1.3 Linear Functions

A **Linear function** is a function of the form $y = f(x) = ax + b$ where a and b are constants.

Example: 1) $f(x) = 2x + 4$

2) $y = -x + 4$

3) $h(x) = 2x - 5$

4) $4x = 2 - y$

Graphing Linear functions:

To graph a linear function we need two points (x_1, y_1), (x_2, y_2). The best two points to chose are the **intercepts** (x - intercept and y - intercept)

Example: graph the linear function $f(x) = 2x + 4$

- we need two points:

x - intercept : put $y = 0 \rightarrow$

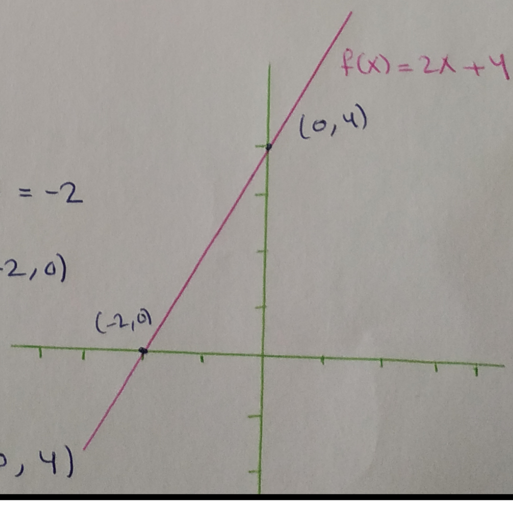
$$0 = 2x + 4 \quad \rightarrow \quad \frac{-4}{2} = \frac{2x}{2} \rightarrow x = -2$$
$$-4 \qquad\; -4$$

$x = -2$ is the x-intercept $\rightarrow (-2, 0)$

y - intercept : Put $x = 0 \rightarrow$

$y = 2(0) + 4 \quad \rightarrow y = 4$

y - intercept is $y = 4 \rightarrow (0, 4)$



Example: Find the x-intercept and the y-intercept and use them to sketch the graph of the equation $4x = 2 - y$.

$4x = 2 - y \rightarrow 4x + y = 2 \rightarrow \boxed{y = 2 - 4x}$

$+y \quad +y \qquad -4x \quad -4x$

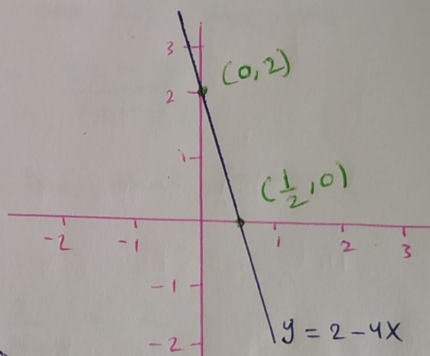
x-intercept: $y = 0 \rightarrow 0 = 2 - 4x \rightarrow \frac{4x}{4} = \frac{2}{4} \rightarrow \boxed{x = \frac{1}{2}}$

$+4x \quad +4x$

$\rightarrow$ first point $(\frac{1}{2}, 0)$

y-intercept: $x = 0 \rightarrow y = 2 - 4(0) \rightarrow \boxed{y = 2}$

$\rightarrow$ second point $(0, 2)$



Example: Sketch the graph of $y = 2x$.

x-intercept: $y = 0 \rightarrow \frac{0}{2} = \frac{2x}{2} \rightarrow x = 0$

the point $(0,0)$

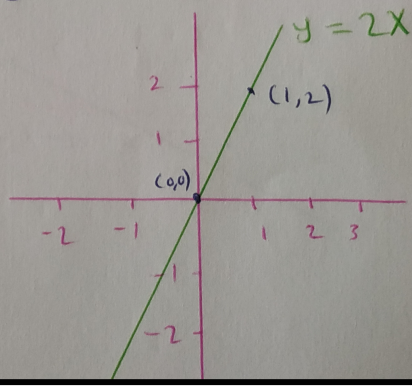
y-intercept: $x = 0 \rightarrow y = 2(0) \rightarrow y = 0$

the point $(0,0)$

we need 2 different points:

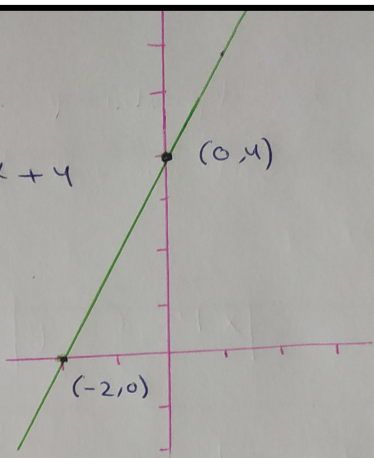
put $x = 1 \rightarrow y = 2(1) \rightarrow y = 2$

the point $(1,2)$



Rate of change (slope ميل الخط):

In the first example $y = f(x) = 2x + 4$



x-intercept point $(-2,0)$

y-intercept point $(0,4)$

$(-2,0) \longrightarrow (0,4)$

y-values changes 4 units $(4-0)$,

whereas the x-values changes 2 units $(0 - {}^-2)$

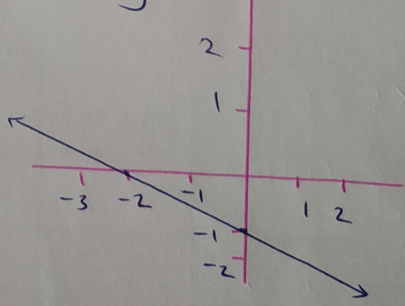
Thus the rate of change $= \dfrac{y_2 - y_1}{x_2 - x_1} = \dfrac{4-0}{0-{}^-2} = \dfrac{4}{2} = 2$

If the points (x_1, y_1) and (x_2, y_2) are on the line, then the slope of the line $m = \dfrac{y_2 - y_1}{x_2 - x_1}$

Example: Find the slope of the line that passing through $(-2,1)$ and $(4,3)$.

Slope $= m = \dfrac{y_2 - y_1}{x_2 - x_1} = \dfrac{3-1}{4-{}^-2} = \dfrac{2}{4+2} = \dfrac{2}{6} = \dfrac{1}{3}$

Example: Find the slope of the following line.



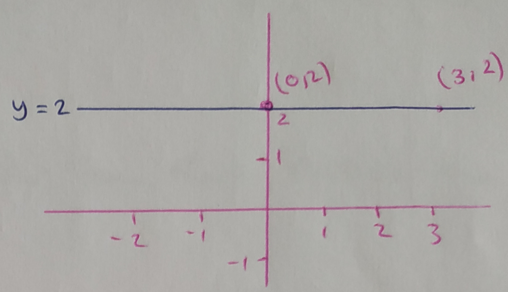
take 2 points $(-2,0)$, $(0,-1)$

Slope $= \dfrac{y_2 - y_1}{x_2 - x_1} = \dfrac{-1-0}{0-{}^-2} = \dfrac{-1}{2}$

* Horizontal lines: $y = b$

Example: sketch the graph of $y = 2$ and find the slope.

Take points $(0,2)$, $(3,2)$

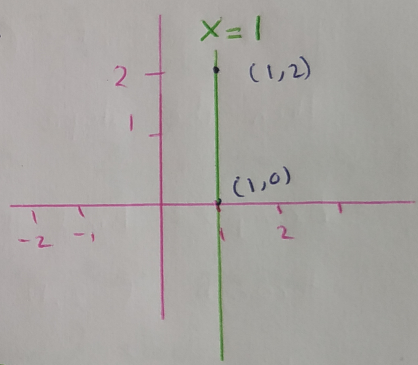


slope $= m = \dfrac{2-2}{3-0} = \dfrac{0}{3} = 0$

The slope of any horizontal line is zero

* Vertical lines: $x = a$

Example: sketch the graph of $x = 1$ and find the slope.



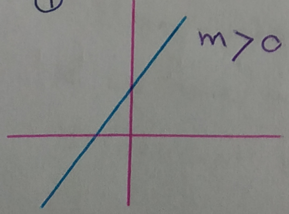
Take points $(1,0)$, $(1,2)$

slope $= m = \dfrac{2-0}{1-1} = \dfrac{2}{0}$ undefined

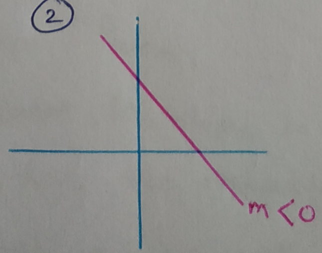
The slope of any ~~hori-~~ vertical line is undefined. Note that $x = a$ is not a function.

Summary: we have 4 cases

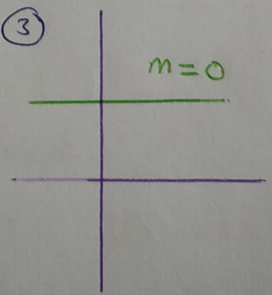
1



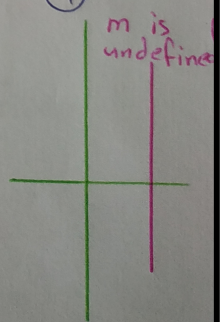
2



3



4



Writing equation of lines:

1) point - slope - form:

If we have a point (x_1, y_1) and a slope m, then the equation is $y - y_1 = m(x - x_1)$

Example: Write the equation for line that passes through $(1, -2)$ and has slope $\frac{2}{3}$.

$$y - y_1 = m(x - x_1) \longrightarrow y - (-2) = \frac{2}{3}(x - 1)$$

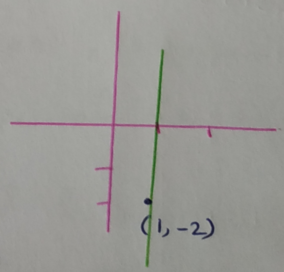
$$\underset{-2}{y + 2} = \frac{2}{3}x - \underset{-2}{\frac{2}{3}} \rightarrow y = \frac{2}{3}x - \frac{2}{3} - \frac{2 \cdot 3}{1 \cdot 3}$$

$$\rightarrow y = \frac{2}{3}x - \frac{2}{3} - \frac{6}{3} \rightarrow y = \frac{2}{3}x - \frac{8}{3} \rightarrow y = \frac{2x - 8}{3}$$

Example: Write the equation for line that passes through $(1, -2)$ and has undefined slope.

undefined slope means that the line is vertical

So the equation $x = 1$



2) Slope - intercept form:

If we have a ~~point (x,~~ slope m and y-intercept b, then the equation is $y = mx + b$

Example:- Write the equation of the line with slope $\frac{1}{2}$ and y-intercept -3.

$$y = \frac{1}{2}x + -3 = \frac{1}{2}x - 3$$

Example: Find the slope and y-intercept of the line whose equation is $x + 2y = 8$.

$$\begin{array}{l} x + 2y = 8 \\ -x \quad\;\; -x \end{array} \rightarrow \frac{2y}{2} = \frac{8 - x}{2} \rightarrow y = \frac{8 - x}{2}$$

$$\rightarrow y = \frac{8}{2} - \frac{x}{2} \rightarrow y = -\frac{1}{2}x + 4$$

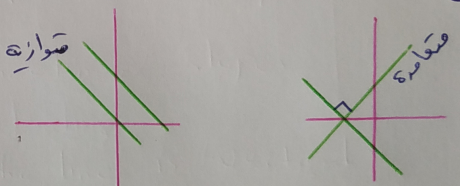
slope $= -\frac{1}{2}$, y-intercept $= +4$

3) Vertical line: $x = a$ where the line passing through (a, y_1)

4) Horizontal line: $y = b$ where the line passing through (x_1, b)

Parallel lines: خطوط متوازية

Two distinct nonvertical lines are parallel if and only if their slopes are equal.



Perpendicular lines: خطوط متعامدة

~~Two distinct~~ A line l_1 with slope m $(m \neq 0)$, is perpendicular to the line l_2 if and only if the slope of l_2 is $\left(-\frac{1}{m}\right)$

Example: Suppose l_1 has slope $m_1 = 5$ and l_2 has slope m_2.

1) If l_1 is perpendicular $(\perp)$ to l_2, Find m_2.

$$m_2 = \frac{-1}{m_1} = -\frac{1}{5}$$

2) If l_1 is parallel $(/\!/)$ to l_2, find m_2.

$$m_2 = m_1 = 5$$