

### 0.3 Integral Exponents

Consider the expression  $(1.5)(1.5)(1.5)$ . This can be written in exponential form as  $(1.5)^3$

In general, if  $a$  is a real number and  $n$  is positive integer, then:-  $a^n = a \cdot a \cdot a \cdot \dots \cdot a$  ( $n$  times)  
Where  $a$  is the base,  $n$  is the exponent.

#### Rules of exponents.

For any real numbers  $a$  and  $b$  and any integers  $m$  and  $n$ :-

①  $a^m \cdot a^n = a^{m+n}$  عند الصرب جمع الأسس

Examples:  $3^2 \cdot 3^{-4} = 3^{2+(-4)} = 3^{-2}$

$x \cdot x^2 = x^{1+2} = x^3$

②  $\frac{a^m}{a^n} = a^{m-n}$ ,  $a \neq 0$  عند القسمة تطرح الأسس

Examples:  $\frac{4^7}{4^4} = 4^{7-4} = 4^3$

$\frac{y^2}{y^{-1}} = y^{2-(-1)} = y^{2+1} = y^3$

③  $(ab)^m = a^m \cdot b^m$

Example:-  $(3x)^2 = (3)^2 x^2 = 9x^2$

$(x \cdot y)^5 = x^5 \cdot y^5$

④  $(a^m)^n = a^{m \cdot n}$  عند الرفع تضرب الأس

Example:-  $(3^2)^3 = 3^{2 \cdot 3} = 3^6 = 729$

$$(X^2)^2 = X^4$$

⑤  $\left(\frac{a}{b}\right)^m = \frac{a^m}{b^m}$  ,  $b \neq 0$

Example:-  $\left(\frac{-2}{5}\right)^3 = \frac{(-2)^3}{5^3} = \frac{-8}{125}$

$$\left(\frac{X^2}{X^{-1}}\right)^3 = \frac{(X^2)^3}{(X^{-1})^3} = \frac{X^6}{X^{-3}} = X^{6-(-3)} = X^9$$

⑥  $a^0 = 1$  ,  $a \neq 0$   
 $0^0$  is undefined.

→ Example:-  $(-0.1256)^0 = 1$   
 $X^0 = 1$

⑦  $a^{-n} = \frac{1}{a^n}$  ,  $a \neq 0$

Example:-  $3^{-2} = \frac{1}{3^2} = \frac{1}{9}$

$$(-4)^{-3} = \frac{1}{(-4)^3} = \frac{1}{-64}$$

⑧  $\left(\frac{a}{b}\right)^{-n} = \left(\frac{b}{a}\right)^n$  ,  $a, b \neq 0$

Example:-  $\left(\frac{X^2}{3}\right)^{-2} = \left(\frac{3}{X^2}\right)^2 = \frac{(3)^2}{(X^2)^2} = \frac{9}{X^4}$

Example:- Simplify the following with positive exponents

بسط المقادير التالية بحيث الجواب يحتوي على قوى موجبة

$$a) 2(X^2)^{-2} = 2X^{-4} = \frac{2}{X^4}$$

عند الرفع نضرب القوى

عند القوى السالبة نعكس مكانها

$$b) X^{-2} \cdot X^{-5} = X^{-2+(-5)} = X^{-7} = \frac{1}{X^7}$$

عند الضرب نجمع القوى

$$c) \frac{X^{-8}}{X^{-4}} = X^{-8-(-4)} = X^{-8+4} = X^{-4} = \frac{1}{X^4}$$

عند القسمة نطرح القوى

$$d) \left( \frac{2X^3}{3X^{-5}} \right)^{-2}$$

$$= \left( \frac{2X^{3-(-5)}}{3} \right)^{-2} = \left( \frac{2X^8}{3} \right)^{-2}$$

عند القسمة نطرح القوى

$$= \left( \frac{3}{2X^8} \right)^2 = \frac{(3)^2}{(2)^2 (X^8)^2} = \frac{9}{4X^{16}}$$

**Example:** Simplify the following so all exponents are positive.

$$\begin{aligned} \text{a) } & (2^3 x^{-4} y^5)^{-2} \\ &= (2^3)^{-2} \cdot (x^{-4})^{-2} \cdot (y^5)^{-2} \\ &= 2^{-6} \cdot x^8 \cdot y^{-10} \\ &= \frac{x^8}{2^6 y^{10}} = \frac{x^8}{64 y^{10}} \end{aligned}$$

$$\begin{aligned} \text{b) } & \frac{2x^4 (x^2 y)^0}{(4x^{-2} y)^2} \\ &= \frac{2x^4}{(4x^{-2} y)^2} = \frac{2x^4}{(4)^2 \cdot (x^{-2})^2 \cdot (y)^2} \\ &= \frac{2x^4}{8 \cdot 16 x^{-4} y^2} \\ &= \frac{x^{4-(-4)}}{8 y^2} = \frac{x^8}{8 y^2} \end{aligned}$$

**Evaluate:**  $0^4 = 0$

$$0^{-4} = \frac{1}{0^4} = \frac{1}{0} \text{ meaningless (undefined)}$$