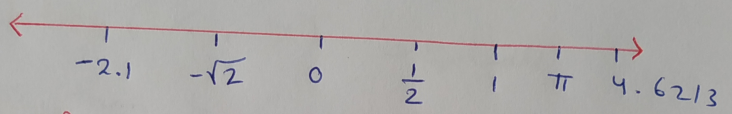
0.2 The real numbers
الأعداد الحقيقية

1) The set of the real numbers $\mathbb{R}$ can be represented by the real number line where each point on the line represents a real number



Subsets of the set of real numbers :

2) The Natural numbers $\mathbb{N} = \{1, 2, 3, 4, \ldots\}$ الأعداد الطبيعية

note that: the Natural numbers $\subseteq$ Real numbers.

3) The Set of the integer numbers $\mathbb{Z}$

$\mathbb{Z} = \{\ldots, -2, -1, 0, 1, 2, 3, \ldots\}$ الأعداد الصحيحة

note that $\mathbb{N} \subseteq \mathbb{Z} \subseteq \mathbb{R}$

4) The set of the Rational numbers $\mathbb{Q}$: all real numbers that can be written as the ratio of two integers $\frac{a}{b}$ where $b \neq 0$.

Examples: 3 , -5 , 0.5 , 3.465

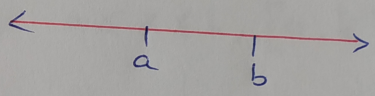
$2.\overline{3}$, 4.153

5) Irrational numbers: any real number that is not rational (can not be written as $\frac{a}{b}$, where a and b are integers)

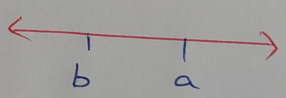
Example:- $\sqrt{2}$, π كسر عشري , $-\sqrt{2}$, $\pi \times 10$, e

* Inequalities ($<$, $>$) المتباينات

1) a is less than b ($a < b$) if a is to the left of b on the number line



2) a is greater than b ($a > b$) if a is to the right of b on the number line

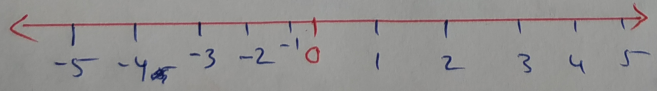


Examples:

$3 < 5$, $3 > -5$, $-2 < 0$

$-2 > -4.5$, $2.3 > 2.222$, $\frac{9}{5} > \frac{11}{8}$

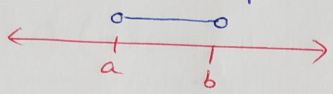
$\frac{9}{5} \downarrow 1.8$ $\quad$ $\frac{11}{8} \downarrow 1.375$



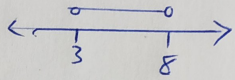
* Intervals الفترات

1) The subset $\{X : a<X<b\}$ is an open interval which can be expressed as $a<X<b$ or (a,b)

or

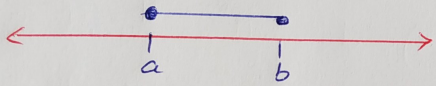


Example:- $3<X<8 \rightarrow (3,8)$



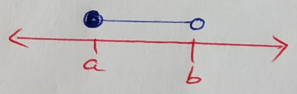
2) The subset $\{X : a \leq X \leq b\}$ is a closed interval which can be expressed as $a \leq X \leq b$, $[a,b]$

or



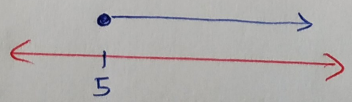
Example: $-5 \leq X \leq 2 \rightarrow [-5,2]$

3) The subset $\{X : a \leq X \leq b\}$ is half-open interval can be expressed as $a \leq X < b$

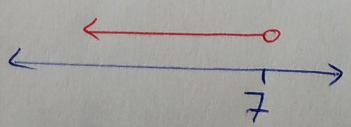


Question: write the types of the following intervals and express them in a second and third way:-

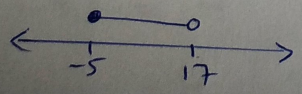
① $X \geq 5 \rightarrow [5,\infty) \rightarrow$



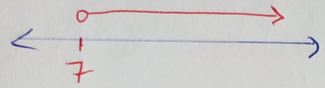
② $(-\infty,7) \rightarrow X \leq 7 \rightarrow$



③



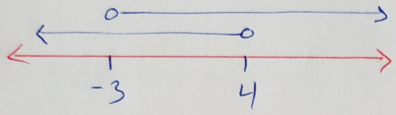
$\rightarrow [-5,17) \rightarrow -5 \leq X < 17$

4) 

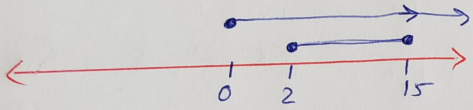
$\rightarrow x > 7 \rightarrow (7, \infty)$

Question :- Perform the following operations and write your answer in interval notations :-

1) $(-\infty, 4) \cap (-3, \infty) = (-3, 4)$

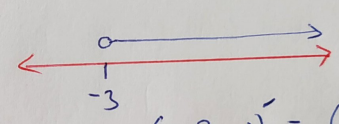


2) $2 \leq x \leq 15 \cup x \geq 0 = [0, \infty)$



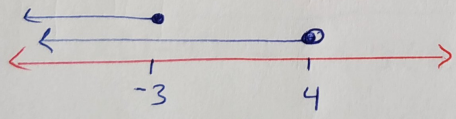
3) $(-\infty, 4) - (-3, \infty)$

$= (-\infty, 4) \cap (-3, \infty)'$



$(-3, \infty)' = (-\infty, -3]$

$= (-\infty, 4) \cap (-\infty, -3]$



$= (-\infty, -3]$

* Absolute Value : القيمة المطلقة

Evaluate the following :

a) $|-4| = 4$

b) $|0| = 0$

c) $|2| = 2$

d) $|-5 - |-3|| = |-5 - 3|$
$= |-8| = 8$

Order of operations :

1) Parentheses ( ) الأقواس

2) powers القوى

3) multiplications (الضرب) and divisions (القسمة) (from left to right)

4) additions (الجمع) and subtractions (الطرح) (from left to right)

Example:- Evaluate the expressions:

1) $3 + 8(4-2) \div 2$

$3 + 8(2) \div 2$

$3 + 16 \div 2$

$3 + 8 = 11$

2) $$\frac{(-3)^2 + |2-5|}{2(1-3) + 6 \div 3(2)}$$

$$= \frac{9 + |-3|}{2(-2) + 2(2)}$$

$$= \frac{9 + 3}{-4 + 4} = \frac{12}{0}$$ meaningless.