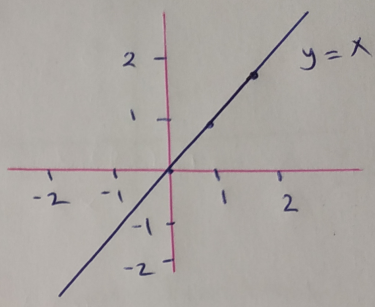
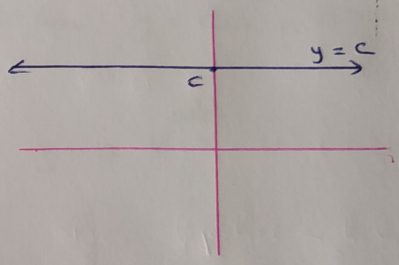
2.4 Special Functions and Their Graphs

1) Identity function : $y = x$



2) Constant function : $y = c$

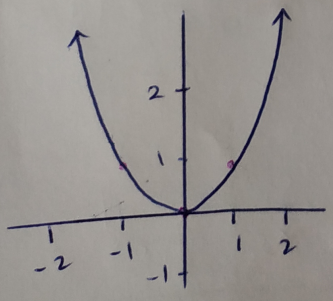
Example :- $y = 4$, $y = -1$, ...



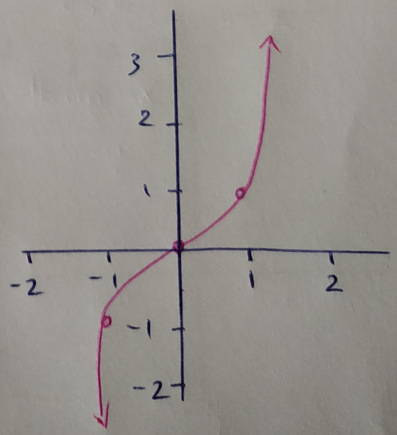
3) Power functions :- $y = a x^b$, $b > 0$

Examples :

① $y = x^2$

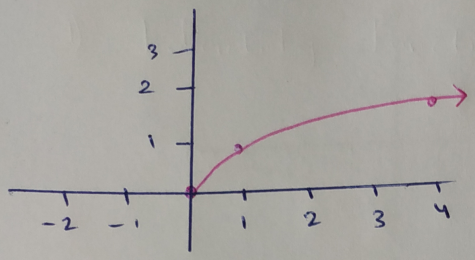


② $y = x^3$

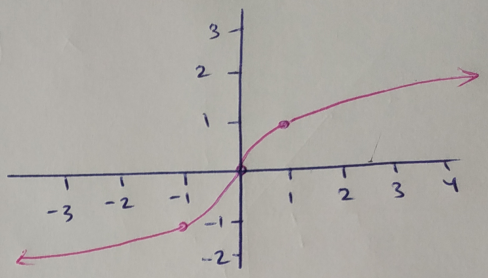


* $y = x^2$ and $y = x^3$ are called polynomial functions.

③ $y = \sqrt{x} = x^{\frac{1}{2}}$



④ $y = \sqrt[3]{x} = x^{\frac{1}{3}}$

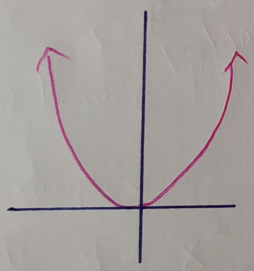


* $y = \sqrt{x}$ and $y = \sqrt[3]{x}$ are called root functions

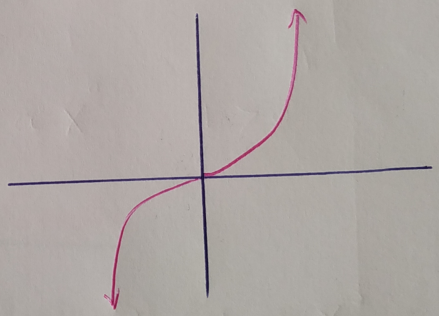
The graphs for any power function $y = ax^b$, $b > 0$:

~~if~~ ~~[illegible]~~ :

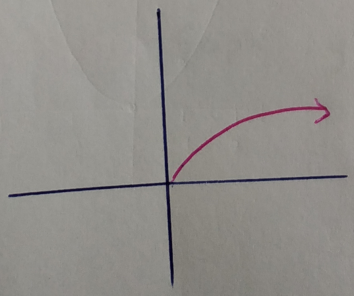
① $y = ax^{\text{even}}$



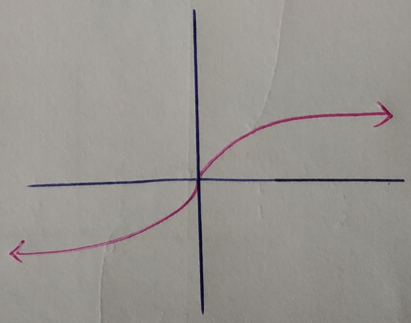
② $y = a x^{\text{odd}}$



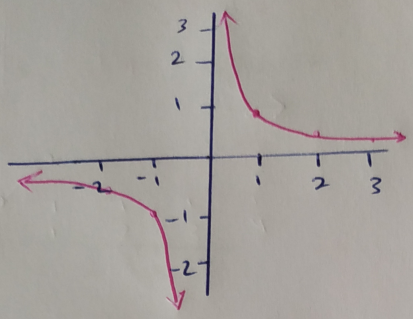
③ $y = a\sqrt[\text{even}]{x}$



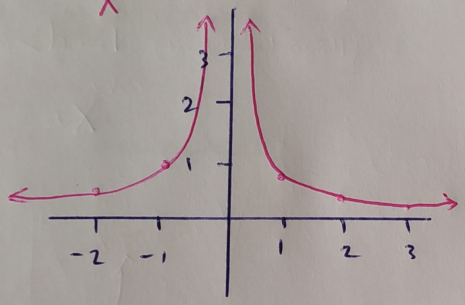
④ $y = a\sqrt[\text{odd}]{x}$



4) $y = \frac{1}{x} = x^{-1}$ (Rational function)



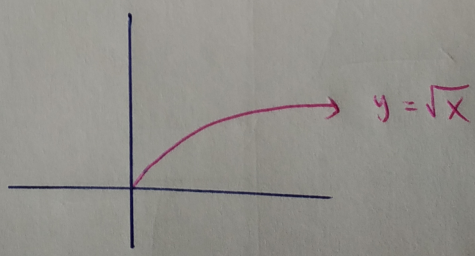
5) $y = \frac{1}{x^2} = x^{-2}$ (Rational function)



Shifts of Graphs

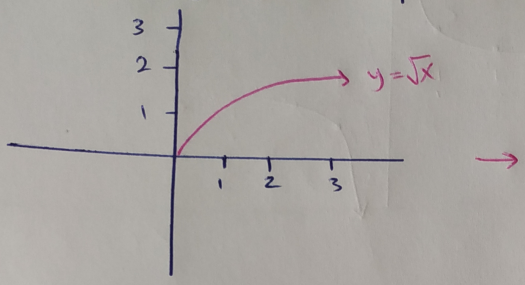
The graph of $y = f(x-h)+k$ is the graph of $y=f(x)$ shifted h units in the x-direction (+ left, − right) and k units in the y-direction (+ up, − down)

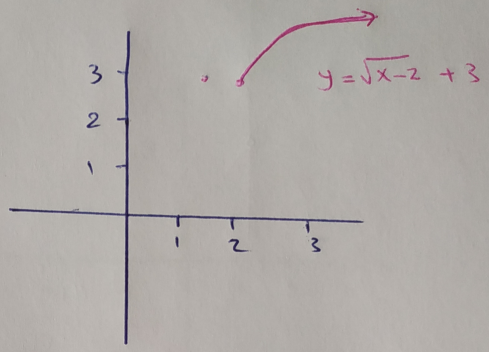
Example:- Graph $y = \sqrt{x}$



Graph $y = \sqrt{x-2} + 3$

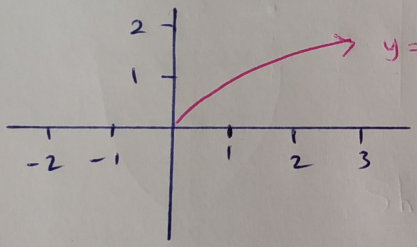
is the graph of $y = \sqrt{x}$ shifted 2 units to the right and 3 units upward

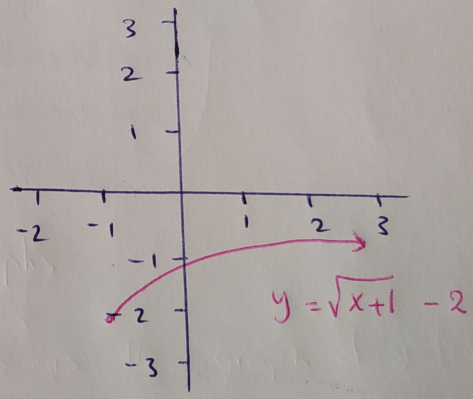




Graph $y = \sqrt{x+1} - 2$

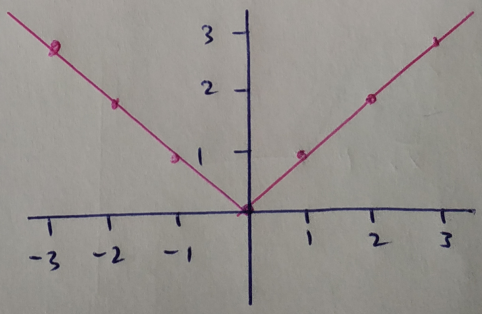
is the graph of $y = \sqrt{x}$ shifted 1 unit to the left and 2 units downward





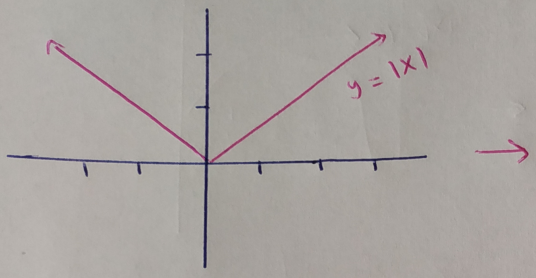
6) Absolute value function: $y = |x|$

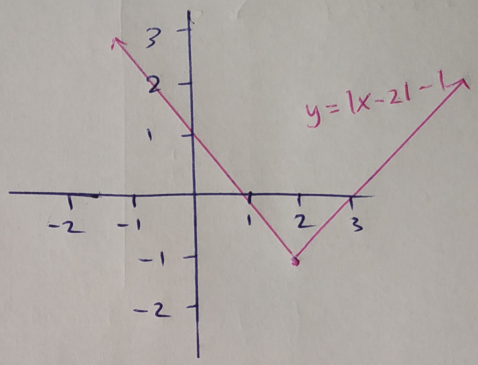
$$f(x) = \begin{cases} x & \text{if } x \geq 0 \\ -x & \text{if } x < 0 \end{cases}$$



Graph $f(x) = |x-2| - 1$

is graph of $f(x) = |x|$ shifted 2 units to the right and 1 unit downward





Find the x-intercepts of $y = |x-2| - 1$

x-intercepts (put $y = 0$)

$0 = (x-2) - 1$ or $0 = -(x-2) - 1$

$0 = x - 3$ $0 = -x + 2 - 1$

$0 = -x + 1$

$\boxed{x = 3}$ $\boxed{x = 1}$

* Piecewise Defined Functions: (متعدد القاعدة)

Graph $f(x) = \begin{cases} x^2 & \text{if } x \leq 2 \\ 4 & \text{if } x > 2 \end{cases}$