1.1 Solutions of linear Equations and Inequalities in One variable.

An equation is a statement that two algebraic expressio are equal

Example: ① $x+1=-6$

② $6(3-x)=5+\frac{x-1}{2}$

③ $x^2+x=2x+6$

The values of x that makes the statements true are called solutions of the equation

Example:- $x=-7$ is a solution of the equation $x+1=-6$

✓ $-7+1=-6$

Solving the equation means Finding the solutions

Procedure:

1) If the equation contains fractions, multiply both sides by LCD
2) Remove the parentheses.
3) Perform any additions or subtractions in the equation
4) Divide both sides of the equation by the coefficient of the variable.
6) check the solution

$x + 1 = 6 \rightarrow x = 5$

$-1 \quad -1$

Example : Solve $4x - 7 = 8x + 2$.

$4x - 7 = 8x + 2 \rightarrow -7 = 4x + 2 \rightarrow$

$-8x \quad -8x \qquad -2 \qquad -2$

$\rightarrow -9 = 4x \rightarrow x = \frac{-9}{4}$

Example : $x + 8 = 8(x + 1)$.

$x + 8 = 8(x + 1)$

$x + 8 = 8x + 8 \rightarrow 8 = 7x + 8 \rightarrow 0 = 7x$

$-x \quad -x \qquad -8 \quad -8$

$\rightarrow x = \frac{0}{7} = 0$

Example : $\frac{2x + 1}{x - 3} = 4 + \frac{5}{x - 3}$

LCD : $(x - 3)$

$(x-3) \cdot \frac{2x+1}{(x-3)} = 4 \cdot (x-3) + \frac{5}{(x-3)} \cdot (x-3)$

$2x + 1 = 4(x - 3) + 5$

$2x + 1 = 4x - 12 + 5 \rightarrow 2x + 1 = 4x - 7$

$-2x \quad -2x$

$1 = 2x - 7 \rightarrow 2x = 8 \rightarrow x = \frac{8}{2} = 4$

$+7 \quad +7$

Check:- $\frac{2(4)+1}{4-3} \overset{??}{=} 4 + \frac{5}{4-3}$

$\frac{9}{1} \overset{??}{=} 4 + \frac{5}{1}$

$9 = 9$ ✓ the solution $x = 4$

Example: $\frac{2x}{3} - 1 = \frac{x-2}{2}$

LCD $= (2)\cdot(3) = 6$

$\frac{2x}{3} \cdot 6 - 1 \cdot 6 = \frac{(x-2)}{2} \cdot 6$

$2x \cdot 2 - 6 = (x-2) \cdot 3$

$4x - 6 = 3x - 6$

$-3x \qquad -3x$

$x - 6 = -6 \rightarrow x = 0$

$+6 \qquad +6$

Check $\frac{2(0)}{3} - 1 \overset{??}{=} \frac{0-2}{2}$

$-1 = -1$ ✓

Example: Solve $\frac{3x}{2x+10} = 1 + \frac{1}{x+5}$

$\frac{3}{2(x+5)} = 1 + \frac{1}{x+5}$

LCD $= 2(x+5)$

$$\frac{3x}{2(x+5)} \cdot 2(x+5) = 1 \cdot 2(x+5) + \frac{1}{(x+5)} \cdot 2(x+5)$$

$$3x = 2(x+5) + 2$$

$$3x = 2x + 10 + 2$$

$$3x = 2x + 12 \rightarrow x = 12$$

$$-2x \quad -2x$$

Check :- $\frac{3(12)}{2(12)+10} \stackrel{??}{=} 1 + \frac{1}{12+5}$

$$\frac{36}{34} \stackrel{??}{=} 1 + \frac{1}{17}$$

$$\frac{18}{17} = \frac{18}{17} \checkmark$$

Solution is $x = 12$

Example : Solve $\frac{2x}{x-3} = 4 + \frac{6}{x-3}$

LCD $= (x-3)$

$$\frac{2x}{x-3} \cdot (x-3) = 4 \cdot (x-3) + \frac{6}{(x-3)} \cdot (x-3)$$

$$2x = 4(x-3) + 6$$

$$2x = 4x - 12 + 6 \rightarrow 2x = 4x - 6$$

$$-4x \quad -4x$$

$$0 \neq \cancel{4x} \quad -2x = -6 \rightarrow x = \frac{-6}{-2} = 3$$

Check :- $\frac{2(3)}{3-3} \stackrel{??}{=} 4 + \frac{6}{3-3}$

$$\frac{6}{0} \stackrel{??}{=} 4 + \frac{6}{0}$$

undefined expressions

no solution

Example: Solve $2x + 5 = -3 + 2x$

$2x + 5 = -3 + 2x$

$-2x \qquad -2x$

$\rightarrow \quad 5 = -3$

no solution

Example:- $2x + 6 = 2(3 + x)$

$2x + 6 = 6 + 2x \rightarrow 6 = 6$

$-2x \qquad -2x$

infinitely many solutions

Solve linear equation of two variables :

Example: Solve the following equation for y:-

$4x + 3y = 12 \rightarrow 3y = 12 - 4x$

$-4x \qquad -4x$

$y = \frac{12 - 4x}{3}$ infinite number of solutions.

2) Solve $9x + \frac{3}{2}y = 11$ for y

LCD = 2

$\frac{3}{2} = 1.5 \rightarrow$

$9x \cdot (2) + \frac{3}{2}y \cdot (2) = 11 \cdot (2)$

$18x + 3y = 22 \rightarrow 3y = 22 - 18x$

$-18x \qquad -18x$

$y = \frac{22 - 18x}{3}$

infinite number of solutions

* Linear Inequalities. المتباينات

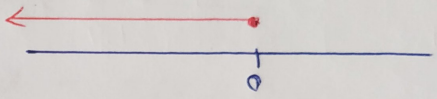
Inequality is a statement that one quantity is greater than or less than another quantity.

Example: Solve $3X + 2 \leq 2$

$3X + 2 \leq 2 \rightarrow 3X \leq 0 \rightarrow 3X \leq \frac{0}{3}$

$-2 \quad -2$

$\rightarrow X \leq 0$

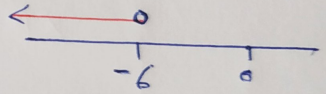


Example: Solve $2X - 1 > 3X + 5$

$-2X \quad -2X$

$-1 > X + 5 \rightarrow -6 > X$

$-5 \quad -5$



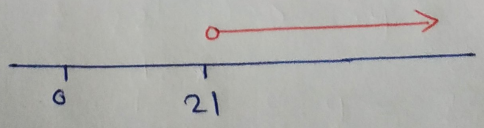
تذكر انه عند ضرب المتباينة بعدد سالب تقلب اشارة المتباينة

وعند قسمة المتباينة على عدد سالب تقلب اشارة المتباينة

Example: $17 - X < -4 \rightarrow -X < -21$

$-17 \quad -17$

$X > \frac{-21}{-1} \rightarrow X > 21$



Example: Solve $\frac{3x}{4} - \frac{1}{6} \geq x - \frac{2(x-1)}{3}$

Find LCD:-

$4 = 2(2)$
$6 = 2(3)$
$3 = 3(1)$

$LCD = 2(2)(3) = 12$

$\frac{3x}{4} \cdot (12) - \frac{1}{6} \cdot (12) \geq x \cdot (12) - \frac{2(x-1)}{3} \cdot (12)$

$3x(3) - \frac{1}{6} \cdot (2) \geq 12x - 2(x-1) \cdot (4)$

$9x - 2 \geq 12x - 8(x-1)$

$9x - 2 \geq 12x - 8x + 8$

$9x - 2 \geq 4x + 8$

$-4x \qquad -4x$

$5x - 2 \geq 8 \rightarrow 5x \geq 10$

$+2 \qquad +2$

$x \geq \frac{10}{5} \rightarrow x \geq 2$