4.1 Linear Inequalities in Two Variables

The solutions of one inequality in two variables are the ordered pairs (x,y) that satify the inequality (all these ordered pairs are called region)

Example:- $y < x$ is a linear inequality such that $(1,0)$, $(3,2)$, $(0,-1)$ and $(-2,-5)$ are solutions of the inequality $y<x$

but $(3,7)$, $(-4,-3)$ and $(2,2)$ are not.

Example:- Graph the inequality $y < x$.

x	y
0	0
2	2

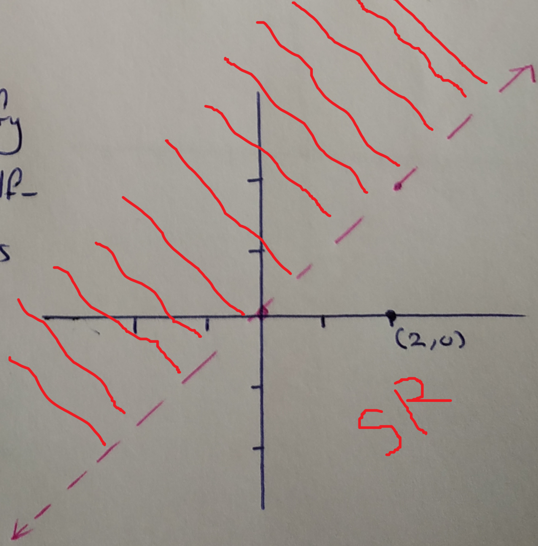
① Graph the line $y = x$ (as متقطع dashed line)

② Pick a test point that is not on the line.

Let's choose. $(\overset{x}{2},\overset{y}{0})$

$0 < 2$ ✓ so $(2,0)$ satisfy the inequality → the half-plane that contains $(2,0)$ is the solution region

المنطقة التي تحتوي (2,0) هي
منطقة الحل للمتباينة



If we had chosen $(\overset{x}{0}, \overset{y}{4})$

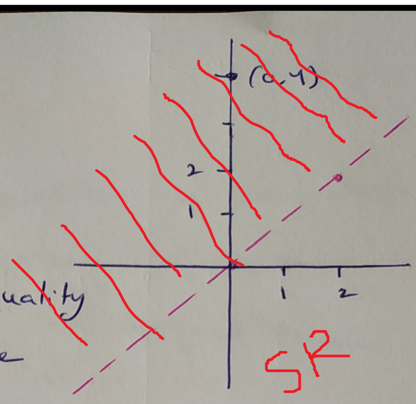
$y < x$

$4 \not< 0$

$(0,4)$ do not satisfy the inequality

So the other half-plane is the

Solution region.



Example:- Graph the inequality $4x - 2y \leq 6$.

① Graph the line $4x - 2y = 6$ (as a solid line) متواصل

if $x = 0 \rightarrow 4(0) - 2y = 6$

$\rightarrow y = -\frac{6}{2} = -3$

if $y = 0 \rightarrow 4x - 2(0) = 6$

$4x = 6 \rightarrow x = \frac{6}{4} = \frac{3}{2}$

x	y
0	-3
$\frac{3}{2}$	0

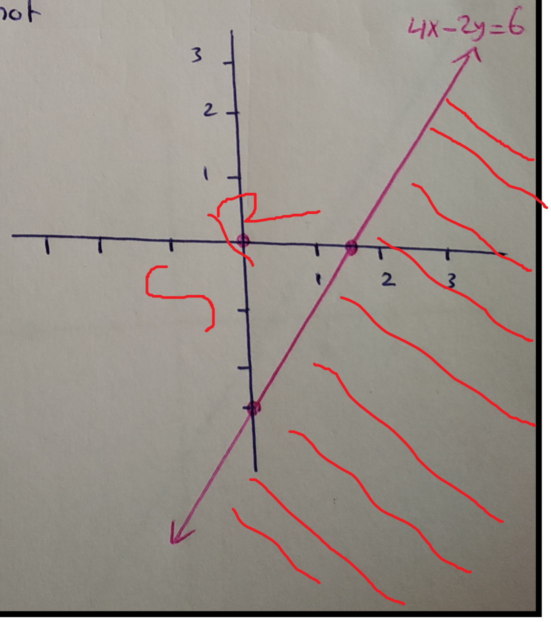
② Pick a test point that is not on the line

take $(0,0)$

$4x - 2y \leq 6$

$4(0) - 2(0) \overset{?}{\leq} 6$

$0 \overset{?}{\leq} 6$ ✓



So the point $(0,0)$ satisfy the inequality $4x - 2y \leq 6$

System of Linear Inequalities :

If we have two or more inequalities in two variables, then the solution of this system is the intersection of the solution sets of all the inequalities.

نبحث عن منطقة الحل المشتركة لجميع المتباينات.

Example:- Graph the solution of the system

$$\begin{cases} 3x - 2y \geq 4 \\ x + y - 3 > 0 \end{cases}$$

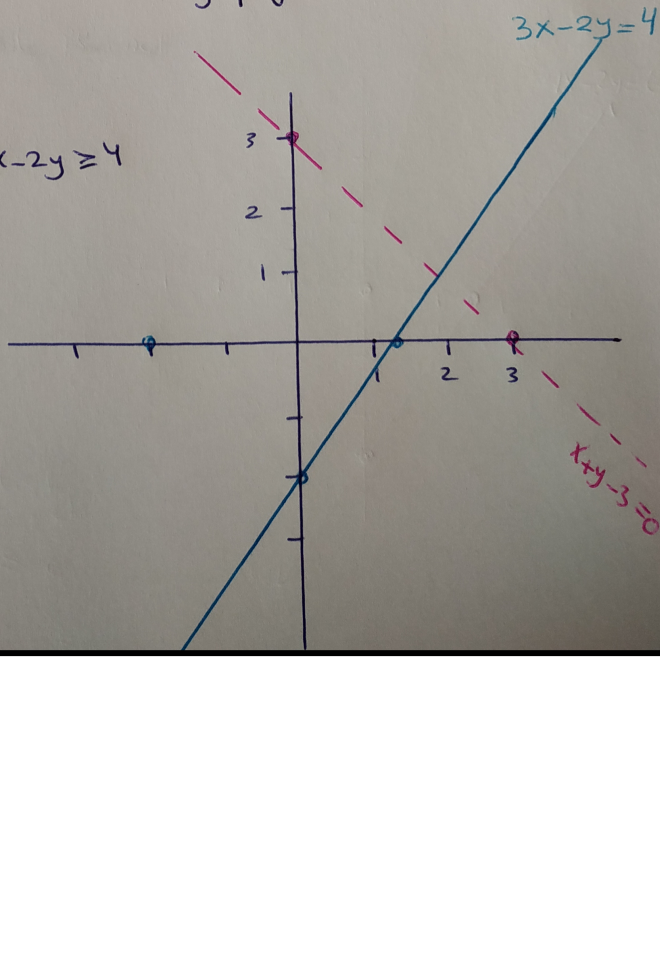
① Graph the lines $3x-2y=4$ (solid) and $x+y-3=0$ (dashed)

$3x - 2y = 4$

x	y
0	-2
$\frac{4}{3}$	0

$x + y - 3 = 0$

x	y
0	3
3	0



② Pick a test point for $3x-2y \geq 4$

take $(0,0)$

$3(0) - 2(0) \overset{??}{\geq} 4$

$0 \ngeq 4$

So the other half-plane is the solution region for $3x - 2y \geq 4$

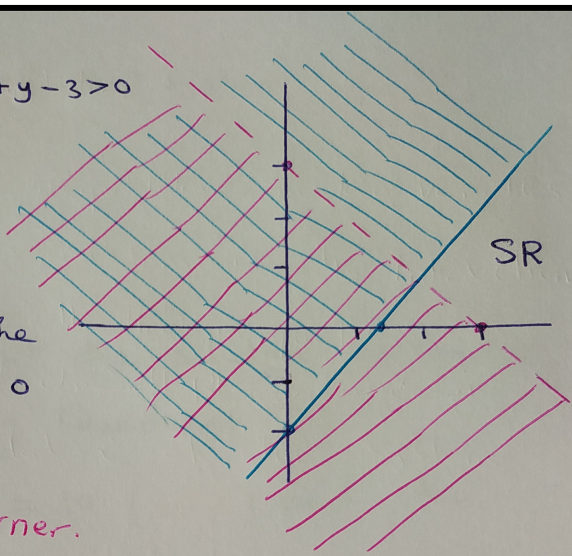
③ Pick a test point for $x+y-3>0$

take $(0,0)$

$0+0-3 \overset{??}{>} 0$

$-3 \not> 0$

So the other half-plane is the solution region for $x+y-3>0$



The two regions form a corner.

to find the corner solve $\begin{cases} 3x-2y=4 \\ (x+y-3=0)_{\times -3} \end{cases}$

$$
\begin{aligned}
3x - 2y &= 4 \\
-3x - 3y + 9 &= 0 \\
\hline
-5y + 9 = 4 \;\rightarrow\; -5y = -5 \;&\rightarrow\; \boxed{y=1}
\end{aligned}
$$

$3x - 2y = 4$

$3x - 2(1) = 4 \;\rightarrow\; 3x = 6 \;\rightarrow\; \boxed{x=2}$

The point $(2,1)$ is the corner.

Example:- Graph the solution of the system

$$\begin{cases} x + 2y \leq 10 \\ 2x + y \leq 14 \\ x \geq 0 \;, \; y \geq 0 \end{cases}$$

① $x \geq 0$ and $y \geq 0$ restrict the solution to Quadrant I (and the axes bounding Quadrant I)

② Graph the line $x + 2y = 10$ (solid) and the line $2x + y = 14$ (solid).

$x + 2y = 10$

x	y
0	5
10	0

$2x + y = 14$

x	y
0	14
7	0

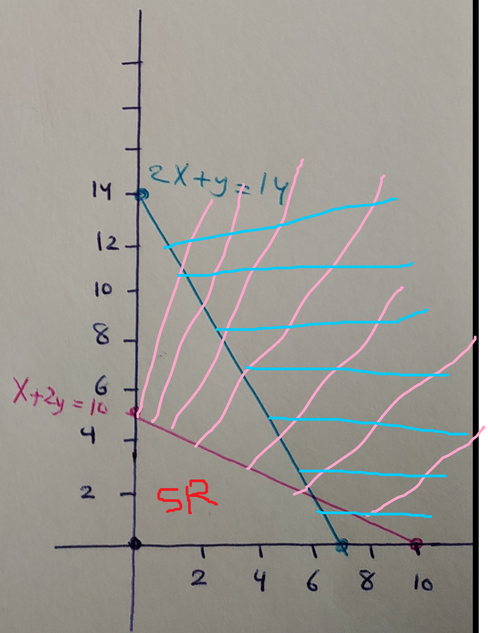
③ Pick a test point for $x + 2y \leq 10$

take $(\overset{x}{0}, \overset{y}{0})$

$0 + 2(0) \overset{??}{\leq} 10$

$0 \leq 10$ ✓

So this half-plane is the solution region for $x + 2y \leq 10$



④ Pick a test point for $2x + y \leq 14$

take $(0, 0)$

$2(0) + 0 \overset{??}{\leq} 14$

$0 \leq 14$ ✓

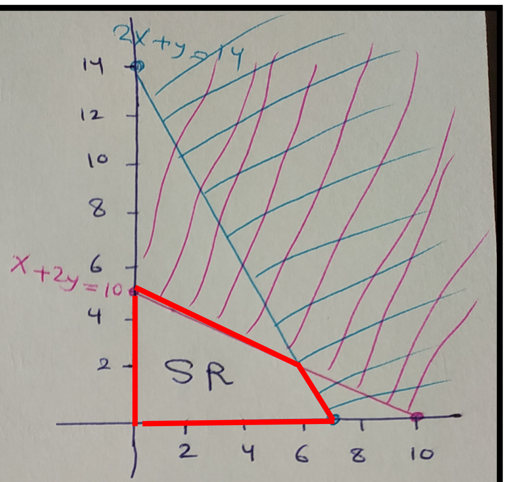
So this half-plane is the solution region for $2x + y \leq 14$

We have 4 Corners

(0,0), (0,5), (7,0)

The last corner is found by

Solving $\begin{cases} (x + 2y \leq 10) * -2 \\ 2x + y = 14 \end{cases}$



$$-2x - 4y = -20$$
$$2x + y = 14$$
$$-3y = -6 \rightarrow \boxed{y = 2}$$

$$x + 2y = 10$$
$$x + 2(2) = 10 \rightarrow x + 4 = 10 \rightarrow \boxed{x = 6}$$

So (6,2) is the fourth corner.