}$



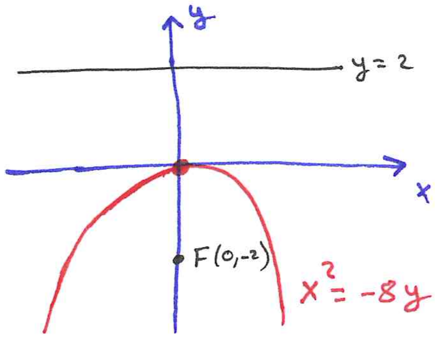
3. $x^2 = -8y$

   $4p = 8 \Leftrightarrow p = 2$

   Focus is $(0,-2)$

   Directrix is $y = 2$

   Vertex is $(0,0)$



4. $y = 4x^2$

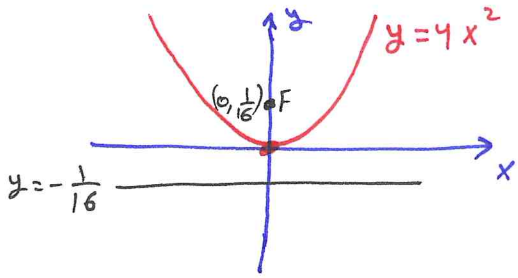
   $x^2 = \frac{1}{4} y$

   $4p = \frac{1}{4} \Leftrightarrow p = \frac{1}{16}$

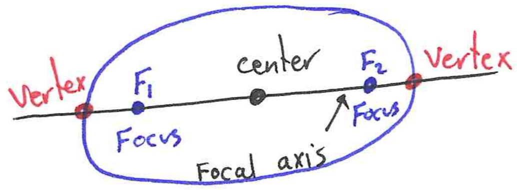
   Focus is $(0, \frac{1}{16})$

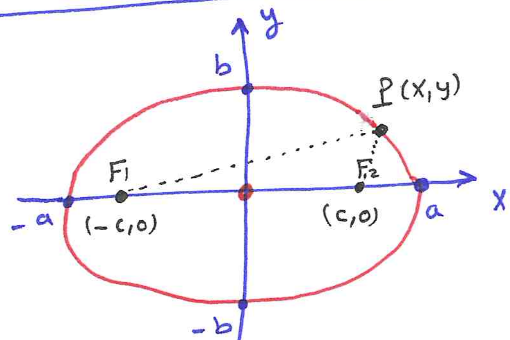
   Directrix is $y = -\frac{1}{16}$

   Vertex is $(0,0)$



Ellipse





$\boxed{a>b}$ *

The ellipse defined by the equation $PF_1 + PF_2 = 2a$ is the graph of

$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ where $c^2 = a^2 - b^2$ is the Center-to-focus distance.

$a \longrightarrow$ is the semimajor axis

$b \longrightarrow$ is the semiminor axis

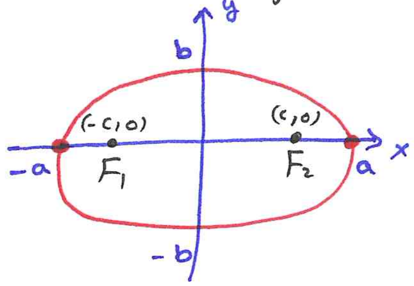
* • Major axis of the ellipse is the line segment of length $2a$
* • Minor axis of the ellipse is the line segment of length $2b$.

Standard-Form Equations for Ellipse Centered at the Origin:

* Foci on the x-axis: $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$

$\boxed{a>b}$

- Center-to-focus distance $c=\sqrt{a^2-b^2}$
- Foci: $(\pm c, 0)$
- Vertices: $(\pm a, 0)$



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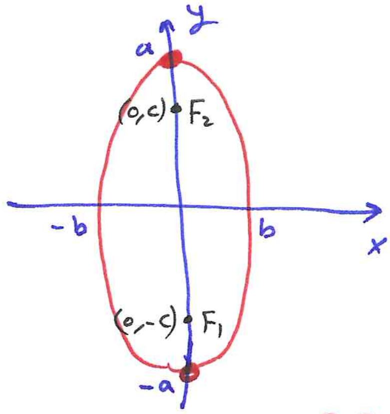
* Foci on the y-axis: $\frac{x^2}{b^2}+\frac{y^2}{a^2}=1$

$\boxed{a>b}$

- Center-to-focus distance $c=\sqrt{a^2-b^2}$
- Foci: $(0, \pm c)$
- Vertices: $(0, \pm a)$



if $a=b \Rightarrow c=0 \Rightarrow$ we get a circle

Exp Put each of the following equations in the standard form. Sketch the ellipse and include the foci.

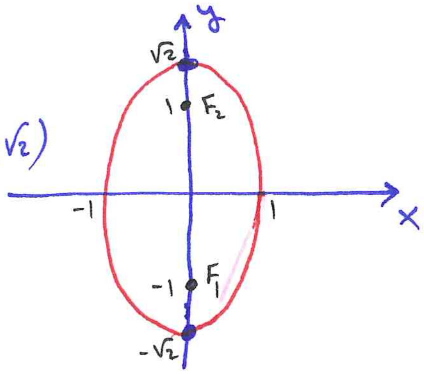
1. $2x^2+y^2=2$

$$\boxed{x^2+\frac{y^2}{2}=1}$$

$c=\sqrt{a^2-b^2}=\sqrt{2-1}=1$

Vertices: $(0, \pm\sqrt{2})$

Foci: $(0, \pm 1)$



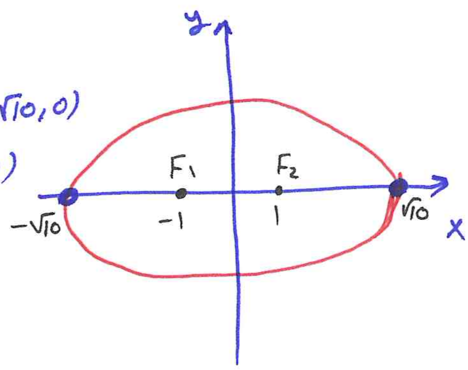
2. $9x^2+10y^2=90$

$$\boxed{\frac{x^2}{10}+\frac{y^2}{9}=1}$$

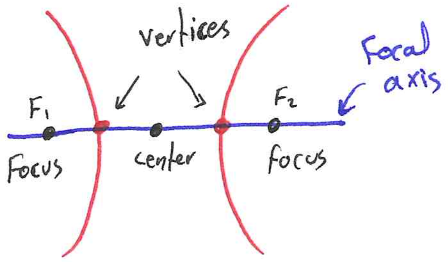
$c=\sqrt{a^2-b^2}=\sqrt{10-9}=1$

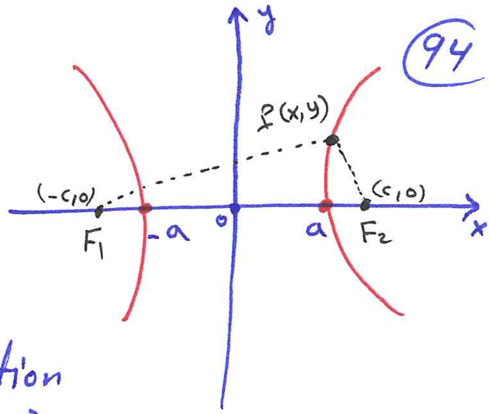
Vertices $(\pm\sqrt{10}, 0)$

Foci: $(\pm 1, 0)$



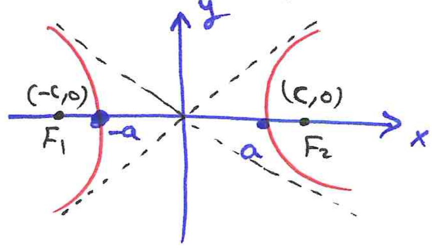
Hyperbolas





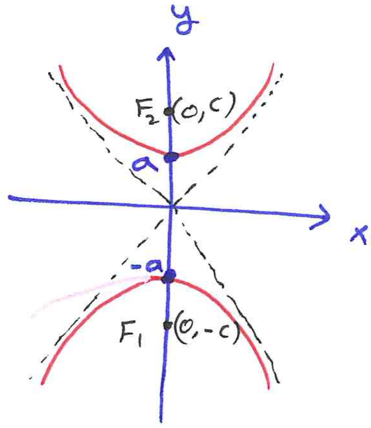
The hyperbola defined by the equation $PF_1 - PF_2 = 2a$ is the graph of $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ where $c = \sqrt{a^2+b^2}$

Standard form Equations for Hyperbolas Centered at Origin:



* Foci on the x-axis: $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$

- Center-to-focus distance $c = \sqrt{a^2+b^2}$
- Foci: $(\pm c, 0)$
- Vertices: $(\pm a, 0)$
- Asymptotes: $y = \pm \frac{b}{a} x$



* Foci on the y-axis: $\frac{y^2}{a^2} - \frac{x^2}{b^2} = 1$

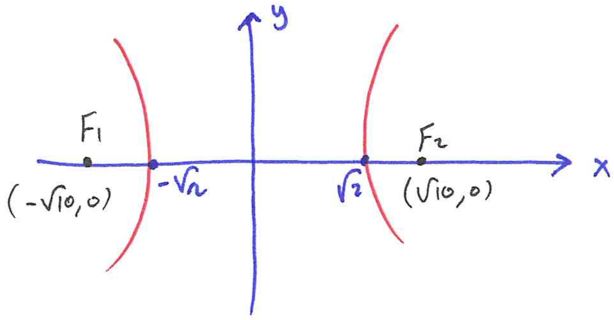
- Center-to-focus distance $c = \sqrt{a^2+b^2}$
- Foci: $(0, \pm c)$ Vertices: $(0, \pm a)$
- Asymptotes: $y = \pm \frac{a}{b} x$

Exp Put each of the following equations in the standard form. Sketch the hyperbola and include the foci and asymptotes.

1. $8x^2 - 2y^2 = 16 \Leftrightarrow \frac{x^2}{2} - \frac{y^2}{8} = 1$ $a = \sqrt{2}$ $b = 2\sqrt{2}$

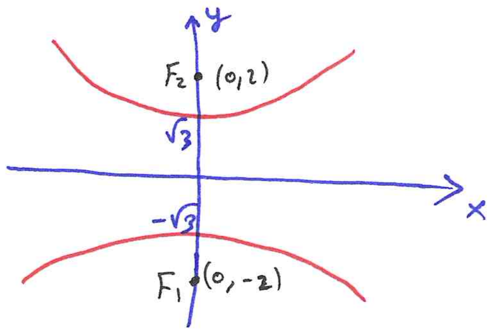
$c = \sqrt{a^2+b^2} = \sqrt{2+8} = \sqrt{10}$

Asymptotes $y = \pm 2x$ $\frac{b}{a} = 2$

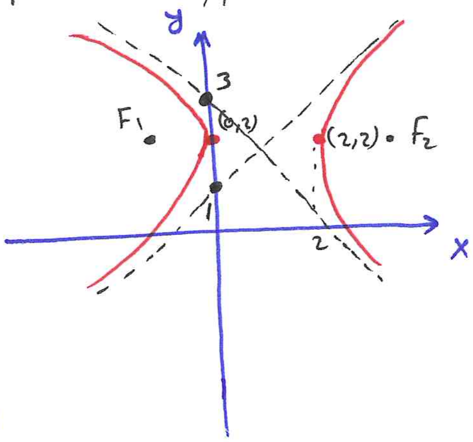


2. $y^2 - 3x^2 = 3 \Leftrightarrow \frac{y^2}{3} - x^2 = 1$

$c = \sqrt{3+1} = 2$, Asymptotes $y = \pm\sqrt{3}x$



Exp show that $x^2 - y^2 - 2x + 4y = 4$ represents a hyperbola. Find it's center, asymptotes and foci.



$$(x-1)^2 - 1 - (y-2)^2 + 4 = 4$$

$$(x-1)^2 - (y-2)^2 = 1$$

- center $(1, 2)$
- $c = \sqrt{a^2+b^2} = \sqrt{2}$ $\Rightarrow$ foci : $(1 \pm \sqrt{2}, 2)$
- Asymptotes $y - 2 = \pm (x-1)$
- Vertices : $(0,2)$ and $(2,2)$: $(1 \pm \underset{\downarrow 1}{a}, 2)$

$$\frac{(y-3)^2}{3} - (x-1)^2 = 1$$

- Center $(1, 3)$
- Vertices $(1, 3 \pm a) = (1, 3 \pm \sqrt{3})$
- foci : $(1, 3 \pm c)$, $c = \sqrt{a^2+b^2}$
  $(1,1), (1,5)$ $= \sqrt{3+1} = 2$
- Asymptotes $y - 3 = \pm \sqrt{3}\,(x-1)$
