2.2 Quadratic Equations

المعادلات التربيعية

A quadratic equation has a general form of

$y = f(x) = ax^2 + bx + c$, where $a \neq 0$

Example: 1) $f(x) = x^2 + 2x - 3$

2) $f(x) = 6 - 4x - 2x^2$

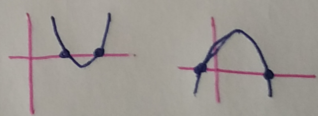
3) $y = (x+2)^2 - 3 = x^2 + 4x + 4 - 3 = x^2 + 4x + 1$

* The graph of the quadratic function is a parabola (قطع مكافئ) opens upward $\cup$ if $a > 0$ or downward $\cap$ if $a < 0$.

* We will learn how to:

1) Find the vertex (الرأس) of the parabola

2) Find the zeros of the quadratic ~~eq~~ function (x- intercepts) if they are real



3) Graph the quadratic functions.

The vertex of a parabola is the point:

$$\left(-\frac{b}{2a},\ f\left(-\frac{b}{2a}\right)\right)$$

The value of $f\left(-\frac{b}{2a}\right)$ is the optimal value of the function.

The optimal value is a maximum (قيمة عظمى) if the parabola opens downward , and it is a minimum (صغرى) if the parabola is upward

Example : Find the vertices and determine if they are maximum points or minimum points.

a) $f(x) = x^2 + 2x - 3 \rightarrow a = 1 , b = 2 , c = -3$

$a = 1$ ↓ opes upward

vertex $\left(\frac{-b}{2a} , f\left(\frac{-b}{2a}\right) \right)$

$$\frac{-b}{2a} = \frac{-2}{2(1)} = \frac{-2}{2} = -1$$

$$f(-1) = (-1)^2 + 2(-1) - 3 = 1 - 2 - 3 = -4$$

The vertex : $(-1 , -4)$

The parabola opens upward So the vertex is minimum point.

b) $f(x) = 6 - 4x - 2x^2 \rightarrow a = -2 , b = -4 , c = 6$

$a = -2$ ↓ opens downward.

vertex : $\left(\frac{-b}{2a} , f\left(\frac{-b}{2a}\right) \right)$

$$\frac{-b}{2a} = -\frac{(-4)}{2(-2)} = \frac{+4}{-4} = -1$$

$$f(-1) = 6 - 4(-1) - 2(-1)^2 = 6 + 4 - 2 = 8$$

The vertex is $(-1, 8)$

8 is a maximum since the parabola opens downward

* Zeros of a function

Zeros are the x- intercepts (to find them, set $y = 0$)

Example: Find the zeros of the following functions

a) $f(x) = x^2 + 2x - 3$

$$0 = x^2 + 2x - 3$$

$$0 = (x+3)(x-1)$$

So $x = -3$ or $x = 1$ Zeros are $-3, 1$

b) $f(x) = 6 - 4x - 2x^2$

$$0 = 6 - 4x - 2x^2$$

use the quadratic form $a = -2,\ b = -4,\ c = 6$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$b^2 - 4ac = (-4)^2 - 4(-2)(6) = 16 + 48 = 64 > 0$

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$$x = \frac{-(-4) \pm \sqrt{64}}{2(-2)} = \frac{4 \pm 8}{-4}$$

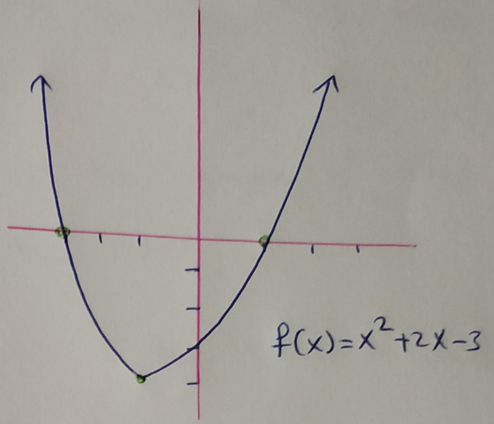
$x_1 = \frac{4+8}{-4} = \frac{12}{-4} = -3$, $x_2 = \frac{4-8}{-4} = \frac{-4}{-4} = 1$

Zeros are $-3, 1$

Graph the function $f(x) = x^2 + 2x - 3$

The vertex is $(-1, 4)$

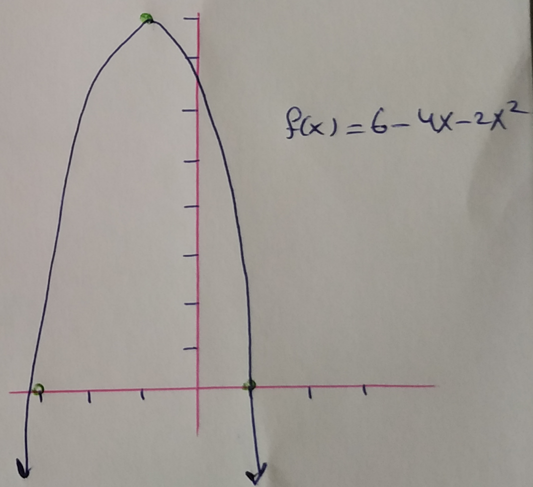
The zeros are $\{-3, 1$



Graph the function $f(x) = 6 - 4x - 2x^2$

The vertex is $(-1, 8)$

The zeros are $-3, 1$

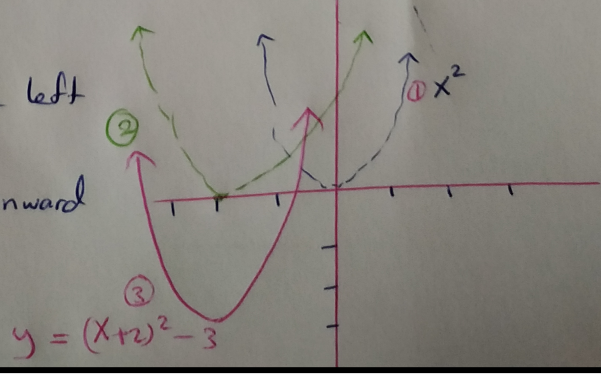


Graph $y = (x+2)^2 - 3$

① Graph $y = x^2$

② shift 2 units to the left
You get $(x+2)^2$

③ shift 3 units downward



* Shifting:

1) The graph of $f(x-c)$ is the graph of $f(x)$ shifted c units to the right $\rightarrow$

2) The graph of $f(x+c)$ is the graph of $f(x)$ shifted c units to the Left. $\leftarrow$

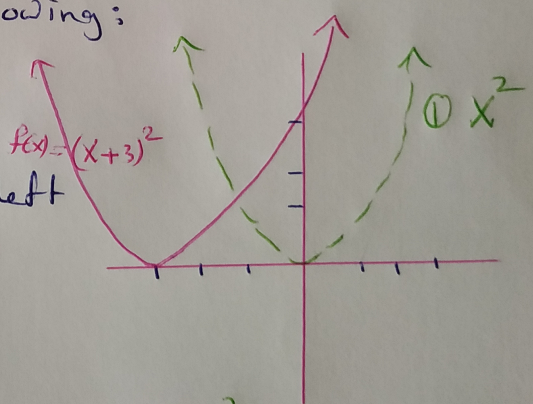
3) The graph of $f(x)+c$ is the graph of $f(x)$ shifted c units upward. $\uparrow$

4) The graph of $f(x)-c$ is the graph of $f(x)$ shifted c units downward $\downarrow$

Example: Graph the following:

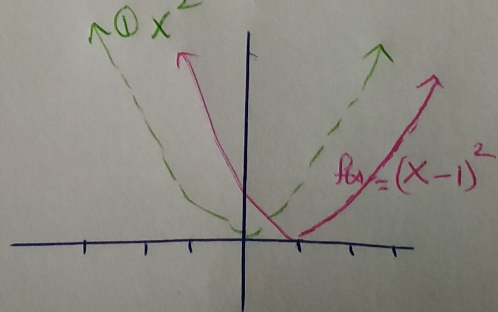
a) $f(x) = (x+3)^2$

x^2 shifted 3 units to the left



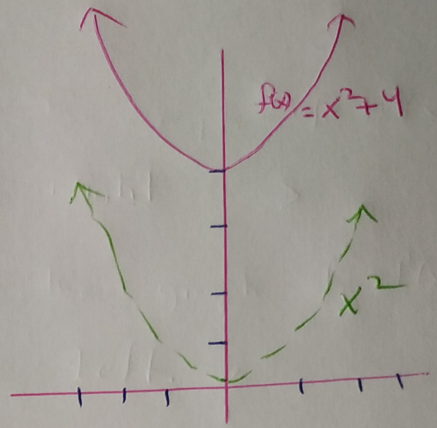
b) $f(x) = (x-1)^2$

x^2 shifted 1 unit to the right



c) $f(x) = x^2 + 4$

x^2 shifted 4 units upward



d) $f(x) = x^2 - 2$

x^2 shifted 2 units downward