

0.5 Operations with Algebraic Expressions.

- **Variable:** Letter used to represent real numbers

- **Algebraic expressions:** additions, subtractions, multiplications, divisions or roots with one or more real numbers or variables (Letters)

Example: ① $3x + 6y^2$ 2 terms

② $\sqrt{x} + 7$ note that $x \geq 0$ 2 terms

③ $\frac{x^2y - x}{x + 1}$ note that $x \neq -1$ 4 terms

Term: is any product of a real number (coefficient) and one or more variables to powers.

Polynomial: is the sum of a finite number of terms with nonnegative integer powers on the variables

Example: ① $x^2 - 3y + \frac{1}{2}x + x^2y^5$ is a polynomial

② $x^2 + y^3 + x^{-1}$ not a polynomial

③ $y^4 - 2\sqrt{x} = y^4 - 2x^{1/2}$ not a polynomial

If a polynomial contains only one variable x , then it is a polynomial in x .

Example: ① $x^3 - x + 6$ a polynomial in x

② $3x^7 - x^2 + x - 1$ a polynomial in x

③ $x^2 + y^3 + 1$ a polynomial of 2 variables x and y

In general, the form of polynomial in x is

$$a_n x^n + a_{n-1} x^{n-1} + \dots + a_1 x^1 + a_0 x^0$$

The degree of a polynomial: is the degree of the term with the highest degree.

Leading coefficient: the coefficient of the term with the highest degree.

Example: ① $x^3 - x + 6$ the degree is 3

the general form: $a_3 x^3 + a_2 x^2 + a_1 x + a_0 x^0$
 $= 1 \cdot x^3 + 0 \cdot x^2 - 1 \cdot x + 6 \cdot x^0$
Leading coefficient

② $3x^7 - x^2 + x - 1$: the degree is 7

the general form: $a_7 x^7 + a_6 x^6 + a_5 x^5 + a_4 x^4 + a_3 x^3 + a_2 x^2$
 $+ a_1 x^1 + a_0 x^0$
 $= 3x^7 + 0x^6 + 0x^5 + 0x^4 + 0x^3$
 $- x^2 + x - 1$

- polynomial with one term is called **monomial**
 $3X^2$, xy^3 , $4y^2zX$
- polynomial with two terms is called **binomial**
 $X^5 + X^2$, $2Xy + 3$, $-X^3 - y^2$
- polynomial with three terms is called **trinomial**
 $2X^5 + X^3 - 2$, $xy - y^2 + X$

* Operations with Algebraic Expressions.

Example: Compute:

$$\begin{aligned} \text{a) } (4xy + 3x) + (5xy - 2x) &= 4xy + 3x + 5xy - 2x \\ &= 9xy + x \end{aligned}$$

$$\begin{aligned} \text{b) } (3x^2 + 4xy + 5y^2 + 1) - (6x^2 - 2xy + 4) \\ &= 3x^2 + 4xy + 5y^2 + 1 - 6x^2 + 2xy - 4 \\ &= -3x^2 + 6xy + 5y^2 - 3 \end{aligned}$$

Example: Perform the indicated operations.

$$\begin{aligned} \text{a) } (8xy^3)(2x^3y)(-3xy^2) &= 8 \cdot 2 \cdot (-3) \cdot x \cdot x^3 \cdot x \cdot y^3 \cdot y \cdot y^2 \\ &= -48x^5y^6 \end{aligned}$$

$$\begin{aligned} \text{b) } -15x^2y^3 \div (3xy^5) &= \frac{-15x^2y^3}{3xy^5} = -5x^1y^{-2} \\ &= -\frac{5x}{y^2} \end{aligned}$$

Example: Find the following products.

a) $-4ab(3a^2b + 4ab^2 - 1)$

$$= -4ab \cdot 3a^2b + (-4ab)(4ab^2) + (-4ab)(-1)$$

$$= -12a^3b^2 - 16a^2b^3 + 4ab$$

b) $(4a + 5b + c)ac$

$$= 4a \cdot ac + 5b \cdot ac + c \cdot ac$$

$$= 4a^2c + 5abc + ac^2$$

Example: Multiply the following

a) $(x-4)(x+3)$

$$= x \cdot x + x \cdot 3 + (-4) \cdot x + (-4) \cdot 3$$

$$= x^2 + \underline{3x} - \underline{4x} - 12 = x^2 - x - 12$$

b) $(3x+2)(2x+5)$

$$= 3x \cdot 2x + 3x \cdot 5 + 2 \cdot 2x + 2 \cdot 5$$

$$= 6x^2 + \underline{15x} + \underline{4x} + 10 = 6x^2 + 19x + 10$$

Special products :-

- * $(x+a)^2 = x^2 + 2ax + a^2$ binomial squared
- * $(x-a)^2 = x^2 - 2ax + a^2$ binomial squared
- * $(x+a)(x-a) = x^2 - a^2$ difference of 2 squared.

$$* (x+a)^3 = x^3 + 3ax^2 + 3a^2x + a^3 \quad \text{binomial cubed}$$

$$* (x-a)^3 = x^3 - 3ax^2 + 3a^2x - a^3 \quad \text{binomial cubed}$$

Example: Multiply the following:

$$\begin{aligned} \text{a) } (x+5)^2 &= x^2 + 2 \cdot 5 \cdot x + (5)^2 \\ &= x^2 + 10x + 25 \end{aligned}$$

$$\begin{aligned} \text{b) } (3x^2 - 4y)^2 &= (3x^2)^2 - 2 \cdot (3x^2)(4y) + (4y)^2 \\ &= 9x^4 - 24x^2y + 16y^2 \end{aligned}$$

$$\begin{aligned} \text{c) } (x+4)^3 &= x^3 + 3 \cdot 4 \cdot x^2 + 3 \cdot (4)^2 \cdot x + (4)^3 \\ &= x^3 + 12x^2 + 48x + 64 \end{aligned}$$

Example: Divide $(4x^3 - 12x - 22)$ by $(x-3)$, $x \neq 3$

$$= \frac{4x^3 - 12x - 22}{x-3}$$

$$= 4x^2 + 12x + 24 + \frac{50}{x-3}$$

$$\begin{array}{r} \text{الناتج} \rightarrow 4x^2 + 12x + 24 \\ \text{المقسوم عليه} \rightarrow x-3 \overline{) 4x^3 + 0x^2 - 12x - 22} \\ \underline{+ 4x^3} \\ 12x^2 - 12x - 22 \\ \underline{+ 12x^2 + 36x} \\ 24x - 22 \\ \underline{+ 24x + 72} \\ \text{رابطي} \rightarrow 50 \end{array}$$