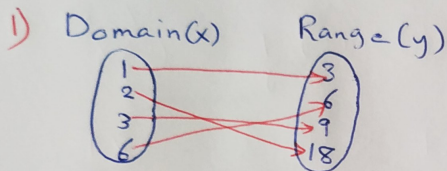


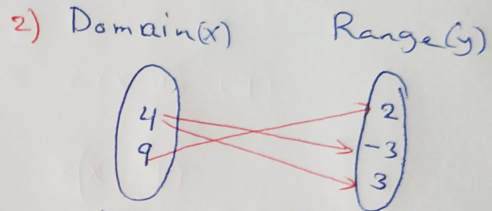
1.2 Functions

A function is a relation between two sets (domain and range) such that for each element in the domain there corresponds only one element of the range.

Example:



This is a function



This relation does not express y as a function of x .

A function may be defined by a set of ordered pairs (أزواج مرتبة), a table, a graph or an equation.

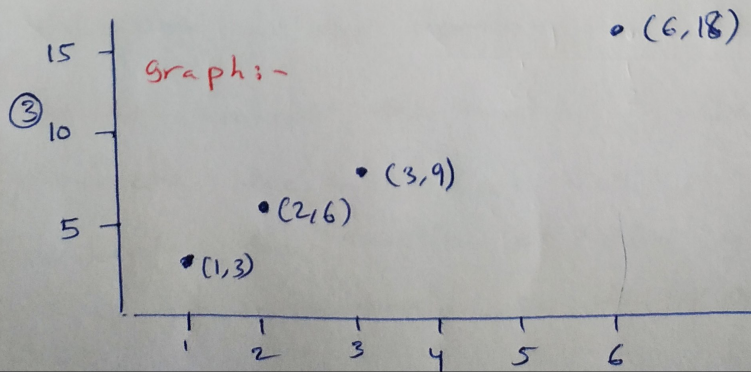
Example:

① table:-

Domain:	Independent variable (x)	1	2	3	6
Range:	dependent variable (y)	3	6	9	18

ordered pairs:-

② $\{(1,3), (2,6), (3,9), (6,18)\}$

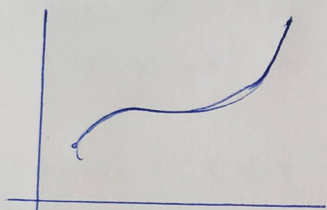


④ Equation: $y = 3x$, $x = 1, 2, 3, 6$

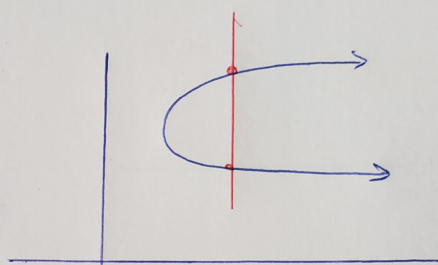
Vertical-line Test :- اختبار الخط العمودي

If no vertical line exists that intersects the graph at more than one point, then the graph is that of a function.

Example:



اي خط عمودي رح يقطع المنحنى
بنقطة واحدة فقط اذن هو اقتزان



ليس اقتزان لانه يوجد خط عمودي
قطع الاقتزان في اكثر من نقطة

Letters other than f may also be used to denote functions

For example $y = g(x)$ or $y = h(x)$.

If the domain is not specified then it is the set of all real numbers (\mathbb{R}).

The range is the set of all outputs obtained from evaluating all the domain.

Example: Consider the function $y = 3x$, $x = 1, 2, 3, 6$

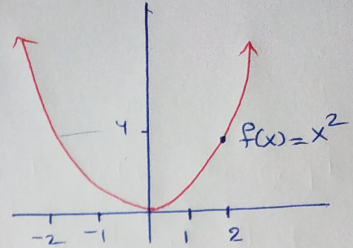
- 1) Evaluate the function at $x=2$, $y = 3(2) = 6$
- 2) evaluate the function at $x=6$, $y = 3(6) = 18$
- 3) 9 is not in the domain, so we can not evaluate for 9

Example: Find the domain and the range of $f(x) = x^2$

Domain: \mathbb{R} or $(-\infty, \infty)$

Range: $[0, \infty)$ or $y \geq 0$

Find the range for $x=2$: $f(2)=4$



Example: Find the domain of $f(x) = \sqrt{4-x}$

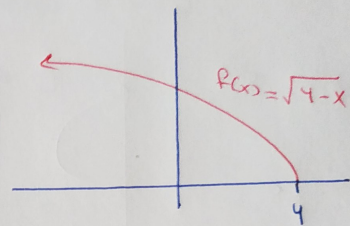
لا تأخذ ما تحت الجذر سالب

$$4 - x \geq 0 \rightarrow 4 \geq x$$

+x +x

Domain: $x \leq 4$ or $(-\infty, 4]$

Range: $y \geq 0$ or $[0, \infty)$



Example: Find the domain of $f(x) = 1 + \frac{1}{x-2}$

Domain: $(-\infty, \infty) / \{2\}$

ممنوع المقام صفر
ممنوع تعرض فيه 2

Example: State the domain and find $f(10)$, $f(-1)$

$$f(x) = \frac{\sqrt{x-1}}{x-2}$$

Domain: $\left. \begin{array}{l} \text{① طاقت الجذر موجبة} \\ \text{② ممنوع المقام صفر} \end{array} \right\} \begin{array}{l} x-1 \geq 0 \rightarrow x \geq 1 \\ x-2=0 \rightarrow x=2 \end{array} \left. \vphantom{\begin{array}{l} \text{①} \\ \text{②} \end{array}} \right\} \text{Domain: } [1, \infty) / \{2\}$

$$f(10) = \frac{\sqrt{10-1}}{10-2} = \frac{\sqrt{9}}{8} = \frac{3}{8}$$

$$f(-1) = \frac{\sqrt{-1-1}}{-1-2} = \frac{\sqrt{-2}}{-3} \quad \text{not Real.}$$

Operations for functions:

$$\textcircled{1} (f+g)(x) = f(x) + g(x)$$

$$\textcircled{2} (f-g)(x) = f(x) - g(x)$$

$$\textcircled{3} (f \cdot g)(x) = f(x) \cdot g(x)$$

$$\textcircled{4} \left(\frac{f}{g}\right)(x) = \frac{f(x)}{g(x)} \quad \text{if } g(x) \neq 0$$

Example: If $f(x) = 3x + 3$, $g(x) = 8 - 2x$.

Find $\textcircled{1} (f+g)(2) = f(2) + g(2)$

$$= 3(2) + 3 + 8 - 2(2)$$

$$= 6 + 3 + 8 - 4 = 9 + 4 = 13$$

$\textcircled{2} (f-g)(1) = f(1) - g(1)$

$$= 3(1) + 3 - (8 - 2(1))$$

$$= 3 + 3 - (6) = 6 - 6 = 0$$

$\textcircled{3} (f \cdot g)(0) = f(0) \cdot g(0) = (3(0) + 3) \cdot (8 - 2(0))$

$$= (0 + 3)(8 - 0) = 3(8) = 24$$

$\textcircled{4} \left(\frac{f}{g}\right)(-1) = \frac{f(-1)}{g(-1)} = \frac{3(-1) + 3}{8 - 2(-1)}$

$$= \frac{-3 + 3}{8 + 2} = \frac{0}{10} = 0$$

$\textcircled{5} \left(\frac{g}{f}\right)(-1) = \frac{g(-1)}{f(-1)}$

$$= \frac{8 - 2(-1)}{3(-1) + 3} = \frac{8 + 2}{-3 + 3} = \frac{10}{0} \text{ undefined}$$

* Composite Functions: الأقرانات المركبة

- f composed of g written as $(f \circ g)(x) = f(g(x))$

- g composed of f written as $(g \circ f)(x) = g(f(x))$

Example: If $f(x) = 3x + 3$, $g(x) = 8 - 2x$. Find:

$$\begin{aligned} 1) (f \circ g)(x) &= f(g(x)) = f(8 - 2x) \\ &= 3(8 - 2x) + 3 \\ &= 24 - 6x + 3 = 27 - 6x \end{aligned}$$

$$\begin{aligned} 2) (g \circ f)(x) &= g(f(x)) = g(3x + 3) \\ &= 8 - 2(3x + 3) \\ &= 8 - 6x - 6 = 2 - 6x \end{aligned}$$

$$3) (f \circ g)(2) = 27 - 6(2) = 27 - 12 = 15$$

$$4) (g \circ f)(0) = 2 - 6(0) = 2 - 0 = 2$$

$$\begin{aligned} \text{or } (g \circ f)(0) &= g(f(0)) = g(3(0) + 3) = g(0 + 3) \\ &= g(3) = 8 - 2(3) = 8 - 6 = 2 \end{aligned}$$

$$\begin{aligned} 5) (f \circ f)(1) &= f(f(1)) = f(3(1) + 3) = f(6) \\ &= 3(6) + 3 = 18 + 3 = 21 \end{aligned}$$