

* Summary of tests:-

• $\{a_n\}$ (sequence)

→ If $\lim_{n \rightarrow \infty} a_n = C$ (any finite number) then $\{a_n\}$ converges to C

→ If $\lim_{n \rightarrow \infty} a_n \begin{cases} \rightarrow \infty \\ \rightarrow \text{DNE} \end{cases}$ then $\{a_n\}$ diverges.

• $\sum a_n$ (series)

① p-series: $\sum \frac{1}{n^p}$

→ if $p > 1$ converges, if $p \leq 1$ diverges.

② Geometric series: $\sum_{n=0}^{\infty} a r^n = a + ar + ar^2 + \dots$

→ if $|r| < 1$, converges to $\frac{a}{1-r}$
if $|r| \geq 1$, diverges.

تقارب السلسلة الجبرية
إذا $|r| < 1$

③ The nth term test:

→ if $\lim_{n \rightarrow \infty} a_n \neq 0$ then $\sum a_n$ diverges.

if $\lim_{n \rightarrow \infty} a_n = 0$ «test fails».

④ The nth partial sum:

→ if $\lim S_n = L$ then $\sum a_n$ converges.

telescoping series

$S_n = a_1 + a_2 + \dots + a_n$
سلسلة جزئية المجموع

if $\lim_{n \rightarrow \infty} S_n \rightarrow \begin{cases} \neq \infty \\ \text{DNE} \end{cases}$, $\sum a_n$ diverges.

④ Series with nonnegative terms:

- Comparison test («D.C.T.»)
- Limit comparison test («L.C.T.»)
- Integral test («true, conti., decreasing»)
- Ratio test.
- n-th root test.

⑤ Alternating series test: $\sum (-1)^{n+1} U_n$

- U_n true, decreasing, $\lim U_n = 0 \rightarrow \sum (-1)^{n+1} U_n$ converges by alternating series test.
- If $\lim U_n \neq 0 \rightarrow$ diverges by n-th term test.