2.1 Quadratic Equations

A quadratic equation of one variable has a general form $ax^2 + bx + c = 0$, where $a \neq 0$

Example: 1) $x^2 - 3x + 2 = 0$

2) $3x^2 + 6x + 12 = 0$

3) $4x^2 - 9 = 0$

4) $2x^2 + 1 = x^2 - x \xrightarrow{\text{إعادة ترتيب}} x^2 + x + 1 = 0$

Solving quadratic equations:

1) Using quadratic formula:

Once the equation in the general form ($ax^2 + bx + c = 0$) then the solution(s) can be found using the quadratic formula: $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$b^2 - 4ac$ is called the discriminant (المميز)

Discriminant	Solutions
Discriminant > 0	$\dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$
Discriminant $= 0$	$-\dfrac{b}{2a}$
Discriminant < 0	no real solutions

Example: Solve the following system of Linear equations

$$\begin{cases} 3x + 4y = 1 & \text{---- } ① \\ 2x - 3y = 12 & \text{---- } ② \end{cases}$$

By elimination: multiply eq ① by 3 and multiply eq. ② by 4

You have
$$9x + 12y = 3$$
$$8x - 12y = 48$$

add the two equations: $\frac{17x}{17} = \frac{51}{17} \rightarrow \boxed{x = 3}$

Substitute $x = 3$ in eq ②: $8x - 12y = 48$

$8(3) - 12y = 48 \rightarrow 24 - 12y = 48$
$\quad -24 \quad\quad -24$

$\rightarrow \frac{-12y}{-12} = \frac{24}{-12} \rightarrow \boxed{y = -2}$

The solution is $(3, -2)$

check the solution: substitute $(3, -2)$ in eq①

$9x + 12y = 3 \rightarrow 9(3) + 12(-2) \overset{??}{=} 3$

$27 - 24 \overset{??}{=} 3$

$3 = 3$ ✓

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Example: Solve the following equation using the quadratic formula $x^2 - 3x + 2 = 0$ $\rightarrow a = 1$
$b = -3$
$c = 2$

discriminant : $b^2 - 4ac$

$$= (-3)^2 - 4(1)(2) = 9 - 8 = 1 > 0$$

Solutions : $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \dfrac{-(-3) \pm \sqrt{1}}{2(1)}$

$$x = \frac{3 \pm 1}{2}$$

$$x_1 = \frac{3+1}{2} = \frac{4}{2} = 2 \quad , \quad x_2 = \frac{3-1}{2} = \frac{2}{2} = 1$$

Solutions : 1, 2

Example: Solve the equation using the quadratic formula $3x^2 + 6x + 12 = 0$ $a = 3$, $b = 6$, $c = 12$

~~x_2~~ discriminant $= b^2 - 4ac = (6)^2 - 4(3)(12)$

$$= 36 - 144 = -108 < 0$$

The equation has no real solutions.

Example :- Solve the following using the quadratic formula $x^2+2x+1=0$ $\rightarrow a=1,\ b=2,\ c=1$

$$\text{discriminant} = b^2-4ac$$
$$= (2)^2-4(1)(1)$$
$$= 4-4=0 \quad \text{one real solution}$$

$$x = \frac{-b \pm \sqrt{b^2-4ac}}{2a} = \frac{-b \pm 0}{2a} = \frac{-b}{2a}$$

$$= \frac{-2}{2(1)} = \frac{-2}{2} = \boxed{-1}$$

2) Using Factoring

If the quadratic equation can be factored, then we can use the zero product property.

if $ab=0$, then $a=0$ or $b=0$ or both.

Example :- Solve using factoring $x^2-3x+2=0$

$$x^2-3x+2=0$$
$$(x-2)(x-1)=0$$

$$x-2=0 \quad \text{or} \quad x-1=0$$
$$+2 \quad +2 \qquad\qquad +1 \quad +1$$
$$\boxed{x=2} \quad \text{or} \quad \boxed{x=1}$$

Example: Solve the equations using factoring

1) $x^2 + 2x + 1 = 0$

$(x + 1)(x + 1) = 0$

$x + 1 = 0$ or $x + 1 = 0$

$-1 \quad -1$ $\qquad$ $-1 \quad -1$

$x = -1$ or $x = -1$ one solution (-1)

2)