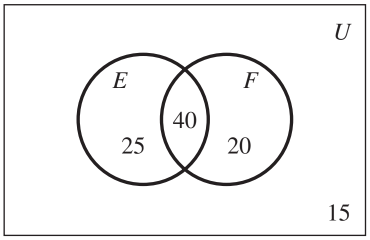
Answers

Below are the answers to odd-numbered Section Exercises and all the Chapter Review and Chapter Test problems.

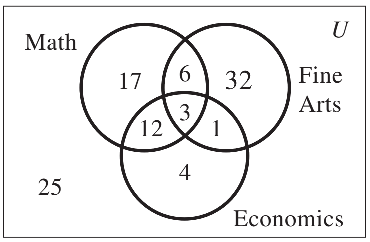
0.1 EXERCISES

1. $\in$ 3. $\notin$ 5. $\{1, 2, 3, 4, 5, 6, 7\}$
7. $\{x: x$ is a natural number greater than 2 and less than 8$\}$
9. $\varnothing, A, B$ 11. no 13. $D \subseteq C$
15. $A \subseteq B$ or $B \subseteq A$ 17. yes 19. no
21. A and B, B and D, C and D
23. $A \cap B = \{4, 6\}$ 25. $A \cap B = \varnothing$
27. $A \cup B = \{1, 2, 3, 4, 5\}$
29. $A \cup B = \{1, 2, 3, 4\} = B$
31. $A' = \{4, 6, 9, 10\}$
33. $A \cap B' = \{1, 2, 5, 7\}$ 35. $(A \cup B)' = \{6, 9\}$
37. $A' \cup B' = \{1, 2, 4, 5, 6, 7, 9, 10\}$
39. $\{1, 2, 3, 5, 7, 9\}$ 41. $\{4, 6, 8, 10\}$
43. $A - B = \{1, 7\}$
45. $A - B = \varnothing$ or $\{\ \}$
47. (a) $L = \{00, 01, 04, 05, 06, 07\}$
$H = \{00, 01, 06, 07, 08\}$
$C = \{01, 02, 03, 08, 09\}$
(b) no
(c) C' is the set of years when the percent change from low to high was 35% or less.
(d) $\{00, 02, 03, 04, 05, 06, 07, 09\}$ = the set of years when the high was 11,000 or less or the percent change was 35% or less.
(e) $\{02, 03, 08, 09\}$ = the set of years when the low was 8000 or less and the percent change exceeded 35%.
49. (a) 130 (b) 840 (c) 520
51.



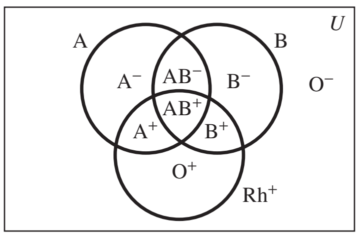
(a) 40
(b) 85
(c) 25

53.



(a) 25
(b) 43
(c) 53

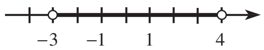
55. (a) and (b)



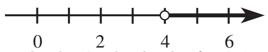
(c) A^+: 34%; B^+: 9%; O^+: 38%; AB^+: 3%; O^-: 7%; A^-: 6%; B^-: 2%; AB^-: 1%

0.2 EXERCISES

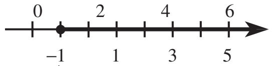
1. (a) irrational (b) rational, integer
(c) rational, integer, natural (d) meaningless
3. (a) Commutative (b) Distributive
5. (a) Multiplicative identity (b) Additive inverse
7. $<$ 9. $<$ 11. $<$ 13. $>$
15. 11 17. 4 19. 2 21. $\frac{-4}{3}$ 23. 3 25. $\frac{17}{11}$
27. entire line 29. $(1, 3]$; half open 31. $(2, 10)$; open
33. $-3 \le x < 5$ 35. $x > 4$
37. $(-3, 4)$



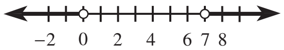
39. $(4, +\infty)$



41. $[-1, +\infty)$



43. $(-\infty, 0) \cup (7, +\infty)$



45. -0.000038585
47. 9122.387471 49. 3240.184509
51. (a) \$1088.91 (b) \$258.62 (c) \$627.20
53. (a) Formula (2) is slightly more accurate, giving 15.762%.
(b) (1): 17.665%; (2): 28.983%
55. (a) $82{,}401 \le I \le 171{,}850$; $171{,}851 \le I \le 373{,}650$; $I > 373{,}650$
(b) $T = \$4681.25$ for $I = \$34{,}000$
$T = \$16{,}781.25$ for $I = \$82{,}400$
(c) $[4681.25, 16{,}781.25]$

0.3 EXERCISES

1. 256 3. -64 5. $\frac{1}{9}$ 7. $-\frac{9}{4}$
9. 6^8 11. $\frac{1}{10}$ 13. $9^0 = 1$ 15. 3^9
17. $\left(\frac{3}{2}\right)^2 = \frac{9}{4}$ 19. $1/x^6$ 21. x/y^2 23. x^7
25. $x^{-2} = 1/x^2$ 27. x^4 29. y^{12} 31. x^{12}
33. x^2y^2 35. $16/x^{20}$ 37. $x^8/(16y^4)$
39. $-16a^2/b^2$ 41. $2/(xy^2)$ 43. $1/(x^9y^6)$
45. $(a^{18}c^{12})/b^6$
47. (a) $1/(2x^4)$ (b) $1/(16x^4)$ (c) $1/x^4$ (d) 8
49. x^{-1} 51. $8x^3$ 53. $\frac{1}{4}x^{-2}$
55. $-\frac{1}{8}x^3$ 57. 2.0736 59. 0.1316872428
61. $S = \$2114.81$; $I = \$914.81$
63. $S = \$9607.70$; $I = \$4607.70$
65. \$7806.24
67. (a) 5, 15, 22
(b) \$3421.6, \$6070.0, \$9066.9 billion
(c) \$16,085 billion
(d) yes
69. (a) 456, 971, 1143
(b) 439

(c) Two possibilities might be more environmental protections and the fact that there are only a limited number of species.
(d) There are only a limited number of species. Also, below some threshold level the ecological balance might be lost, perhaps resulting in an environmental catastrophe (which the equation could not predict). Upper limit = 1883

71. (a) 10 (b) \$1385.5 billion (c) \$2600.8 billion (d) \$4304.3 billion

0.4 EXERCISES

1. (a) $\frac{16}{3} \approx 5.33$ (b) 1.2
3. (a) 8 (b) not real
5. $\frac{9}{4}$
7. (a) 4 (b) $\frac{1}{4}$
9. $(6.12)^{4/9} \approx 2.237$
11. $m^{3/2}$
13. $(m^2n^5)^{1/4}$
15. $2\sqrt{x}$
17. $\sqrt[6]{x^7}$
19. $-1/(4\sqrt[4]{x^5})$
21. $y^{3/4}$
23. $z^{19/4}$
25. $1/y^{5/2}$
27. x
29. $1/y^{21/10}$
31. $x^{1/2}$
33. $1/x$
35. $8x^2$
37. $8x^2y^2\sqrt{2y}$
39. $2x^2y\sqrt[3]{5x^2y^2}$
41. $6x^2y\sqrt{x}$
43. $42x^3y^2\sqrt{x}$
45. $2xy^5/3$
47. $2b\sqrt[4]{b}/(3a^2)$
49. $1/9$
51. 7
53. $\sqrt{6}/3$
55. $\sqrt{mx}/x$
57. $\sqrt[3]{mx^2}/x^2$
59. $-\frac{2}{3}x^{-2/3}$
61. $3x^{3/2}$
63. $(3\sqrt{x})/2$
65. $1/(2\sqrt{x})$
67. (a) $10^{8.5} = 10^{17/2} = \sqrt{10^{17}}$ (b) $10^{9.0} = 1{,}000{,}000{,}000$ (c) $10^{2.1} \approx 125.9$
69. (a) $S = 1000\sqrt{\left(1 + \frac{r}{100}\right)^5}$ (b) \$1173.26 (nearest cent)
71. (a) $P = 0.924\sqrt[100]{t^{13}}$ (b) 2005 to 2010; 0.1074 billion vs. 0.0209 billion. By 2045 and 2050 the population will be much larger than earlier in the 21st century, and there is a limited number of people that any land can support—in terms of both space and food.
73. 74 kg
75. 39,491
77. (a) 10 (b) 259

0.5 EXERCISES

1. (a) 2 (b) -1 (c) 10 (d) one
3. (a) 5 (b) -14 (c) 0 (d) several
5. (a) 5 (b) 0 (c) 2 (d) -5
7. -12
9. -296
11. $\frac{-7}{31}$
13. 87.4654
15. $21pq - 2p^2$
17. $m^2 - 7n^2 - 3$
19. $3q + 12$
21. $x^2 - 1$
23. $35x^5$
25. $3rs$
27. $2ax^4 + a^2x^3 + a^2bx^2$
29. $6y^2 - y - 12$
31. $12 - 30x^2 + 12x^4$
33. $16x^2 + 24x + 9$
35. $0.01 - 16x^2$
37. $36x^2 - 9$
39. $x^4 - x^2 + \frac{1}{4}$
41. $0.1x^2 - 1.995x - 0.1$
43. $x^3 - 8$
45. $x^8 + 3x^6 - 10x^4 + 5x^3 + 25x$
47. (a) $9x^2 - 21x + 13$ (b) 5
49. $3 + m + 2m^2n$
51. $8x^3y^2/3 + 5/(3y) - 2x^2/(3y)$
53. $x^3 + 3x^2 + 3x + 1$
55. $8x^3 - 36x^2 + 54x - 27$
57. $x^2 - 2x + 5 - 11/(x + 2)$
59. $x^2 + 3x - 1 + (-4x + 2)/(x^2 + 1)$
61. $x + 2x^2$
63. $x - x^{1/2} - 2$
65. $x - 9$
67. $4x^2 + 4x$
69. $55x$
71. (a) $49.95 + 0.49x$ (b) \$114.63
73. (a) $4000 - x$ (b) $0.10x$ (c) $0.08(4000 - x)$ (d) $0.10x + 0.08(4000 - x)$
75. $(15 - 2x)(10 - 2x)x$

0.6 EXERCISES

1. $3b(3a - 4a^2 + 6b)$
3. $2x(2x + 4y^2 + y^3)$
5. $(7x^2 + 2)(x - 2)$
7. $(6 + y)(x - m)$
9. $(x + 2)(x + 6)$
11. $(x - 16)(x + 1)$
13. $(7x + 4)(x - 2)$
15. $(x - 5)^2$
17. $(7a + 12b)(7a - 12b)$
19. (a) $(3x - 1)(3x + 8)$ (b) $(9x + 4)(x + 2)$
21. $x(4x - 1)$
23. $(x^2 - 5)(x + 4)$
25. $(x - 3)(x + 2)$
27. $2(x - 7)(x + 3)$
29. $2x(x - 2)^2$
31. $(2x - 3)(x + 2)$
33. $3(x + 4)(x - 3)$
35. $2x(x + 2)(x - 2)$
37. $(5x + 2)(2x + 3)$
39. $(5x - 1)(2x - 9)$
41. $(y^2 + 4x^2)(y + 2x)(y - 2x)$
43. $(x + 2)^2(x - 2)^2$
45. $(2x + 1)(2x - 1)(x + 1)(x - 1)$
47. $x + 1$
49. $1 + x$
51. $(x + 1)^3$
53. $(x - 4)^3$
55. $(x - 4)(x^2 + 4x + 16)$
57. $(3 + 2x)(9 - 6x + 4x^2)$
59. $P(1 + rt)$
61. $m(c - m)$
63. (a) $p(10{,}000 - 100p)$; $x = 10{,}000 - 100p$ (b) 6200
65. (a) $R = x(300 - x)$ (b) $300 - x$

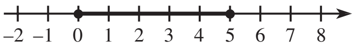
0.7 EXERCISES

1. $2y^3/z$
3. $\frac{1}{3}$
5. $(x - 1)/(x - 3)$
7. $20x/y$
9. $\frac{32}{3}$
11. $3x + 9$
13. $\dfrac{-(x + 1)(x + 3)}{(x - 1)(x - 3)}$
15. $15bc^2/2$
17. $5y/(y - 3)$
19. $\dfrac{-x(x - 3)(x + 2)}{x + 3}$
21. $\dfrac{1}{x + 1}$
23. $\dfrac{4a - 4}{a(a - 2)}$
25. $\dfrac{-x^2 + x + 1}{x + 1}$
27. $\dfrac{16a + 15a^2}{12(x + 2)}$
29. $\dfrac{79x + 9}{30(x - 2)}$
31. $\dfrac{9x + 4}{(x - 2)(x + 2)(x + 1)}$
33. $(7x - 3x^3)/\sqrt{3 - x^2}$
35. $\frac{1}{6}$
37. xy
39. $\dfrac{x + 1}{x^2}$
41. $\dfrac{1}{\sqrt{a}} = \dfrac{\sqrt{a}}{a}$
43. $\dfrac{x - 2}{(x - 3)\sqrt{x^2 + 9}}$

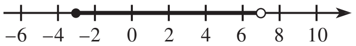
45. (a) -12 (b) $\frac{25}{36}$
47. $2b^2 - a$
49. $(1 - 2\sqrt{x} + x)/(1 - x)$
51. $1/(\sqrt{x + h} + \sqrt{x})$
53. $(bc + ac + ab)/abc$
55. (a) $\dfrac{0.1x^2 + 55x + 4000}{x}$
(b) $0.1x^2 + 55x + 4000$
57. $\dfrac{t^2 + 9t}{(t + 3)^2}$

CHAPTER 0 REVIEW EXERCISES

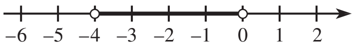
1. yes 2. no 3. no
4. $\{1, 2, 3, 4, 9\}$ 5. $\{5, 6, 7, 8, 10\}$
6. $\{1, 2, 3, 4, 9\}$
7. yes, $(A' \cup B')' = \{1, 3\} = A \cap B$
8. (a) Commutative Property of Addition
(b) Associative Property of Multiplication
(c) Distributive Law
9. (a) irrational (b) rational, integer
(c) meaningless
10. (a) $>$ (b) $<$ (c) $>$
11. 6 12. 142 13. 10
14. 5/4 15. 9 16. -29
17. 13/4 18. -10.62857888
19. (a) $[0, 5]$, closed



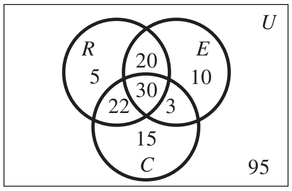
(b) $[-3, 7)$, half-open



(c) $(-4, 0)$, open



20. (a) $-1 < x < 16$ (b) $-12 \le x \le 8$
(c) $x < -1$
21. (a) 1 (b) $2^{-2} = 1/4$ (c) 4^6 (d) 7
22. (a) $1/x^2$ (b) x^{10} (c) x^9 (d) $1/y^8$
(e) y^6
23. $-x^2y^2/36$ 24. $9y^8/(4x^4)$ 25. $y^2/(4x^4)$
26. $-x^8z^4/y^4$ 27. $3x/(y^7z)$ 28. $x^5/(2y^3)$
29. (a) 4 (b) 2/7 (c) 1.1
30. (a) $x^{1/2}$ (b) $x^{2/3}$ (c) $x^{-1/4}$
31. (a) $\sqrt[3]{x^2}$ (b) $1/\sqrt{x} = \sqrt{x}/x$ (c) $-x\sqrt{x}$
32. (a) $5y\sqrt{2x}/2$ (b) $\sqrt[3]{x^2y}/x^2$
33. $x^{5/6}$ 34. y
35. $x^{17/4}$ 36. $x^{11/3}$
37. $x^{2/5}$ 38. x^2y^8
39. $2xy^2\sqrt{3xy}$ 40. $25x^3y^4\sqrt{2y}$
41. $6x^2y^4\sqrt[3]{5x^2y^2}$ 42. $8a^2b^4\sqrt{2a}$
43. $2xy$ 44. $4x\sqrt{3xy}/(3y^4)$
45. $-x - 2$ 46. $-x^2 - x$
47. $4x^3 + xy + 4y - 4$ 48. $24x^5y^5$
49. $3x^2 - 7x + 4$ 50. $3x^2 + 5x - 2$
51. $4x^2 - 7x - 2$ 52. $6x^2 - 11x - 7$
53. $4x^2 - 12x + 9$ 54. $16x^2 - 9$
55. $2x^4 + 2x^3 - 5x^2 + x - 3$
56. $8x^3 - 12x^2 + 6x - 1$
57. $x^3 - y^3$
58. $(2/y) - (3xy/2) - 3x^2$
59. $3x^2 + 2x - 3 + (-3x + 7)/(x^2 + 1)$
60. $x^3 - x^2 + 2x + 7 + 21/(x - 3)$ 61. $x^2 - x$
62. $2x - a$ 63. $x^3(2x - 1)$
64. $2(x^2 + 1)^2(1 + x)(1 - x)$ 65. $(2x - 1)^2$
66. $(4 + 3x)(4 - 3x)$ 67. $2x^2(x + 2)(x - 2)$
68. $(x - 7)(x + 3)$ 69. $(3x + 2)(x - 1)$
70. $(x - 3)(x - 2)$ 71. $(x - 12)(x + 2)$
72. $(4x + 3)(3x - 8)$ 73. $(2x + 3)^2(2x - 3)^2$
74. $x^{2/3} + 1$
75. (a) $\dfrac{x}{(x + 2)}$ (b) $\dfrac{2xy(2 - 3xy)}{2x - 3y}$
76. $\dfrac{x^2 - 4}{x(x + 4)}$ 77. $\dfrac{(x + 3)}{(x - 3)}$
78. $\dfrac{x^2(3x - 2)}{(x - 1)(x + 2)}$ 79. $(6x^2 + 9x - 1)/(6x^2)$
80. $\dfrac{4x - x^2}{4(x - 2)}$ 81. $-\dfrac{x^2 + 2x + 2}{x(x - 1)^2}$
82. $\dfrac{x(x - 4)}{(x - 2)(x + 1)(x - 3)}$
83. $\dfrac{(x - 1)^3}{x^2}$ 84. $\dfrac{1 - x}{1 + x}$
85. $3(\sqrt{x} + 1)$ 86. $2/(\sqrt{x} + \sqrt{x - 4})$
87. (a)



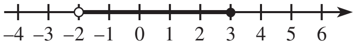
R: recognized
E: exercise
C: community involvement

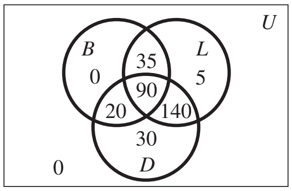
(b) 10 (c) 100
88. 52.55%
89. 16
90. (a) \$4115.27 (b) \$66,788.69
91. (a) 939,577 (b) 753,969
92. (a) 1.1 inch, about quarter-sized
(b) 104 mph
93. (a) $10{,}000\left[\dfrac{(0.0065)(1.0065)^n}{(1.0056)^n - 1}\right]$ (b) \$243.19 (for both)
94. (a) $S = k\sqrt[3]{A}$ (b) $\sqrt[3]{2.25} \approx 1.31$
95. $26x - 300 - 0.001x^2$
96. $1{,}450{,}000 - 3625x$
97. $(50 + x)(12 - 0.5x)$
98. (a) $\dfrac{12{,}000p}{100 - p}$
(b) \$0. It costs nothing if no effort is made to remove pollution.

(c) \$588,000
(d) Undefined. Removing 100% would be impossible, and the cost of getting close would be enormous.

99. $\dfrac{56x^2 + 1200x + 8000}{x}$

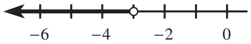
CHAPTER 0 TEST

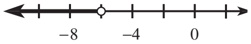
1. (a) {3, 4, 6, 8} (b) {3, 4}; {3, 6}; or {4, 6}
   (c) {6} or {8}
2. 21
3. (a) 8 (b) 1 (c) $\frac{1}{2}$ (d) -10
   (e) 30 (f) $\frac{5}{6}$ (g) $\frac{2}{3}$ (h) -3
4. (a) $\sqrt[5]{x}$ (b) $\dfrac{1}{\sqrt[4]{x^3}}$
5. (a) $\dfrac{1}{x^5}$ (b) $\dfrac{x^{21}}{y^6}$
6. (a) $\dfrac{\sqrt{5x}}{5}$ (b) $2a^2b^2\sqrt{6ab}$ (c) $\dfrac{1 - 2\sqrt{x} + x}{1 - x}$
7. (a) 5 (b) -8 (c) -5
8. $(-2, 3]$ 

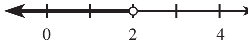
9. (a) $2x^2(4x - 1)$ (b) $(x - 4)(x - 6)$
   (c) $(3x - 2)(2x - 3)$ (d) $2x^3(1 + 4x)(1 - 4x)$
10. (c); -2
11. $2x + 1 + \dfrac{2x - 6}{x^2 - 1}$
12. (a) $19y - 45$ (b) $-6t^6 + 9t^9$
    (c) $4x^3 - 21x^2 + 13x - 2$ (d) $-18x^2 + 15x - 2$
    (e) $4m^2 - 28m + 49$ (f) $\dfrac{x^4}{3x + 9}$ (g) $\dfrac{x^7}{81}$
    (h) $\dfrac{6 - x}{x - 8}$ (i) $\dfrac{x^2 - 4x - 3}{x(x - 3)(x + 1)}$
13. $\dfrac{y - x}{y + xy^2}$
14. (a) 

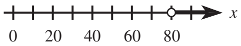
    (b) 0
    (c) 175
15. \$4875.44 (nearest cent)

1.1 EXERCISES

1. $x = -9/4$ **3.** $x = 0$ **5.** $x = -32$
7. $x = -29/2$ **9.** $x = -5$ **11.** $x = 17/13$
13. $x = -1/3$ **15.** $x = 3$ **17.** $x = 5/4$
19. no solution **21.** $x \approx -0.279$
23. $x \approx -1147.362$ **25.** $y = \frac{3}{4}x - \frac{15}{4}$
27. $y = -6x + \frac{22}{3}$ **29.** $t = \dfrac{S - P}{Pr}$
31. $x < 2$ **33.** $x < -4$ **35.** $x \le -1$
37. $x < -3$ 

39. $x < -6$ 

41. $x < 2$ 

43. 145 months
45. \$3356.50 **47.** 440 packs, or 220,000 CDs
49. \$29,600
51. (a) 77.7% (b) $t \approx 14.5$, during 2015
53. 96
55. \$90,000 at 9%; \$30,000 at 13%
57. \$2160/month; 8% increase
59. $x > 80$ 

61. $695 + 5.75x \le 900$; 35 or fewer
63. (a) $t = 15$ (b) $t > 21.7$ (c) in 2017
65. (a) $0.479 \le h \le 1$; $h = 1$ means 100% humidity
(b) $0 \le h \le 0.237$

1.2 EXERCISES

1. (a) To each x-value there corresponds exactly one y-value.
(b) domain: $\{-7, -1, 0, 3, 4.2, 9, 11, 14, 18, 22\}$
range: $\{0, 1, 5, 9, 11, 22, 35, 60\}$
(c) $f(0) = 1, f(11) = 35$
3. yes; to each x-value there corresponds exactly one y-value; domain $= \{1, 2, 3, 8, 9\}$, range $= \{-4, 5, 16\}$
5. The vertical-line test shows that graph (a) represents a function of x, but graph (b) does not.
7. yes **9.** no
11. (a) -10 (b) 6 (c) -34 (d) 2.8
13. (a) -3 (b) 1 (c) 13 (d) 6
15. (a) -251 (b) -128 (c) 22 (d) -4.25
17. (a) 63/8 (b) 6 (c) -6
19. (a) no, $f(2 + 1) = f(3) = 13$ but $f(2) + f(1) = 10$
(b) $1 + x + h + x^2 + 2xh + h^2$
(c) no, $f(x) + f(h) = 2 + x + h + x^2 + h^2$
(d) no, $f(x) + h = 1 + x + x^2 + h$
(e) $1 + 2x + h$
21. (a) $-2x^2 - 4xh - 2h^2 + x + h$
(b) $-4x - 2h + 1$
23. (a) 10 (b) 6
25. (a) $(1, -3)$, yes (b) $(3, -3)$, yes
(c) $b = a^2 - 4a$ (d) $x = 0, x = 4$, yes
27. domain: all reals; range: reals $y \ge 4$
29. domain: reals $x \ge -4$; range: reals $y \ge 0$
31. $x \ge 1, x \ne 2$ **33.** $-7 \le x \le 7$
35. (a) $3x + x^3$ (b) $3x - x^3$ (c) $3x^4$ (d) $\dfrac{3}{x^2}$
37. (a) $\sqrt{2x} + x^2$ (b) $\sqrt{2x} - x^2$
(c) $x^2\sqrt{2x}$ (d) $\dfrac{\sqrt{2x}}{x^2}$
39. (a) $-8x^3$ (b) $1 - 2(x - 1)^3$
(c) $[(x - 1)^3 - 1]^3$ (d) $(x - 1)^6$

41. (a) $2\sqrt{x^4 + 5}$ (b) $16x^2 + 5$
(c) $2\sqrt{2\sqrt{x}}$ (d) $4x$

43. (a) $f(20) = 103{,}000$ means that if \$103,000 is borrowed, it can be repaid in 20 years (of \$800-per-month payments).
(b) no; $f(5 + 5) = f(10) = 69{,}000$, but $f(5) + f(5) = 40{,}000 + 40{,}000 = 80{,}000$
(c) 15 years; $f(15) = 89{,}000$

45. (a) $f(1950) = 16.5$ means that in 1950 there were 16.5 workers supporting each Social Security beneficiary.
(b) 3.4
(c) Through 2050, the graph must be the same. After 2050, the graph might be the same.
(d) domain: $1950 \le t \le 2050$
range: $1.9 \le n \le 16.5$

47. (a) $f(105) \approx 1.45$; $g(105) \approx 0.78$
(b) $f(107) \approx 1.51$ means that at the end of 2007 there were 1.51 million persons in state prisons.
(c) $g(92) = 0.66$ means that at the end of 1992 there were 0.66 million persons on parole.
(d) $(f - g)(107) = f(107) - g(107) = 1.51 - 0.82 = 0.69$ means that at the end of 2007 there were 0.69 million more persons in state prisons than there were on parole.
(e) $(f - g)(105)$ is greater because for 2005 there is a greater distance between the graphs.

49. (a) $s \ge 0$
(b) $f(10) \approx -29.33$ means that if the air temperature is −5°F and there is a 10 mph wind, then the temperature feels like −29.33°F.
(c) $f(0) = 45.694$ from the formula, but $f(0)$ should equal the air temperature, −5°F.

51. (a) $C(10) = \$4210$
(b) $C(100) = \$32{,}200$
(c) $C(100)$ The total cost of producing 100 items is \$32,200.

53. (a) $0 \le p < 100$
(b) \$5972.73; to remove 45% of the particulate pollution would cost \$5972.73.
(c) \$65,700; to remove 90% of the particulate pollution would cost \$65,700.
(d) \$722,700; to remove 99% of the particulate pollution would cost \$722,700.
(e) \$1,817,700; to remove 99.6% of the particulate pollution would cost \$1,817,700.

55. (a) yes (b) $A(2) = 96$; $A(30) = 600$
(c) $0 < x < 50$

57. (a) $(P \circ q)(t) = 180(1000 + 10t) - \dfrac{(1000 + 10t)^2}{100} - 200$
(b) $x = 1150$, $P = \$193{,}575$

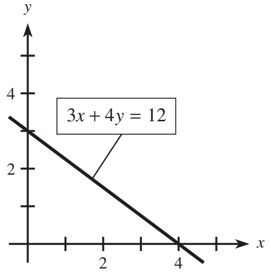
59. (a) yes; the output of g (customers) is the input for f.
(b) no; the output of f is revenue, and this is not the input for g.
(c) $f \circ g$: input (independent variable) is advertising dollars.
output (dependent variable) is revenue dollars.
$f \circ g$ shows how advertising dollars result in revenue dollars.

61. $L = 2x + 3200/x$

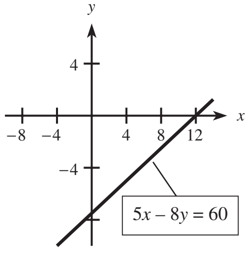
63. $R = (30 + x)(100 - 2x)$

1.3 EXERCISES

1. x-intercept 4
y-intercept 3



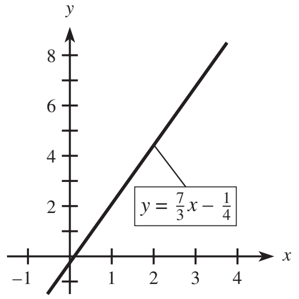
3. x-intercept 12
y-intercept −7.5



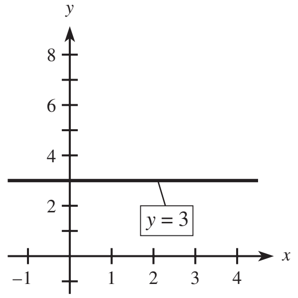
5. $m = 4$ **7.** $m = -1/2$ **9.** $m = 0$ **11.** 0

13. 0 **15.** (a) negative (b) undefined

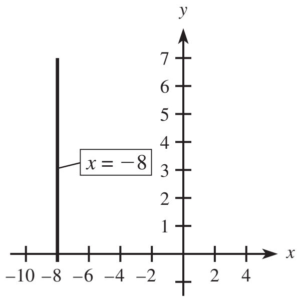
17. $m = 7/3$, $b = -1/4$



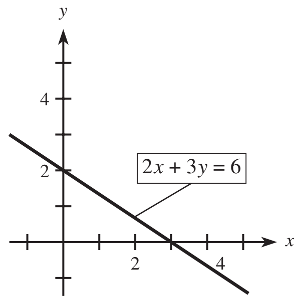
19. $m = 0$, $b = 3$



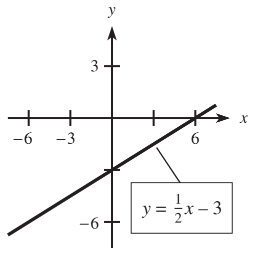
21. undefined slope, no y-intercept



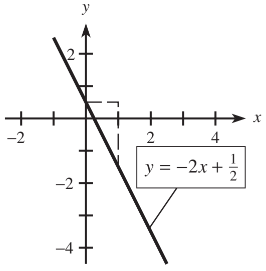
23. $m = -2/3$, $b = 2$



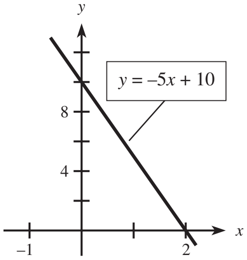
25.



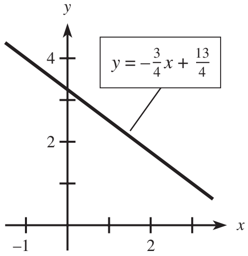
27.



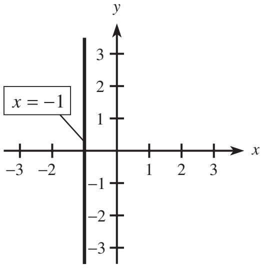
29.



31.



33.

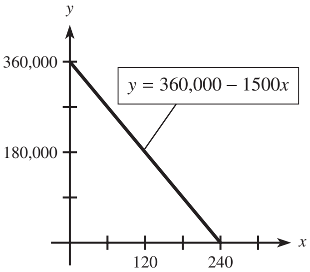


35. $y = 2x - 4$ **37.** $-x + 13y = 32$

39. perpendicular **41.** neither; same line

43. $y = -\frac{3}{5}x - \frac{41}{5}$ **45.** $y = -\frac{6}{5}x + \frac{23}{5}$

47. (a)



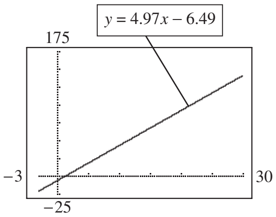
(b) 240 months

(c) After 60 months, the value of the building is $270,000.

49. (a) $m = 4.97$; $b = -6.49$

(b) The percent of the U.S. population with Internet service is changing at the rate of 4.97 percentage points per year.

(c)



51. (a) $m = -0.065$; $b = 31.39$

(b) The F-intercept represents the percent of the world's land that was forest in 1990.

(c) −0.065 percentage points per year. This means that after 1990, the world forest area as a percent of land area changes by −0.065 percentage points per year.

53. (a) $m = 0.78$

(b) This means that the average annual earnings of females increases $0.78 for each $1 increase in the average annual earnings of males.

(c) $45,484

55. $y = 0.0838x + 16.37$

57. (a) $B = 1.05W - 182.80$ (b) $709.70

59. (a) $y = 10.585x - 20{,}898.025$

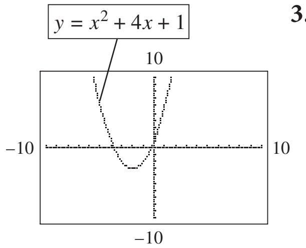
(b) The CPI-U is changing at the rate of $10.59/year.

61. $p = 85{,}000 - 1700x$ **63.** $R = 3.2t - 0.2$

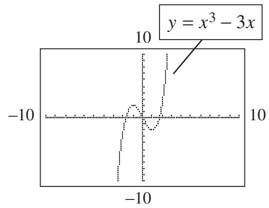
65. $y = 0.48x - 71$

1.4 EXERCISES

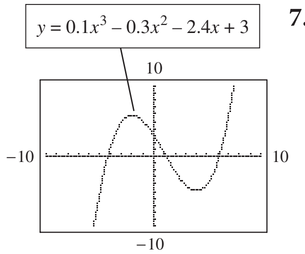
1.



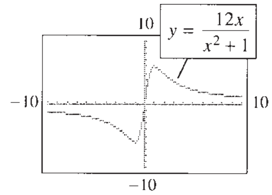
3.



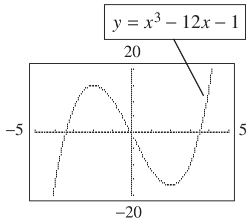
5.



7.

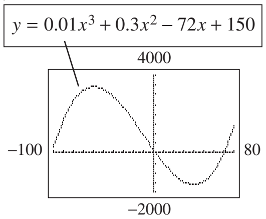


9.

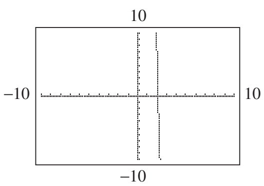


11. $y = (3x + 7)/(x^2 + 4)$

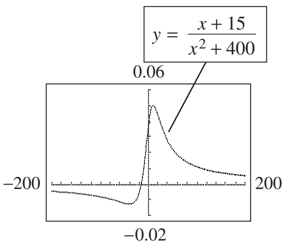
13. (a) $y = 0.01x^3 + 0.3x^2 - 72x + 150$



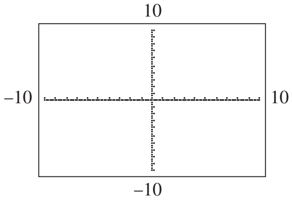
(b) standard window



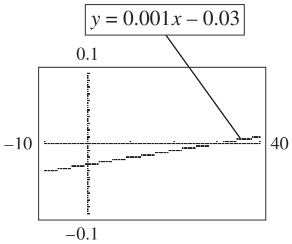
15. (a) $y = \dfrac{x + 15}{x^2 + 400}$



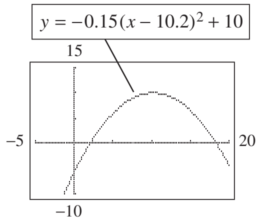
(b) standard window



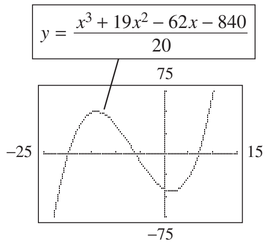
17. (a) x-intercept 30, y-intercept -0.03
(b)



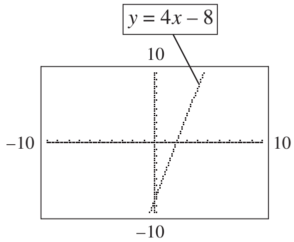
19.



21.

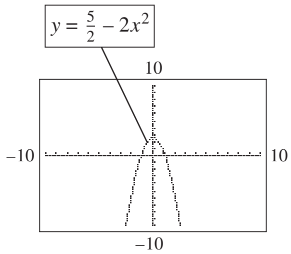


23.



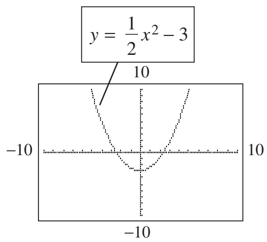
Linear

25.



Not linear

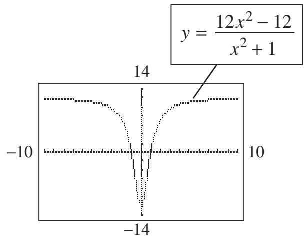
27.



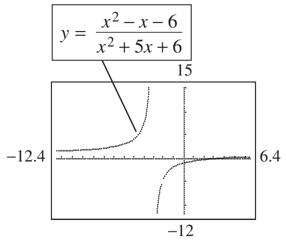
Not linear

29. $f(-2) = -18; f(\frac{3}{4}) = 0.734375$

31.



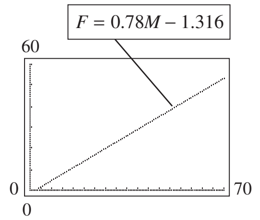
33.



35. $x = 3.5$ **37.** either $x = 5$ or $x = -2$

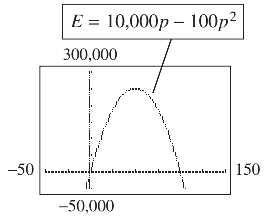
39. (a) $-1.1098, 8.1098$ (b) $-1.1098, 8.1098$

41. (a)



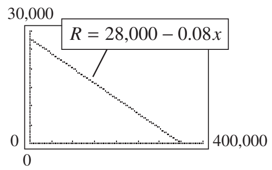
(b) When average annual earnings for males is \$50,000, average annual earnings for females is \$37,684. (c) \$47,434

43. (a)



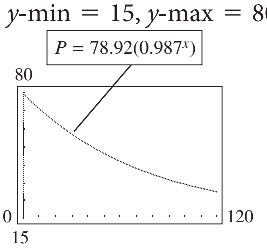
(b) $E \geq 0$ when $0 \leq p \leq 100$

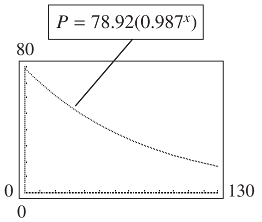
45. (a)



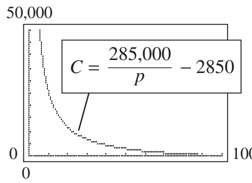
(b) decreasing; as more people become aware of the product, there are fewer to learn about it, so the rate will decrease.

47. (a) x-min $= 0$, x-max $= 120$
(b) y-min $= 15$, y-max $= 80$
(c)

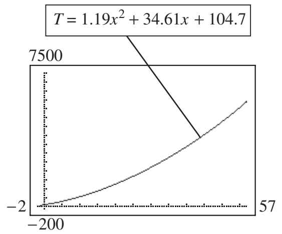


(d)


(e) The percent decreases from 78.9% in 1890 to 16.4% in 2010.

49. (a)


(b) Near $p = 0$, cost grows without bound.
(c) The coordinates of the point mean that obtaining stream water with 1% of the current pollution levels would cost \$282,150.
(d) The p-intercept means that stream water with 100% of the current pollution levels would cost \$0.

51. (a)


(b) increasing; the per capita federal tax burden is increasing.

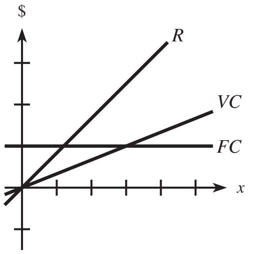
1.5 EXERCISES

1. one solution; $(-1, -2)$
3. infinitely many solutions (each point on the line)
5. $x = 2, y = 2$ **7.** no solution
9. $x = 10/3, y = 2$ **11.** $x = 14/11, y = 6/11$
13. $x = 4, y = -1$
15. $x = 3, y = -2$ **17.** $x = 2, y = 1$
19. $x = -52/7, y = -128/7$ **21.** $x = 1, y = 1$
23. dependent
25. $x = 4, y = 2$ **27.** $x = -1, y = 1$
29. $x = -17, y = 7, z = 5$
31. $x = 4, y = 12, z = -1$
33. $x = 44, y = -9, z = -1/2$
35. $x \approx 2.455$; during 1998; amount $\approx$ \$842.67 billion
37. (a) $x + y = 1800$ (b) $20x$ (c) $30y$
(d) $20x + 30y = 42{,}000$
(e) 1200 tickets at \$20; 600 tickets at \$30
39. \$68,000 at 18%; \$77,600 at 10%
41. 10%: \$27,000; 12%: \$24,000
43. 4 oz of A, $6\frac{2}{3}$ oz of B
45. 4550 of species A, 1500 of species B
47. 7 cc of 20%; 3 cc of 5%
49. 10,000 at \$40; 6000 at \$60
51. 80 cc **53.** 5 oz of A, 1 oz of B, 5 oz of C
55. 200 type A, 100 type B, 200 type C

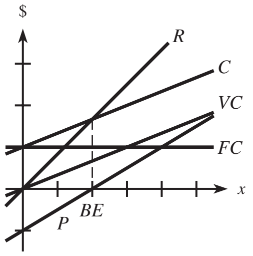
1.6 EXERCISES

1. (a) $P(x) = 34x - 6800$ (b) \$95,200
3. (a) $P(x) = 37x - 1850$
(b) $-\$740$, loss of \$740 (c) 50
5. (a) $m = 5, b = 250$
(b) $\overline{MC} = 5$ means each additional unit produced costs \$5.
(c) slope = marginal cost; C-intercept = fixed costs
(d) 5, 5
7. (a) 27
(b) $\overline{MR} = 27$ means each additional unit sold brings in \$27.
(c) 27, 27
9. (a) $P(x) = 22x - 250$ (b) 22 (c) $\overline{MP} = 22$
(d) Each unit sold adds \$22 to profits at all levels of production, so produce and sell as much as possible.
11. $P = 58x - 8500$, $\overline{MP} = 58$
13. (a) $C(x) = 35x + 6600$ (b) $R(x) = 60x$
(c) $P(x) = 25x - 6600$
(d) $C(200) = 13{,}600$ dollars is the cost of producing 200 helmets.
$R(200) = 12{,}000$ dollars is the revenue from sale of 200 helmets.
$P(200) = -1600$ dollars; will lose \$1600 from production and sale of 200 helmets.
(e) $C(300) = 17{,}100$ dollars is the cost of producing 300 helmets.
$R(300) = 18{,}000$ dollars is the revenue from sale of 300 helmets.
$P(300) = 900$ dollars; will profit \$900 from production and sale of 300 helmets.
(f) $\overline{MP} = 25$ dollars per unit; each additional unit produced and sold increases profit by \$25.
15. (a) Revenue passes through the origin.
(b) \$2000 (c) 400 units
(d) $\overline{MC} = 2.5$; $\overline{MR} = 7.5$
17. 33
19. (a) $R(x) = 12x$; $C(x) = 8x + 1600$
(b) 400 units
21. (a) $P(x) = 4x - 1600$
(b) $x = 400$ units to break even
23. (a) $C(x) = 4.5x + 1045$
(b) $R(x) = 10x$
(c) $P(x) = 5.5x - 1045$
(d) 190 surge protectors
25. (a) $R(x) = 54.90x$ (b) $C(x) = 14.90x + 20{,}200$
(c) 505

27. (a) R starts at origin and is the steeper line.
FC is a horizontal line.
VC starts at origin and is not as steep as R.
(See figure.)



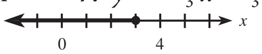
(b) C starts where FC meets the \$-axis and is parallel to VC. Where C meets R is the break-even point (BE). P starts on the \$-axis at the negative of FC and crosses the x-axis at BE. (See figure.)



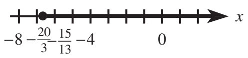
29. Demand decreases.
31. (a) 600 (b) 300 (c) shortage
33. 16 demanded, 25 supplied; surplus
35. $p = -2q/3 + 1060$
37. $p = 0.0001q + 0.5$
39. (a) demand falls; supply rises (b) (30, \$25)
41. (a) $q = 20$ (b) $q = 40$
(c) shortage, 20 units short
43. shortage **45.** $q = 20, p = \$18$
47. $q = 10, p = \$180$ **49.** $q = 100, p = \$325$
51. (a) \$15 (b) $q = 100, p = \$100$
(c) $q = 50, p = \$110$ (d) yes
53. $q = 8; p = \$188$ **55.** $q = 500; p = \$40$
57. $q = 1200; p = \$15$

CHAPTER 1 REVIEW EXERCISES

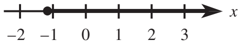
1. $x = \frac{31}{3}$ **2.** $x = -13$ **3.** $x = -\frac{29}{8}$
4. $x = -\frac{1}{9}$ **5.** $x = 8$
6. no solution **7.** $y = -\frac{2}{3}x - \frac{4}{3}$
8. $x \le 3$



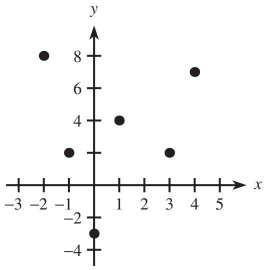
9. $x \ge -20/3$



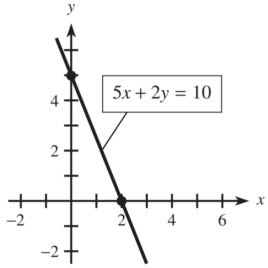
10. $x \ge -15/13$

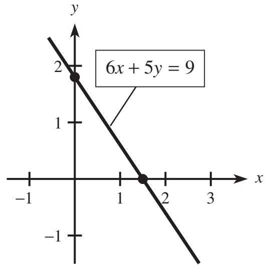


11. yes **12.** no **13.** yes
14. domain: reals $x \le 9$; range: reals $y \ge 0$
15. (a) 2 (b) 37 (c) 29/4
16. (a) 0 (b) 9/4 (c) 10.01
17. $9 - 2x - h$ **18.** yes **19.** no **20.** 4
21. $x = 0, x = 4$
22. (a) domain: $\{-2, -1, 0, 1, 3, 4\}$
range: $\{-3, 2, 4, 7, 8\}$
(b) 7 (c) $x = -1, x = 3$
(d)

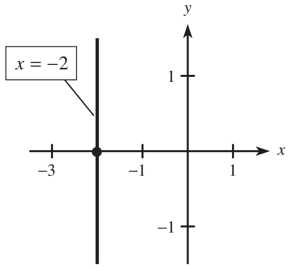


(e) no; for $y = 2$, there are two different x-values, -1 and 3.
23. (a) $x^2 + 3x + 5$ (b) $(3x + 5)/x^2$
(c) $3x^2 + 5$ (d) $9x + 20$
24. x: 2, y: 5 **25.** x: 3/2, y: 9/5

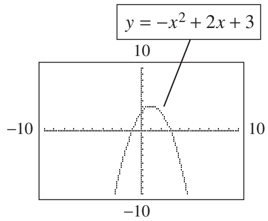




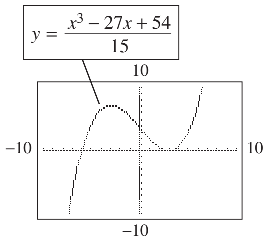
26. x: -2, y: none



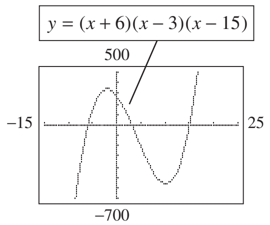
27. $m = 1$ **28.** undefined **29.** $m = -\frac{2}{5}, b = 2$
30. $m = -\frac{4}{3}, b = 2$ **31.** $y = 4x + 2$
32. $y = -\frac{1}{2}x + 3$ **33.** $y = \frac{2}{5}x + \frac{9}{5}$
34. $y = -\frac{11}{8}x + \frac{17}{14}$ **35.** $x = -1$ **36.** $y = 4x + 2$
37. $y = \frac{4}{3}x + \frac{10}{3}$
38.



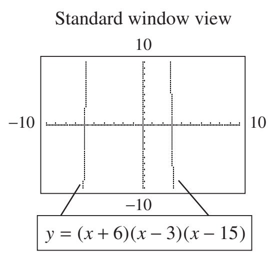
39.



40. (a)

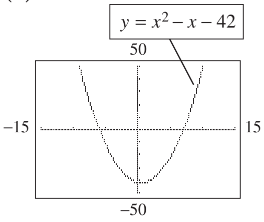


(b)

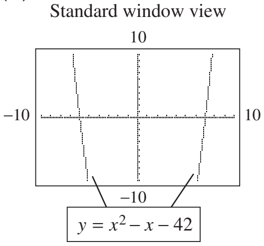


(c) The graph in (a) shows a complete graph. The graph in (b) shows a piece that rises toward the high point and a piece between the high and low points.

41. (a)



(b)



(c) The graph in (a) shows a complete graph. The one in (b) shows pieces that fall toward the minimum point and rise from it.

42. reals $x \geq -3$ with $x \neq 0$

43. $0.2857 \approx 2/7$ **44.** $x = 2, y = 1$

45. $x = 10, y = -1$ **46.** $x = 3, y = -2$

47. no solution **48.** $x = 10, y = -71$

49. $x = 1, y = -1, z = 2$

50. $x = 11, y = 10, z = 9$

51. (a) 1997 (b) 27 (c) $x = 30.0$; in 2010

52. 95%

53. 40,000 miles. He would normally drive more than 40,000 miles in 5 years, so he should buy diesel.

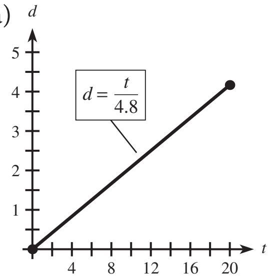
54. (a) yes (b) no (c) 4

55. (a) \$565.44
(b) The monthly payment on a \$70,000 loan is \$494.75.

56. (a) $(P \circ q)(t) = 330(100 + 10t) - 0.05(100 + 10t)^2 - 5000$
(b) $x = 250, P = \$74{,}375$

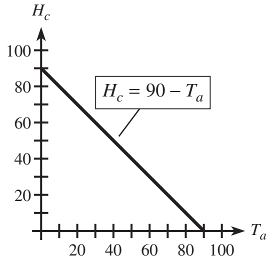
57. $(W \circ L)(t) = 0.03[65 - 0.1(t - 25)^2]^3$

58. (a)



(b) When the time between seeing lightning and hearing thunder is 9.6 seconds, the storm is 2 miles away.

59.

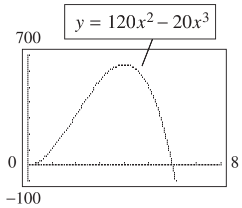


60. (a) $P = 58x - 8500$
(b) The profit increases by \$58 for each unit sold.

61. (a) yes
(b) $m = 427, b = 4541$
(c) In 2000, average annual health care costs were \$4541 per consumer.
(d) Average annual health care costs are changing at the rate of \$427 per year.

62. $F = \frac{9}{5}C + 32$ or $C = \frac{5}{9}(F - 32)$

63. (a)



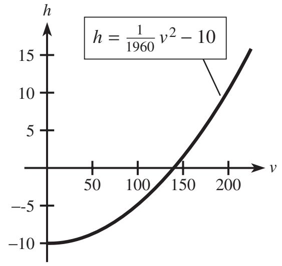
(b) $0 \leq x \leq 6$

64. (a) $v^2 = 1960(h + 10)$

$$\frac{v^2}{1960} = h + 10$$

$$h = \frac{1}{1960}v^2 - 10$$

(b) 12.5 cm

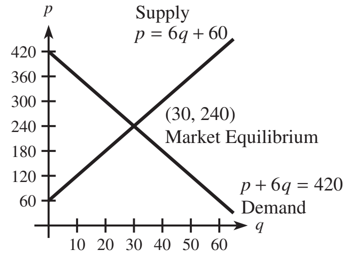


65. \$100,000 at 9.5%; \$50,000 at 11%

66. 2.8 liters of 20%; 1.2 liters of 70%

67. (a) 12 supplied; 14 demanded (b) shortfall
(c) increase

68.



69. (a) 38.80 (b) 61.30 (c) 22.50 (d) 200

70. (a) $C(x) = 22x + 1500$ (b) $R(x) = 52x$
(c) $P(x) = 30x - 1500$ (d) $\overline{MC} = 22$
(e) $\overline{MR} = 52$ (f) $\overline{MP} = 30$ (g) $x = 50$

71. $q = 300, p = \$150$ **72.** $q = 700, p = \$80$

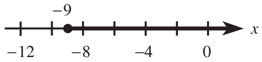
CHAPTER 1 TEST

1. $x = -6$

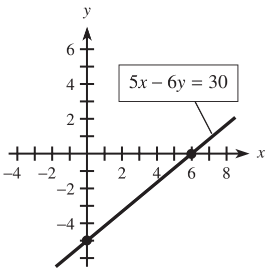
2. $x = 18/7$ **3.** $x = -3/7$ **4.** $x = -38$

5. $5 - 4x - 2h$

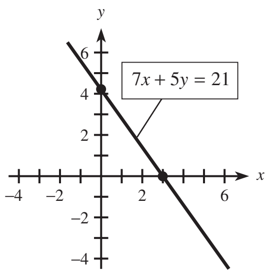
6. $t \geq -9$



7. x: 6; y: −5



8. x: 3; y: 21/5



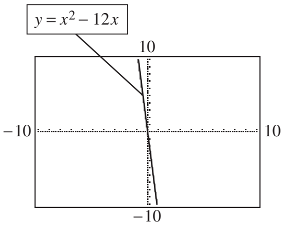
9. (a) domain: $x \geq -4$
range: $f(x) \geq 0$
(b) $2\sqrt{7}$ (c) 6

10. $y = -\frac{3}{2}x + \frac{1}{2}$ **11.** $m = -\frac{5}{4}$; $b = \frac{15}{4}$

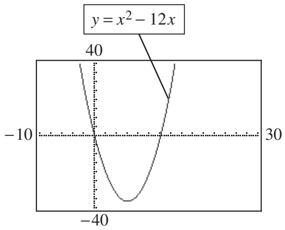
12. (a) $x = -3$ (b) $y = -4x - 13$

13. (a) no; a vertical line intersects the curve twice.
(b) yes; there is exactly one y-value for each x-value.
(c) no; one value of x gives two y-values.

14. (a)



(b)



15. $x = -2$, $y = 2$

16. (a) $5x^3 + 2x^2 - 3x$ (b) $x + 2$
(c) $5x^2 + 7x + 2$

17. (a) 30 (b) $P = 8x - 1200$ (c) 150
(d) $\overline{MP} = 8$; the sale of each additional unit gives \$8 more profit.

18. (a) $R = 50x$
(b) 19,000; it costs \$19,000 to produce 100 units.
(c) 450

19. $q = 200$, $p = \$2500$

20. (a) 720,000; original value of the building
(b) −2000; building depreciates \$2000 per month.

21. 400

22. 12,000 at 9%, 8000 at 6%

2.1 EXERCISES

1. $x^2 + 2x - 1 = 0$ **3.** $y^2 + 3y - 2 = 0$

5. −2, 6 **7.** $\frac{3}{2}, -\frac{3}{2}$ **9.** 0, 1 **11.** $\frac{1}{2}$

13. (a) $2 + 2\sqrt{2}, 2 - 2\sqrt{2}$ (b) 4.83, −0.83

15. no real solutions **17.** $\sqrt{7}, -\sqrt{7}$ **19.** 4, −4

21. 1, −9 **23.** −7, 3 **25.** 8, −4 **27.** $-\frac{7}{4}, \frac{3}{4}$

29. −6, 2 **31.** $(1 + \sqrt{31})/5, (1 - \sqrt{31})/5$

33. −2, 5 **35.** −300, 100 **37.** 0.69, −0.06

39. 8, 1 **41.** 1/2 **43.** −9, −10

45. $x = 20$ or $x = 70$

47. (a) $x = 10$ or $x = 345\frac{5}{9}$
(b) yes; for any $x > 10$ and $x < 345\frac{5}{9}$

49. 6 seconds

51. (a) ± 50
(b) $s = 50$; there is no particulate pollution in the air above the plant.

53. 97.0 mph

55. 2012 **57.** \$80 **59.** $x \approx 27.0$, during 2017

61. (a) 18 (b) ≈ 31
(c) Speed triples, but K changes only by a factor of 1.72.

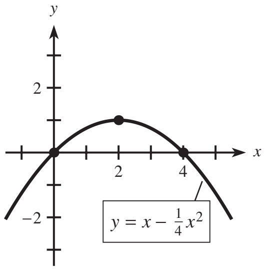
2.2 EXERCISES

1. (a) $(-1, -\frac{1}{2})$ (b) minimum
(c) −1 (d) $-\frac{1}{2}$

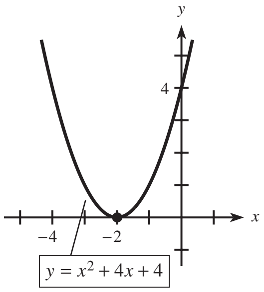
3. (a) (1, 9) (b) maximum (c) 1 (d) 9

5. (a) (3, 9) (b) maximum (c) 3 (d) 9

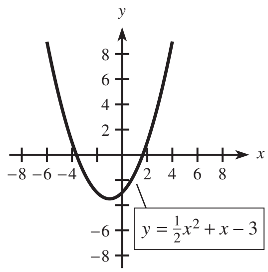
7. maximum, (2, 1); zeros (0, 0), (4, 0)



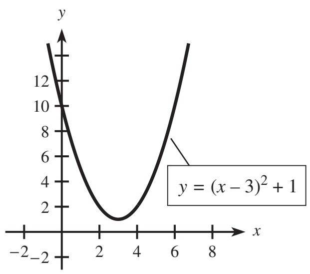
9. minimum, (−2, 0); zero (−2, 0)



11. minimum, $(-1, -3\frac{1}{2})$; zeros $(-1 + \sqrt{7}, 0)$, $(-1 - \sqrt{7}, 0)$

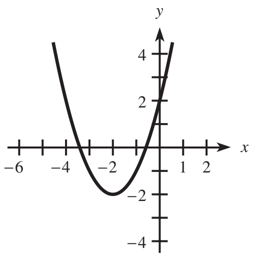


13. (a) 3 units to the right and 1 unit up
(b)

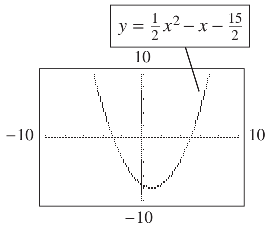


15. $y = (x + 2)^2 - 2$
(a) 2 units to the left and 2 units down

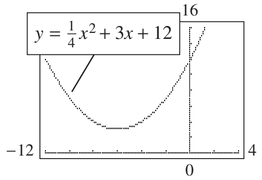
(b)



17. vertex $(1, -8)$; zeros $(-3, 0)$, $(5, 0)$

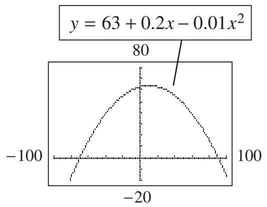


19. vertex $(-6, 3)$; no real zeros

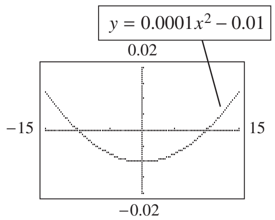


21. -5

23. vertex $(10, 64)$; zeros $(90, 0)$, $(-70, 0)$



25. vertex $(0, -0.01)$; zeros $(10, 0)$, $(-10, 0)$

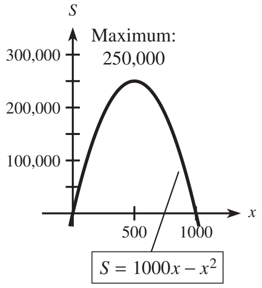


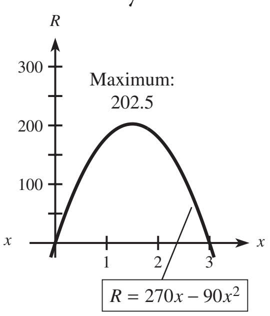
27. (a) $(1, -24)$ (b) $x \approx -0.732, 2.732$

29. (a) $x = 2$ (b) $x - 2$
(c) $(x - 2)(3x - 2)$ (d) $x = 2, \frac{2}{3}$

31. (a) 80 units (b) $540 **33.** 400 trees

35. dosage = 500 mg **37.** intensity = 1.5 lumens





39. equation (a) (384.62, 202.31) (b) (54, 46)
Projectile (a) goes higher.

41. (a) From b to c. The average rate is the same as the slope of the segment. Segment b to c is steeper.
(b) $d > b$ to make segment a to d be steeper (have greater slope).

43. (a)

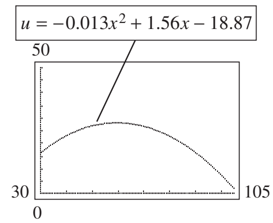
Rent	Number Rented	Revenue
600	50	$30,000
620	49	$30,380
640	48	$30,720

(b) increase
(c) $R = (50 - x)(600 + 20x)$
(d) $800

45. (a) quadratic
(b) $a < 0$ because the graph opens downward.
(c) The vertex occurs after 2004 (when $x > 0$). Hence $-b/2a > 0$ and $a < 0$ means $b > 0$. The value $c = f(0)$, or the y-value in 2004. Hence $c > 0$.

47. Yes; rises at an average rate of $176 billion per year from 2005 to 2010, at $226 billion per year from 2010 to 2015.

49.



51. (a) $x \approx 106.4$; 2007
(b) 2007 and after, when $u(x) < 0$

2.3 EXERCISES

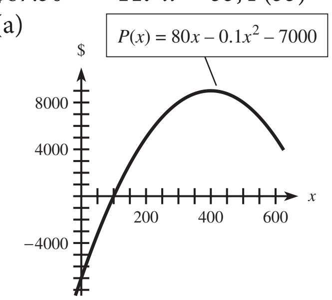
1. $x = 40$ units, $x = 50$ units

3. $x = 50$ units, $x = 300$ units

5. $x = 15$ units; reject $x = 100$ **7.** $41,173.61

9. $87.50 **11.** $x = 55$, $P(55) = \$2025$

13. (a)



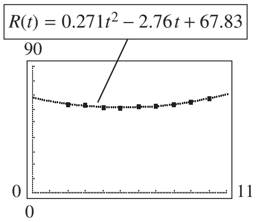
(b) (400, 9000); maximum point (c) positive
(d) negative (e) closer to 0 as a gets closer to 400

15. (a) $P(x) = -x^2 + 350x - 15{,}000$; maximum profit is $15,625
(b) no (c) x-values agree

17. (a) $x = 28$ units, $x = 1000$ units
(b) $651,041.67

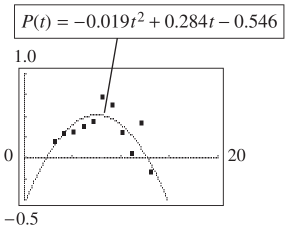
(c) $P(x) = -x^2 + 1028x - 28{,}000$; maximum profit is \$236,196
(d) \$941.60

19. (a) $t \approx 5.1$, in 2009; $R \approx \$60.79$ billion
(b) The data show a smaller revenue, $R = \$60.27$ billion, in 2008.
(c)



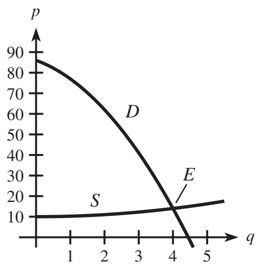
(d) The model fits the data quite well.

21. (a) $P(t) = -0.019t^2 + 0.284t - 0.546$
(b) 2008
(c)



(d) The model projects decreasing profits, and except for 2012, the data support this.
(e) Management would be interested in increasing revenues or reducing costs (or both) to improve profits.

23. (a)

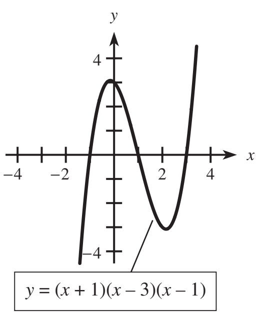


(b) See E on graph. (c) $q = 4, p = \$14$

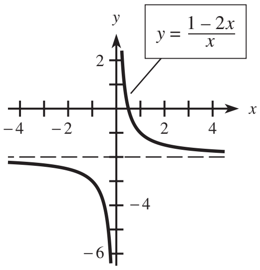
25. $q = 10, p = \$196$ **27.** $q = 216\frac{2}{3}, p = \$27.08$
29. $p = \$40, q = 30$ **31.** $q = 90, p = \$50$
33. $q = 70, p = \$62$

2.4 EXERCISES

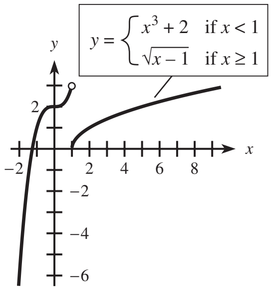
1. l **3.** d **5.** h **7.** i **9.** k
11. a **13.** (a) cubic (b) quartic **15.** d
17. a **19.** c **21.** f
23.



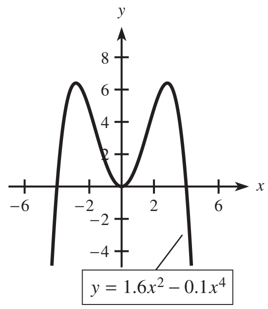
25.



27.

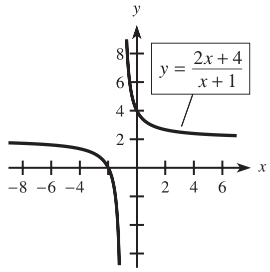


29. (a) 8/3 (b) 9.9 (c) -999.999 (d) no
31. (a) 64 (b) 1 (c) 1000 (d) 0.027
33. (a) 2 (b) 4 (c) 0 (d) 2
35. (a)



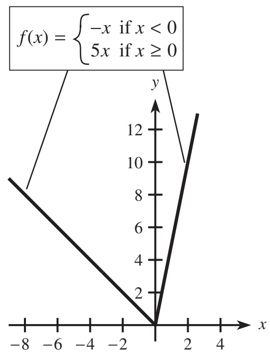
(b) polynomial (c) no asymptotes
(d) turning points at $x = 0$ and approximately $x = -2.8$ and $x = 2.8$

37. (a)



(b) rational
(c) vertical: $x = -1$, horizontal: $y = 2$
(d) no turning points

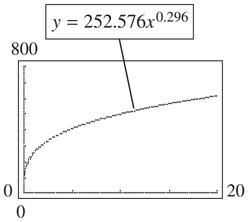
39. (a)



(b) piecewise defined
(c) no asymptotes
(d) turning point at $x = 0$

41. (a) 6800; 11,200 (b) $0 < x < 27$

43. (a) downward (b)



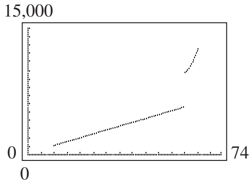
(c) 2010

45. (a) yes; $p = 100$
(b) $p \neq 100$
(c) $0 \leq p < 100$
(d) It increases without bound.

47. (a) $A(2) = 96$; $A(30) = 600$
(b) $0 < x < 50$

49. (a)

$$y = \begin{cases} 90.742x + 210.291 & \text{if } 10 \leq x \leq 60 \\ 66.786x^2 - 7820.9x + 238{,}565.429 & \text{if } 60 \leq x \end{cases}$$



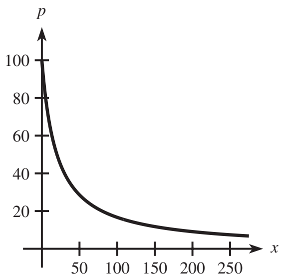
(b) \$3840 billion (\$3.840 trillion)
(c) \$27,669 billion (\$27.669 trillion)

51. (a)

$$p = \begin{cases} 44 & \text{if } 0 < x \leq 1 \\ 64 & \text{if } 1 < x \leq 2 \\ 84 & \text{if } 2 < x \leq 3 \\ 104 & \text{if } 3 < x \leq 4 \end{cases}$$

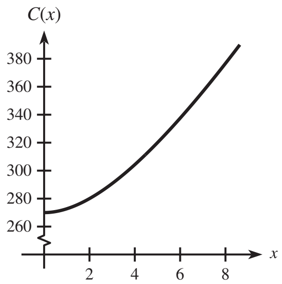
(b) 64; it costs 64 cents to mail a 1.2 oz letter.
(c) Domain $0 < x \leq 4$; Range $\{44, 64, 84, 104\}$
(d) 64 cents and 84 cents

53. (a) $p = \dfrac{200}{2 + 0.1x}$
(b) yes, when $p = \$100$



55. $C(x) = 30(x - 1) + \dfrac{3000}{x + 10}$

(a)



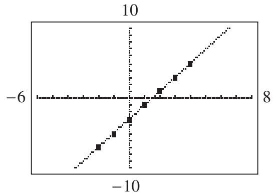
(b) Any turning point would indicate the minimum or the maximum cost. In this case, $x = 0$ gives a minimum.
(c) The y-intercept is the fixed cost of production.

2.5 EXERCISES

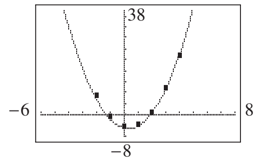
1. linear **3.** quadratic **5.** quartic

7. quadratic or power

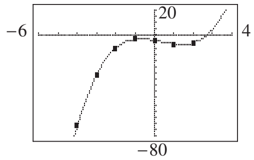
9. $y = 2x - 3$



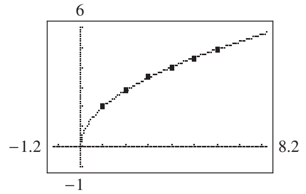
11. $y = 2x^2 - 1.5x - 4$



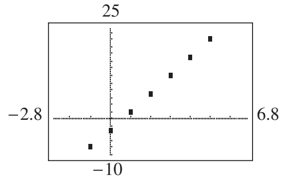
13. $y = x^3 - x^2 - 3x - 4$



15. $y = 2x^{0.5}$

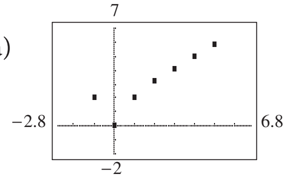


17. (a)



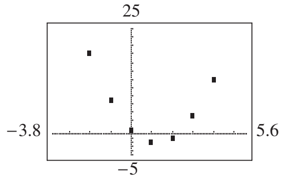
(b) linear
(c) $y = 5x - 3$

19. (a)



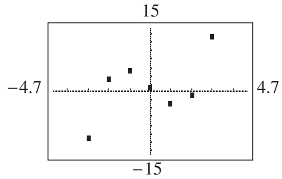
(b) quadratic
(c) $y = 0.09595x^2 + 0.4656x + 1.4758$

21. (a)



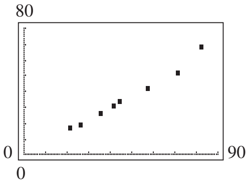
(b) quadratic
(c) $y = 2x^2 - 5x + 1$

23. (a)



(b) cubic
(c) $y = x^3 - 5x + 1$

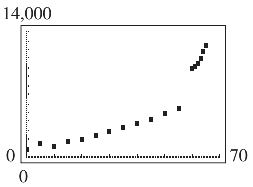
25. (a)



(b) $y = 0.807x - 2.395$
(c) 0.807; Women's annual earnings increase by \$807 for each \$1000 increase in men's earnings.

27. (a) $y = -0.039x^3 + 1.69x^2 - 18.2x + 71.1$
(b) 2002; no, data maximum is in 2000.

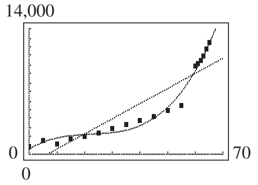
29. (a)



(b) $y = 168.319x - 1061.592$

(c) $y = 0.1195x^3 - 7.9690x^2 + 193.370x + 575.369$

(d)



Cubic model is the better fit.

31. (a) $y = 0.0052x^2 - 0.62x + 15.0$

(b) $x \approx 59.6$

(c) No, it is unresasonable for the temperature to feel warmer for winds greater than 60 mph.

33. (a) quadratic and power

(b) $y = 2.025x^2 - 86.722x + 853.890$, $y = 0.0324x^{2.737}$

(c) Quadratic is better.

(d) \$4708 billion (\$4.708 trillion)

35. (a) power: $y = 0.5125x^{4.038}$
cubic: $y = -4.62x^3 + 900x^2 - 13{,}480x + 51{,}120$

(b) power: 880,320 thousand
cubic: 483,738 thousand

(c) Both exceed the estimate of the total U.S. population in 2015.

CHAPTER 2 REVIEW EXERCISES

1. $x = 0, x = -\frac{5}{3}$ **2.** $x = 0, x = \frac{4}{3}$

3. $x = -2, x = -3$

4. $x = (-5 + \sqrt{47})/2, x = (-5 - \sqrt{47})/2$

5. no real solutions **6.** $x = \frac{\sqrt{3}}{2}, x = -\frac{\sqrt{3}}{2}$

7. $\frac{5}{7}, -\frac{4}{5}$ **8.** $(-1 + \sqrt{2})/4, (-1 - \sqrt{2})/4$

9. 7/2, 100 **10.** 13/5, 90

11. no real solutions **12.** $z = -9, z = 3$

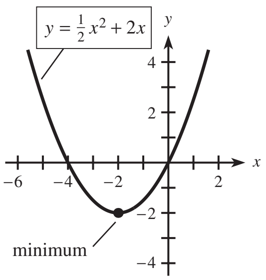
13. $x = 8, x = -2$ **14.** $x = 3, x = -1$

15. $x = (-a \pm \sqrt{a^2 - 4b})/2$

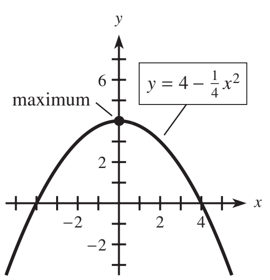
16. $r = (2a \pm \sqrt{4a^2 + x^3c})/x$

17. 1.64, −7051.64 **18.** 0.41, −2.38

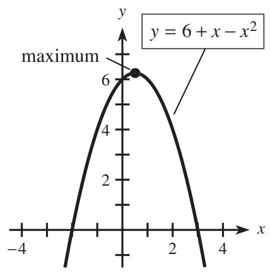
19. vertex $(-2, -2)$; zeros $(0, 0), (-4, 0)$



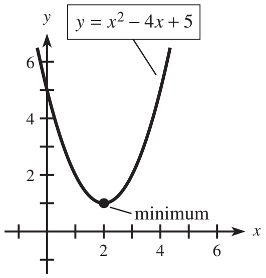
20. vertex $(0, 4)$; zeros $(4, 0), (-4, 0)$



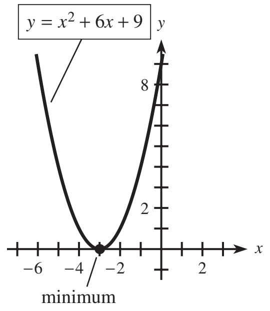
21. vertex $(\frac{1}{2}, \frac{25}{4})$; zeros $(-2, 0), (3, 0)$



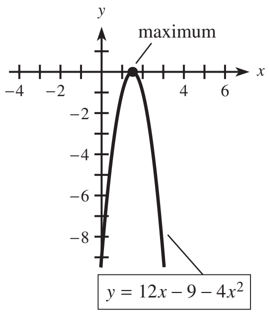
22. vertex (2, 1); no real zeros



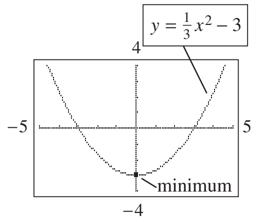
23. vertex $(-3, 0)$; zero $(-3, 0)$



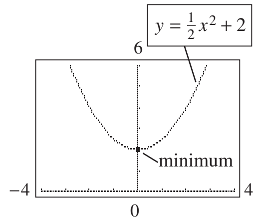
24. vertex $(\frac{3}{2}, 0)$; zero $(\frac{3}{2}, 0)$



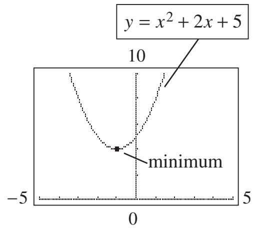
25. vertex $(0, -3)$; zeros $(-3, 0), (3, 0)$



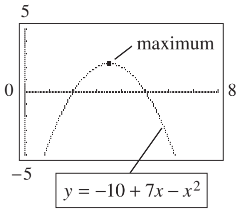
26. vertex (0, 2); no real zeros



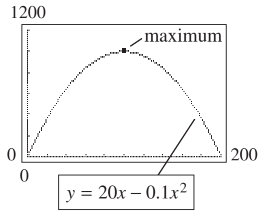
27. vertex $(-1, 4)$; no real zeros



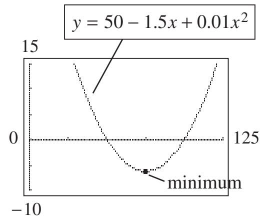
28. vertex $(\frac{7}{2}, \frac{9}{4})$; zeros (2, 0), (5, 0)



29. vertex (100, 1000); zeros (0, 0), (200, 0)



30. vertex $(75, -6.25)$; zeros (50, 0), (100, 0)



31. 20 **32.** 30

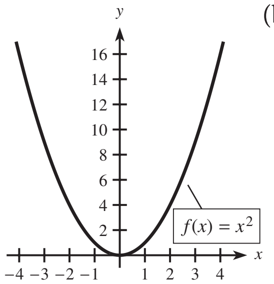
33. (a) $(1, -4\frac{1}{2})$ (b) $x = -2, x = 4$ (c) B

34. (a) (0, 49) (b) $x = -7, x = 7$ (c) D

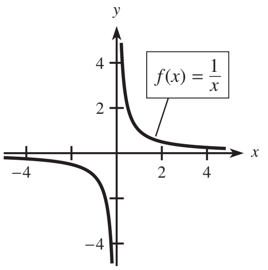
35. (a) (7, 25) approximately, actual is $(7, 24\frac{1}{2})$
(b) $x = 0, x = 14$ (c) A

36. (a) $(-1, 9)$ (b) $x = -4, x = 2$ (c) C

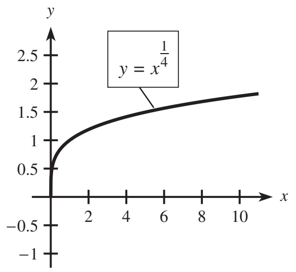
37. (a)



(b)



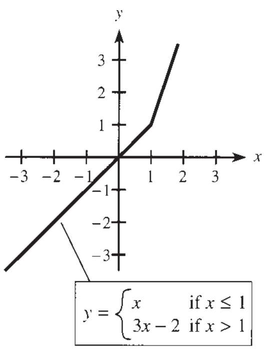
(c)



38. (a) 0 (b) 10,000 (c) -25 (d) 0.1

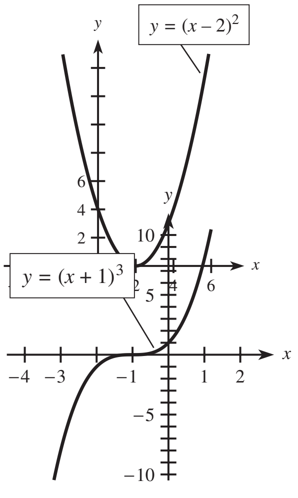
39. (a) -2 (b) 0 (c) 1 (d) 4

40.

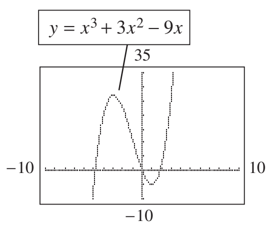


41. (a)

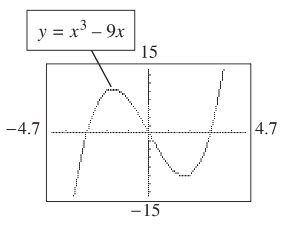
(b)



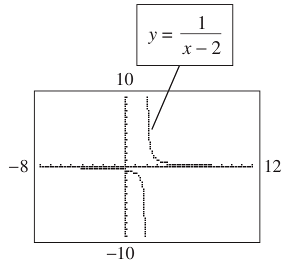
42. Turns: $(1, -5), (-3, 27)$



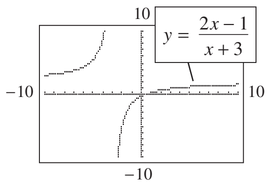
43. Turns: $(1.7, -10.4)$, $(-1.7, 10.4)$



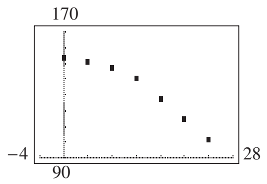
44. VA: $x = 2$;
HA: $y = 0$



45. VA: $x = -3$;
HA: $y = 2$

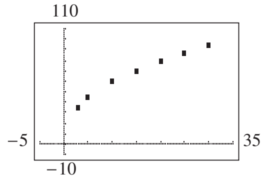


46. (a)



(b) $y = -2.1786x + 159.8571$
(c) $y = -0.0818x^2 - 0.2143x + 153.3095$

47. (a)



(b) $y = 2.1413x + 34.3913$
(c) $y = 22.2766x^{0.4259}$

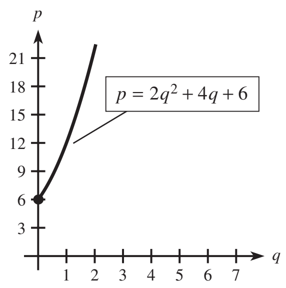
48. (a) $t = -1.65, t = 3.65$ (b) Just $t = 3.65$
(c) at 3.65 seconds

49. $x = 20, x = 800$

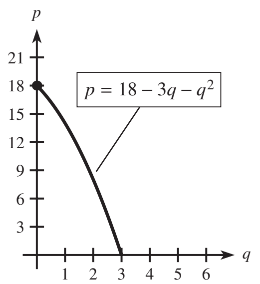
50. (a) 2000 and 2008 (b) 2004; 8.59

51. (a) $x = 200$ (b) A = 30,000 square feet

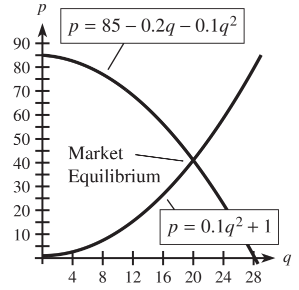
52.



53.



54. (a)



(b) $p = 41, q = 20$

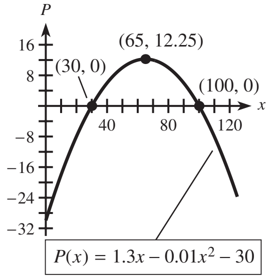
55. $p = 400, q = 10$ **56.** $p = 10, q = 20$

57. $x = 46 + 2\sqrt{89} \approx 64.9, x = 46 - 2\sqrt{89} \approx 27.1$

58. $x = 15, x = 60$

59. max revenue = \$2500; max profit = \$506.25

60. max profit = 12.25; break-even at $x = 100$, $x = 30$

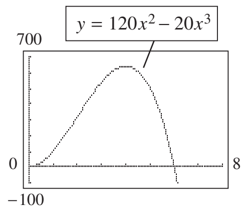


61. $x = 50, P(50) = 640$

62. (a) $C = 15{,}000 + 140x + 0.04x^2$;
$R = 300x - 0.06x^2$
(b) 100, 1500 (c) 2500
(d) $P = 160x - 15{,}000 - 0.1x^2$; max at 800
(e) at 2500: $P = -240{,}000$; at 800: $P = 49{,}000$

63. (a) power (b) 36.7 million
(c) 40.8; the number of HIV infections will be 40.8 million in 2015.

64. (a)

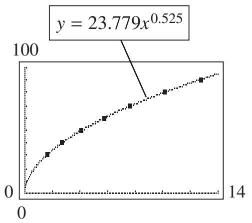


(b) $0 \le x \le 6$

65. (a) rational (b) $0 \le p < 100$
(c) 0; it costs \$0 to remove no pollution.
(d) \$475,200

66. (a) $x = 12$; $C(12) = \$30.68$
(b) $x = 825$; $C(825) = \$1734.70$

67. (a) and (b)



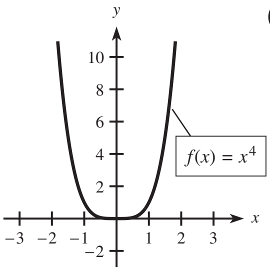
(c) 55 mph (d) 9.9 seconds

68. (a) $y = 2.94x^2 + 32.7x + 640$
(b) $x \approx 23.3$, in 2024
(c) Very little; projections were made before the Health Care Bill of 2010.

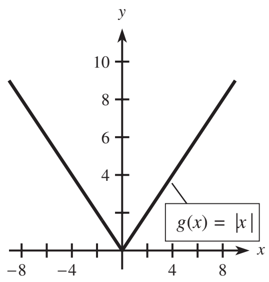
69. (a) $y = 5.66x^{1.70}$
(b) $y = 2.46x^2 - 34.7x + 399$
(c) Both are quite good.

CHAPTER 2 TEST

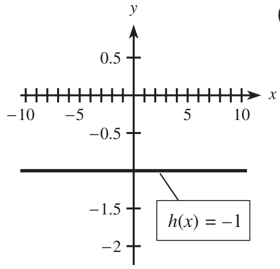
1. (a)



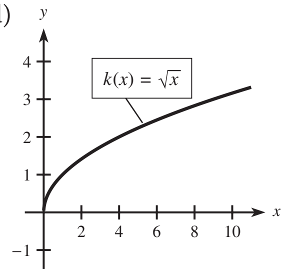
(b)



(c)



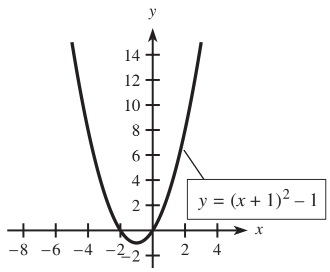
(d)



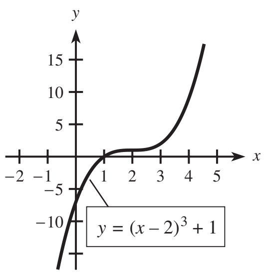
2. b; a

3.

4. (a)



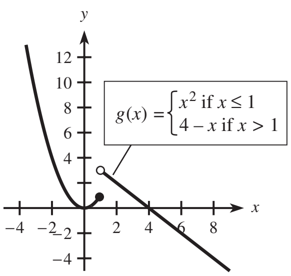
(b)



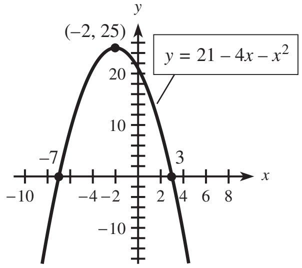
5. b; the function is cubic, $f(1) < 0$

6. (a) -10 (b) $-16\frac{1}{2}$ (c) -7

7.



8. vertex $(-2, 25)$; zeros $-7, 3$



9. $x = 2, x = 1/3$

10. $x = \dfrac{-3 + 3\sqrt{3}}{2}, x = \dfrac{-3 - 3\sqrt{3}}{2}$

11. $x = 2/3$

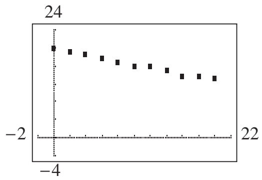
12. c; $g(x)$ has a vertical asymptote at $x = -2$, as does graph c.

13. HA: $y = 0$; VA: $x = 5$

14. 42

15. (a) quartic (b) cubic

16. (a)



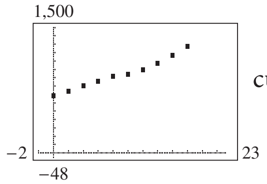
model: $y = -0.3577x + 19.9227$
(b) $f(x) = 5.6$
(c) at $x = 55.7$

17. $q = 300, p = \$80$

18. (a) $P(x) = -x^2 + 250x - 15{,}000$
(b) 125 units, \$625 (c) 100 units, 150 units

19. (a) $f(15) = -19.5$ means that when the air temperature is 0°F and the wind speed is 15 mph, the air temperature feels like -19.5°F.
(b) -31.4°F

20. (a)



(b) linear: $y = 30.65x + 667.43$
cubic: $y = 0.118x^3 - 2.51x^2 + 40.28x + 676$
(c) linear: \$1280.43; cubic: \$1420.70 (d) no

3.1 EXERCISES

1. 3 **3.** A, F, Z **5.** $\begin{bmatrix} -1 & -2 & -3 \\ 1 & 0 & -1 \\ -2 & 3 & 4 \end{bmatrix}$

7. A, C, D, F, G, Z

9. 1 **11.** $\begin{bmatrix} 1 & 3 & 4 \\ 0 & 2 & 0 \\ -2 & 1 & 3 \end{bmatrix}$ **13.** $\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$

15. $\begin{bmatrix} 9 & 5 \\ 4 & 7 \end{bmatrix}$ **17.** $\begin{bmatrix} 0 & -2 & -5 \\ 4 & 2 & 0 \\ 2 & 3 & 7 \end{bmatrix}$ **19.** $\begin{bmatrix} 2 & 3 & 2 \\ 3 & 4 & 1 \\ 2 & 1 & 6 \end{bmatrix}$

21. impossible **23.** $\begin{bmatrix} 3 & 3 & 9 & 0 \\ 12 & 6 & 3 & 3 \\ 9 & 6 & 0 & 3 \end{bmatrix}$

25. $\begin{bmatrix} 28 & 16 \\ 10 & 18 \end{bmatrix}$ **27.** impossible

29. $x = 3, y = 2, z = 3, w = 4$

31. $x = 4, y = 1, z = 3, w = 3$

33. $x = 2, y = 2, z = -3$

35. (a) $A = \begin{bmatrix} 69 & 75 & 13 & 13 & 74 \\ 12 & 14 & 24 & 10 & 65 \end{bmatrix}$

$B = \begin{bmatrix} 256 & 176 & 65 & 8 & 11 \\ 20 & 6 & 16 & 1 & 1 \end{bmatrix}$

(b) $A + B = \begin{bmatrix} 325 & 251 & 78 & 21 & 85 \\ 32 & 20 & 40 & 11 & 66 \end{bmatrix}$

(c) $\begin{bmatrix} 187 & 101 & 52 & -5 & -63 \\ 8 & -8 & -8 & -9 & -64 \end{bmatrix}$
more species in the United States

37. (a) $\begin{bmatrix} 11{,}041.7 & 8978.4 & 6461 \\ 8739.8 & 9159.6 & 6877.3 \\ 9798.1 & 9086.7 & 6448.4 \\ 9696.6 & 8926.7 & 6109.5 \end{bmatrix}$
(b) air pollution

39. (a) $\begin{bmatrix} 825 & 580 & 1560 \\ 810 & 650 & 350 \end{bmatrix}$ (b) $\begin{bmatrix} -75 & 20 & -140 \\ 10 & -50 & 50 \end{bmatrix}$

41. $A = \begin{array}{c} \begin{matrix} \text{M} & \text{F} \end{matrix} \\ \begin{bmatrix} 54.4 & 55.6 \\ 62.1 & 66.6 \\ 67.4 & 74.1 \\ 70.7 & 78.1 \\ 74.9 & 80.1 \\ 75.7 & 80.6 \end{bmatrix} \end{array}$ $B = \begin{array}{c} \begin{matrix} \text{M} & \text{F} \end{matrix} \\ \begin{bmatrix} 45.5 & 45.2 \\ 51.5 & 54.9 \\ 61.1 & 67.4 \\ 63.8 & 72.5 \\ 68.3 & 75.2 \\ 69.7 & 76.5 \end{bmatrix} \end{array}$

$A - B = C = \begin{bmatrix} 8.9 & 10.4 \\ 10.6 & 11.7 \\ 6.3 & 6.7 \\ 6.9 & 5.6 \\ 6.6 & 4.9 \\ 6.0 & 4.1 \end{bmatrix}$

43. (a) $\begin{bmatrix} -823.2 & 121.1 \\ -834.6 & 135.8 \\ -506.9 & 132.0 \end{bmatrix}$ billions of dollars of U.S. trade balance in goods and services for 2007–2009

(b) $\begin{bmatrix} 3144.0 & 855.5 \\ 3444.4 & 932.4 \\ 2643.9 & 872.6 \end{bmatrix}$ total value (in billions of dollars) of U.S. trade for goods and services, 2007–2009

(c) $\begin{bmatrix} 96.70 & 40.69 \\ 108.74 & 44.51 \\ 89.04 & 41.86 \end{bmatrix}$ average monthly value (in billions of dollars) for U.S. exports of goods and services, 2007–2009

45. (a) $\begin{bmatrix} 46.20 & 84.00 & 210.00 & 10.50 \\ 42.00 & 84.00 & 42.00 & 0.00 \\ 58.80 & 147.00 & 94.50 & 0.00 \\ 31.50 & 147.00 & 42.00 & 21.00 \\ 42.00 & 0.00 & 210.00 & 10.50 \end{bmatrix}$

(b) $\begin{bmatrix} 48.40 & 88.00 & 220.00 & 11.00 \\ 44.00 & 88.00 & 44.00 & 0.00 \\ 61.60 & 154.00 & 99.00 & 0.00 \\ 33.00 & 154.00 & 44.00 & 22.00 \\ 44.00 & 0.00 & 220.00 & 11.00 \end{bmatrix}$

47. (a) $A = \begin{bmatrix} 0 & 1 & 1 & 0 \\ 1 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 \\ 0 & 1 & 1 & 0 \end{bmatrix}$

(b) $B = \begin{bmatrix} 0 & 1 & 1 & 1 \\ 1 & 0 & 1 & 1 \\ 0 & 1 & 0 & 1 \\ 1 & 1 & 1 & 0 \end{bmatrix}$ (c) person 2

49. (a) $\begin{bmatrix} 80 & 75 \\ 58 & 106 \end{bmatrix}$ (b) $\begin{bmatrix} 176 & 127 \\ 139 & 143 \end{bmatrix}$

(c) $\begin{bmatrix} 10 & 4 \\ 7 & 2 \end{bmatrix}$ (d) $\begin{bmatrix} -10 & 19 \\ -7 & 20 \end{bmatrix}$ shortage, taken from inventory.

51. (a) 3, 4, 5, 6 (b) 1

53.

Worker 1:	0.9625	Worker 2:	0.9375
Worker 3:	0.9125	Worker 4:	0.8875
Worker 5:	0.85	Worker 6:	0.875
Worker 7:	0.90	Worker 8:	0.925
Worker 9:	0.95		

Worker 5 is least efficient; performs best at center 5

3.2 EXERCISES

1. (a) $[32]$ (b) $[11 \quad 17]$ **3.** $\begin{bmatrix} 29 & 25 \\ 10 & 12 \end{bmatrix}$

5. $\begin{bmatrix} 14 & 2 & 16 \\ 28 & 5 & 12 \end{bmatrix}$

7. $\begin{bmatrix} 7 & 5 & 3 & 2 \\ 14 & 9 & 11 & 3 \\ 13 & 10 & 12 & 3 \end{bmatrix}$ **9.** impossible

11. $\begin{bmatrix} 13 & 9 & 3 & 4 \\ 9 & 7 & 16 & 1 \end{bmatrix}$ **13.** $\begin{bmatrix} 9 & 7 & 16 \\ 5 & 17 & 20 \end{bmatrix}$

15. $\begin{bmatrix} 9 & 0 & 8 \\ 13 & 4 & 11 \\ 16 & 0 & 17 \end{bmatrix}$ **17.** $\begin{bmatrix} 161 & 126 \\ 42 & 35 \end{bmatrix}$ **19.** no

21. no **23.** $\begin{bmatrix} -55 & 88 & 0 \\ -42 & 67 & 0 \\ 28 & -44 & 1 \end{bmatrix}$ **25.** $\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$

27. $\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$ **29.** A **31.** Z

33. no (see Problem 25)

35. (a) $AB = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$, $BA = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

(b) $ad - bc \neq 0$

37. $\begin{bmatrix} 2 - 2 + 2 \\ 6 - 4 - 4 \\ 4 + 0 - 2 \end{bmatrix} = \begin{bmatrix} 2 \\ -2 \\ 2 \end{bmatrix}$; solution

39. $\begin{bmatrix} 1 + 2 + 2 \\ 4 + 0 + 1 \\ 2 + 2 + 1 \end{bmatrix} = \begin{bmatrix} 5 \\ 5 \\ 5 \end{bmatrix}$; solution

41. $\begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}$ (Some entries may appear as decimal approximations of 0.)

43. $\begin{bmatrix} 31{,}680 & 36{,}960 \\ 42{,}500 & 47{,}600 \end{bmatrix}$

The entries represent the dealer's cost for each car.

45. (a) $A = \begin{bmatrix} 2378 & 2071 \\ 2723 & 2980 \\ 5114 & 4850 \\ 1581 & 1627 \\ 2490 & 2573 \\ 1245 & 1289 \\ 824 & 823 \end{bmatrix}$ $B = \begin{bmatrix} 131.0 & 113.0 \\ 47.8 & 40.6 \\ 86.8 & 84.3 \\ 118.0 & 125.0 \\ 107.0 & 116.0 \\ 64.2 & 57.8 \\ 96.3 & 98.3 \end{bmatrix}$

(b) 7×7 (c) the diagonal entries

47. (a) $[0.55 \quad 0.45]$ After 5 years, M has 55% and S has 45% of the population.

(b) 10 years: $(PD)D = PD^2$; 15 years: PD^3

(c) 60% in M and 40% in S. Population proportions are stable.

49. 172, 208, 268, 327, 101, 123, 268, 327, 216, 263, 162, 195, 176, 215, 343, 417

51. (a)

$B = \begin{bmatrix} \frac{3}{4} & \frac{2}{5} & \frac{1}{4} \\ \frac{1}{4} & \frac{3}{5} & \frac{3}{4} \end{bmatrix}$; $D = \begin{bmatrix} 22 & 30 \\ 12 & 20 \\ 8 & 11 \end{bmatrix}$; $BD = \begin{bmatrix} 23.3 & 33.25 \\ 18.7 & 27.75 \end{bmatrix}$

Houston's need for black crude is 23,300 gal and for gold crude is 18,700 gal. Gulfport needs 33,250 gal of black and 27,750 gal of gold.

(b) $PBD = [135.945 \quad 197.3255]$; Houston's cost = \$135,945; Gulfport's cost = \$197,532.50

53. (a) $B = \begin{bmatrix} 0.7 & 8.5 & 10.2 & 1.1 & 5.6 & 3.6 \\ 0.5 & 0.2 & 6.1 & 1.3 & 0.2 & 1.0 \\ 2.2 & 0.4 & 8.8 & 1.2 & 1.2 & 4.8 \\ 251.8 & 63.4 & 81.6 & 35.2 & 54.3 & 144.2 \\ 30.0 & 1.0 & 1.0 & 1.0 & 1.0 & 1.0 \\ 788.9 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$

(b) $A = \begin{bmatrix} 1.11 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0.95 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1.11 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1.11 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0.95 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0.95 \end{bmatrix}$

3.3 EXERCISES

1. $\left[\begin{array}{ccc|c} 1 & -2 & -1 & -7 \\ 0 & 7 & 5 & 21 \\ 4 & 2 & 2 & 1 \end{array}\right]$

3. $\left[\begin{array}{ccc|c} 1 & -3 & 4 & 2 \\ 2 & 0 & 2 & 1 \\ 1 & 2 & 1 & 1 \end{array}\right]$ **5.** $x = 2, y = 1/2, z = -5$

7. $x = -5, y = 2, z = 1$

9. $x = 4, y = 1, z = -2$ **11.** $x = 15, y = -13, z = 2$

13. $x = 15, y = 0, z = 2$

15. $x = 1, y = 3, z = 1, w = 0$

17. no solution

19. (a) $x = (11 + 2z)/3, y = (-1 - z)/3,$ $z =$ any real number

(b) many possibilities, including $x = 11/3, y = -\frac{1}{3},$ $z = 0$ and $x = 13/3, y = -2/3, z = 1$

21. If a row of the matrix has all 0's in the coefficient matrix and a nonzero number in the augment, there is no solution.

23. $x = 0, y = -z, z =$ any real number

25. no solution

27. $x = 1 - z, y = \frac{1}{2}z, z =$ any real number

29. $x = 1, y = -1, z = 1$

31. $x = 2z - 2, y = 1 + z, z =$ any real number

33. $x = \frac{7}{2} - z, y = -\frac{1}{2}, z =$ any real number

35. $x = \frac{26}{5} - \frac{7}{5}z, y = \frac{4}{5} + \frac{2}{5}z, z =$ any real number

37. $x_1 = 20, x_2 = 60, x_3 = 40$

39. $x_1 = 1, x_2 = 0, x_3 = 1, x_4 = 0$

41. $x = 7/5, y = -3/5, z = w, w =$ any real number

43. no solution

45. $x_1 = 1 - 2x_4 - 3x_5, x_2 = 4 + 5x_4 + 7x_5,$ $x_3 = -3 - 3x_4 - 5x_5, x_4 =$ any real number, $x_5 =$ any real number

47. $x = (b_2c_1 - b_1c_2)/(a_1b_2 - a_2b_1)$

49. beef: 2 cups; sirloin: 8 cups

51. (a) \$50,000 at 12%, \$85,000 at 10%, \$100,000 at 8%

(b) \$6000 at 12%, \$8500 at 10%, \$8000 at 8%

53. AP = 1100, DT = 440, CA = 660

55. AF: 2 oz, FP: 2 oz, NMG: 1 oz

57. 2 of portfolio I, 2 of portfolio II

59. $\frac{3}{8}$ pound of red meat, 6 slices of bread, 4 glasses of milk

61. type I = 3(type IV), type II = 1000 − 2(type IV), type III = 500 − type IV, type IV = any integer satisfying 0 ≤ type IV ≤ 500

63. bacteria III = any amount satisfying
1800 ≤ bacteria III ≤ 2300
bacteria I = 6900 − 3 (bacteria III)
bacteria II = $\frac{1}{2}$(bacteria III) − 900

65. (a) $C = 2800 + 0.6R$
$U = 7000 - R$
$R =$ any integer satisfying $0 \le R \le 7000$

(b) $R = 1000$: $C = 3400$
$U = 6000$
$R = 2000$: $C = 4000$
$U = 5000$

(c) Min $C = 2800$ when $R = 0$ and $U = 7000$

(d) Max $C = 7000$ when $R = 7000$ and $U = 0$

67. There are three possibilities:
(1) 4 of I and 2 of II
(2) 5 of I, 1 of II, and 1 of III
(3) 6 of I and 2 of III

3.4 EXERCISES

1. $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ **3.** yes **5.** $\begin{bmatrix} 2 & -7 \\ -1 & 4 \end{bmatrix}$

7. no inverse **9.** $\begin{bmatrix} -\frac{1}{10} & \frac{7}{10} \\ \frac{1}{5} & -\frac{2}{5} \end{bmatrix}$

11. $\begin{bmatrix} \frac{1}{3} & 0 & 0 \\ 0 & \frac{1}{3} & 0 \\ 0 & 0 & \frac{1}{3} \end{bmatrix}$ **13.** $\begin{bmatrix} -1 & 1 & 0 \\ 1 & 0 & 0 \\ -1 & 0 & 1 \end{bmatrix}$

15. $\begin{bmatrix} \frac{1}{3} & -\frac{1}{3} & \frac{1}{3} \\ -\frac{2}{3} & -\frac{1}{3} & \frac{7}{3} \\ \frac{1}{3} & \frac{2}{3} & -\frac{5}{3} \end{bmatrix}$

17. no inverse **19.** no inverse

21. $\begin{bmatrix} 2 & 2 & 0 & 2 & 2 \\ 1 & 0 & 2 & 2 & 1 \\ 0 & 1 & 0 & 2 & 1 \\ 2 & 0 & 2 & 2 & 1 \\ 1 & 0 & 0 & 0 & 2 \end{bmatrix}$ **23.** $\begin{bmatrix} 13 \\ 5 \end{bmatrix}$ **25.** $\begin{bmatrix} 9 \\ 6 \\ 3 \end{bmatrix}$

27. $\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 2 \end{bmatrix}$ **29.** $x = 2, y = 1$

31. $x = 1, y = 2$ **33.** $x = 1, y = 1, z = 1$

35. $x = 1, y = 3, z = 2$

37. $x_1 = 5.6, x_2 = 5.4, x_3 = 3.25, x_4 = 6.1, x_5 = 0.4$

39. (a) −2 (b) inverse exists

41. (a) 0 (b) no inverse

43. (a) −5 (b) inverse exists

45. (a) −19 (b) inverse exists

47. Hang on **49.** Answers in back

51. $x_0 = 2400, y_0 = 1200$

53. (a) A = 5.5 mg and B = 8.8 mg for patient I

(b) A = 10 mg and B = 16 mg for patient II

55. \$68,000 at 18%, \$77,600 at 10%

57. (a) 2 Deluxe, 8 Premium, 32 Ultimate

(b) 22 Deluxe, 8 Premium, 22 Ultimate
[New] = [Old] + 8[Col. 1 of A^{-1}]

59. \$200,000 at 6%, \$300,000 at 8%, \$500,000 at 10%

61. (a) $\begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 1 & 1 \end{bmatrix}$ (b) 108

63. (a) $M = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 1 & 1 \end{bmatrix}$ (b) 30

3.5 EXERCISES

1. (a) 15 (b) 4 **3.** 8 **5.** 40
7. most: raw materials; least: fuels
9. raw materials, manufacturing, service
11. farm products = 200; machinery = 40
13. utilities = 200; manufacturing = 400
15. (a) agricultural products = 244; oil products = 732
(b) agricultural products = 0.4; oil products = 1.2
17. (a) mining = 106; manufacturing = 488
(b) mining = 1.4; manufacturing = 1.2
19. (a)

$$A = \begin{matrix} & \text{EC} & \text{C} \\ & \begin{bmatrix} 0.3 & 0.6 \\ 0.2 & 0.2 \end{bmatrix} & \begin{matrix} \text{Electronic components} \\ \text{Computers} \end{matrix} \end{matrix}$$

(b) electronic components = 1200; computers = 320
21. (a)

$$A = \begin{matrix} & \text{F} & \text{O} \\ & \begin{bmatrix} 0.30 & 0.04 \\ 0.35 & 0.10 \end{bmatrix} & \begin{matrix} \text{Fishing} \\ \text{Oil} \end{matrix} \end{matrix}$$

(b) fishing = 100; oil = 1250
23. development = \$21,000; promotional = \$12,000
25. engineering = \$15,000; computer = \$13,000
27. fishing = 400; agriculture = 500; mining = 400
29. electronics = 1240; steel = 1260; autos = 720
31. service = 90; manufacturing = 200; agriculture = 100
33. products = $\frac{7}{17}$ households;
machinery = $\frac{1}{17}$ households
35. government = $\frac{10}{19}$ households;
industry = $\frac{11}{19}$ households
37. (a)

$$A = \begin{matrix} & \text{M} & \text{U} & \text{H} \\ & \begin{bmatrix} 0.5 & 0.4 & 0.3 \\ 0.4 & 0.5 & 0.3 \\ 0.1 & 0.1 & 0.4 \end{bmatrix} & \begin{matrix} \text{Manufacturing} \\ \text{Utilities} \\ \text{Households} \end{matrix} \end{matrix}$$

(b) manufacturing = 3 households;
utilities = 3 households

39. $\begin{bmatrix} 24 \\ 96 \\ 24 \\ 120 \\ 492 \\ 3456 \end{bmatrix}$ 3456 bolts, 492 braces, 120 panels

41. $\begin{bmatrix} 10 \\ 10 \\ 20 \\ 56 \\ 20 \\ 26 \\ 300 \end{bmatrix}$ 56 2 × 4s, 20 braces, 26 clamps, 300 nails

CHAPTER 3 REVIEW EXERCISES

1. 4 **2.** 0 **3.** *A*, *B* **4.** none **5.** *D*, *F*, *G*, *I*

6. $\begin{bmatrix} -2 & 5 & 11 & -8 \\ -4 & 0 & 0 & -4 \\ 2 & 2 & -1 & -9 \end{bmatrix}$

7. zero matrix **8.** order

9. $\begin{bmatrix} 6 & -1 & -9 & 3 \\ 10 & 3 & -1 & 4 \\ -2 & -2 & -2 & 14 \end{bmatrix}$ **10.** $\begin{bmatrix} 3 & -3 \\ 4 & -1 \\ 2 & -6 \\ 1 & -2 \end{bmatrix}$

11. $\begin{bmatrix} 2 & 1 \\ 5 & 1 \end{bmatrix}$ **12.** $\begin{bmatrix} 12 & -6 \\ 15 & 0 \\ 18 & 0 \\ 3 & 9 \end{bmatrix}$ **13.** $\begin{bmatrix} 4 & 0 \\ 0 & 4 \end{bmatrix}$

14. $\begin{bmatrix} 2 & -12 \\ -8 & -22 \end{bmatrix}$ **15.** $\begin{bmatrix} 9 & 20 \\ 4 & 5 \end{bmatrix}$ **16.** $\begin{bmatrix} 5 & 16 \\ 6 & 15 \end{bmatrix}$

17. $\begin{bmatrix} 2 & 37 & 61 & -55 \\ -2 & 9 & -3 & -20 \\ 10 & 10 & -14 & -30 \end{bmatrix}$ **18.** $\begin{bmatrix} 43 & -23 \\ 33 & -12 \\ -13 & 15 \end{bmatrix}$

19. $\begin{bmatrix} 10 & 16 \\ 15 & 25 \\ 18 & 30 \\ 6 & 11 \end{bmatrix}$ **20.** $\begin{bmatrix} 17 & 73 \\ 7 & 28 \end{bmatrix}$ **21.** $\begin{bmatrix} 3 & 7 \\ 23 & 42 \end{bmatrix}$

22. *F* **23.** *F* **24.** $\begin{bmatrix} -19 & 12 \\ -8 & 5 \end{bmatrix}$ **25.** *F*

26. (a) infinitely many solutions (bottom row of 0's)
(b) $x = 6 + 2z$
$y = 7 - 3z$
$z =$ any real number
Two specific solutions:
If $z = 0$, then $x = 6$, $y = 7$.
If $z = 1$, then $x = 8$, $y = 4$.
27. (a) no solution (last row says $0 = 1$)
(b) no solution
28. (a) Unique coefficient matrix is I_3.
(b) $x = 0$, $y = -10$, $z = 14$
29. (1, 2, 1) **30.** $x = 22$, $y = 9$
31. $x = -3$, $y = 3$, $z = 4$
32. $x = -\frac{3}{2}$, $y = 7$, $z = -\frac{11}{2}$ **33.** no solution
34. $x = 2 - 2z$, $y = -1 - 2z$, $z =$ any real number

35. $x = -2 + 8z$
$y = -2 + 3z$
$z =$ any real number

36. $x_1 = 1, x_2 = 11, x_3 = -4, x_4 = -5$ **37.** yes

38. $\begin{bmatrix} \frac{1}{2} & \frac{1}{4} \\ \frac{5}{2} & \frac{7}{4} \end{bmatrix}$ **39.** $\begin{bmatrix} -1 & -2 & 8 \\ 1 & 2 & -7 \\ 1 & 1 & -4 \end{bmatrix}$

40. $\begin{bmatrix} 2 & 1 & -2 \\ 7 & 5 & -8 \\ -13 & -9 & 15 \end{bmatrix}$

41. $x = -33, y = 30, z = 19$

42. $x = 4, y = 5, z = -13$

43. $A^{-1} = \begin{bmatrix} -41 & 32 & 5 \\ 17 & -13 & -2 \\ -9 & 7 & 1 \end{bmatrix}$; $x = 4, y = -2, z = 2$

44. no **45.** (a) 16 (b) yes, det $\neq 0$

46. (a) 0 (b) no, det $= 0$

47. $\begin{bmatrix} 250 & 140 \\ 480 & 700 \end{bmatrix}$ **48.** $\begin{bmatrix} 1030 & 800 \\ 700 & 1200 \end{bmatrix}$

49. (a) higher in June (b) higher in July

50. $\begin{array}{c} \text{Men} \quad \text{Women} \\ \begin{bmatrix} 865 & 885 \\ 210 & 270 \end{bmatrix} \end{array} \begin{array}{l} \text{Robes} \\ \text{Hoods} \end{array}$ **51.** $\begin{bmatrix} 1750 \\ 480 \end{bmatrix} \begin{array}{l} \text{Robes} \\ \text{Hoods} \end{array}$

52. (a) $\begin{bmatrix} 13{,}500 & 12{,}400 \\ 10{,}500 & 10{,}600 \end{bmatrix}$
(b) Department A should buy from Kink; Department B should buy from Ace.

53. (a) $[0.20 \quad 0.30 \quad 0.50]$
(b) $\begin{bmatrix} 0.013469 \\ 0.013543 \\ 0.006504 \end{bmatrix}$
(c) $[0.20 \ 0.30 \ 0.50] \begin{bmatrix} 0.013469 \\ 0.013543 \\ 0.006504 \end{bmatrix} = 0.20(0.013469) + 0.30(0.013543) + 0.50(0.006504) = 0.0100087$
(d) The historical return of the portfolio, 0.0100087, is the estimated expected monthly return of the portfolio. This is roughly 1% per month.

54. 400 fast food, 700 software, 200 pharmaceutical

55. (a) A $=$ 2C, B $=$ 2000 $-$ 4C, C $=$ any integer satisfying $0 \leq C \leq 500$
(b) yes; A $=$ 500, B $=$ 1000, C $=$ 250
(c) max A $=$ 1000 when B $=$ 0, C $=$ 500

56. (a) 3 passenger, 4 transport, 4 jumbo
(b) 1 passenger, 3 transport, 7 jumbo
(c) column 2

57. (a) shipping $=$ 5680; agriculture $=$ 1960
(b) shipping $=$ 0.4; agriculture $=$ 1.8

58. (a) $A = \begin{array}{c} \text{S} \qquad \text{C} \\ \begin{bmatrix} 0.1 & 0.1 \\ 0.2 & 0.05 \end{bmatrix} \end{array} \begin{array}{l} \text{Shoes} \\ \text{Cattle} \end{array}$
(b) shoes $=$ 1000; cattle $=$ 500

59. mining $=$ 360; manufacturing $=$ 320; fuels $=$ 400

60. government $= \frac{64}{93}$ households; agriculture $= \frac{59}{93}$ households; manufacturing $= \frac{40}{93}$ households

CHAPTER 3 TEST

1. $\begin{bmatrix} 3 & 1 & 5 \\ 1 & 3 & 6 \end{bmatrix}$ **2.** $\begin{bmatrix} -1 & 2 & 2 \\ 1 & -1 & 6 \end{bmatrix}$

3. $\begin{bmatrix} -12 & -16 & -155 \\ 5 & 12 & 87 \end{bmatrix}$ **4.** $\begin{bmatrix} 23 & 6 \\ 182 & 45 \\ 21 & 1 \end{bmatrix}$

5. $\begin{bmatrix} 0 & -7 \\ 26 & 1 \end{bmatrix}$ **6.** $\begin{bmatrix} -43 & -46 & -207 \\ 39 & 30 & -77 \\ 17 & 5 & -216 \end{bmatrix}$

7. $\begin{bmatrix} -2 & 3/2 \\ 1 & -1/2 \end{bmatrix}$ **8.** $\begin{bmatrix} -3 & 2 & 2 \\ 1 & 0 & -1 \\ 1/2 & -1/2 & 0 \end{bmatrix}$

9. $\begin{bmatrix} 5 \\ 14 \\ 15 \end{bmatrix}$ **10.** $x = -0.5, y = 0.5, z = 2.5$

11. $x = 4 - 1.8z, y = 0.2z, z =$ any real number

12. no solution **13.** $x = 2, y = 2, z = 0, w = -2$

14. $x = 6w - 0.5, y = 0.5 - w, z = 2.5 - 3w, w =$ any real number

15. (a) $B = \$45{,}000, E = \$40{,}000$
(b) $\$0 \leq H \leq \$25{,}000$ (so $B \geq 0$)
(c) min $E = \$20{,}000$ when $H = \$0$ and $B = \$75{,}000$

16. (a) $\begin{bmatrix} 0.08 & 0.22 & 0.12 \\ 0.10 & 0.08 & 0.19 \\ 0.05 & 0.07 & 0.09 \\ 0.10 & 0.26 & 0.15 \\ 0.12 & 0.04 & 0.24 \end{bmatrix}$
(b) 0.08, 0.22, 0.12 consumed by carnivores 1, 2, 3
(c) plant 5 by 1, plant 4 by 2, plant 5 by 3

17. (a) $[1000 \quad 4000 \quad 2000 \quad 1000]$
(b) $[45{,}000 \quad 55{,}000 \quad 90{,}000 \quad 70{,}000]$
(c) $\begin{bmatrix} 5 \\ 3 \\ 4 \\ 4 \end{bmatrix}$ (d) [\$1,030,000] (e) $\begin{array}{c} A \\ B \\ C \\ D \end{array} \begin{array}{c} \$ \\ \begin{bmatrix} 65 \\ 145 \\ 125 \\ 135 \end{bmatrix} \end{array}$

18. (a) 121, 46, 247, 95, 261, 99, 287, 111, 179, 69, 169, 64
(b) Frodo lives

19. growth, 2000; blue-chip, 400; utility, 400

20. (a) agriculture $=$ 245; minerals $=$ 235
(b) agriculture $=$ 7; minerals $=$ 1
(c) agriculture $=$ 0.5; minerals $=$ 1.5

21. profit $=$ households
nonprofit $= \frac{2}{3}$ households

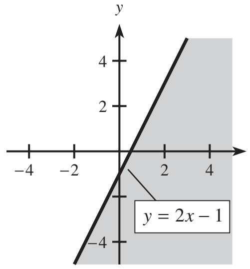
22.
$$\begin{array}{c} \\ \end{array}\begin{matrix} \text{Ag} & \text{M} & \text{F} & \text{S} \end{matrix}$$
$$\begin{bmatrix} 0.2 & 0.1 & 0.1 & 0.1 \\ 0.3 & 0.2 & 0.2 & 0.2 \\ 0.2 & 0.2 & 0.3 & 0.3 \\ 0.1 & 0.4 & 0.2 & 0.2 \end{bmatrix} \begin{matrix} \text{Agriculture} \\ \text{Machinery} \\ \text{Fuel} \\ \text{Steel} \end{matrix}$$

23. agriculture: 5000; machinery: 8000; fuel: 8000; steel: 7000

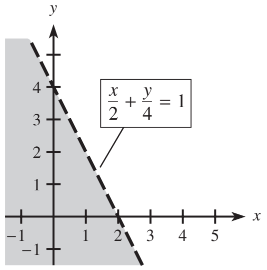
24. agriculture: $\frac{520}{699}$ households; steel: $\frac{236}{233}$ households; fuel: $\frac{159}{233}$ households

4.1 EXERCISES

1.



3.



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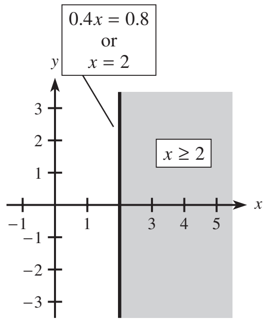
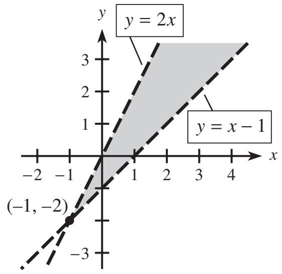


7. (0, 0), (20, 10), (0, 15), (25, 0)

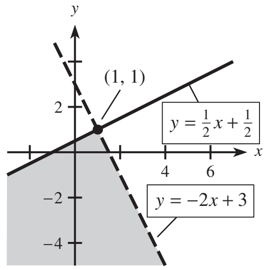
9. (5, 0), (15, 0), (6, 9), (2, 6)

11. (0, 5), (1, 2), (3, 1), (6, 0)

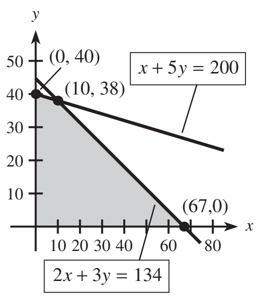
13.



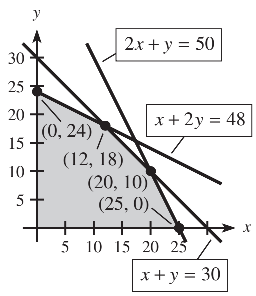
15.



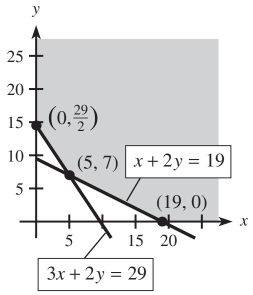
17.



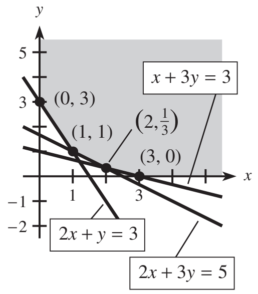
19.



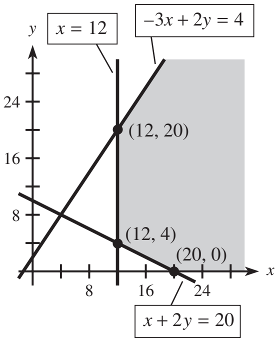
21.



23.



25.



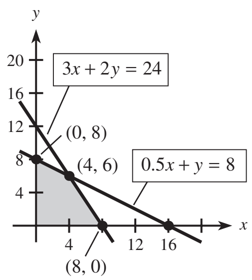
27. (a) Let x = the number of deluxe models and y = the number of economy models.

$3x + 2y \leq 24$

$\frac{1}{2}x + y \leq 8$

$x \geq 0, y \geq 0$

(b)



Corners: (0, 0), (8, 0), (4, 6), (0, 8)

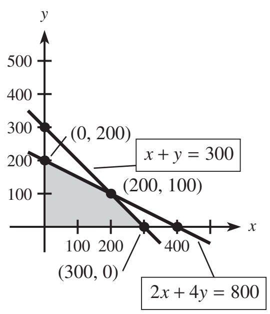
29. (a) Let x = the number of cord-type trimmers and y = the number of cordless trimmers. Constraints are

$x + y \leq 300$

$2x + 4y \leq 800$

$x \geq 0, y \geq 0$

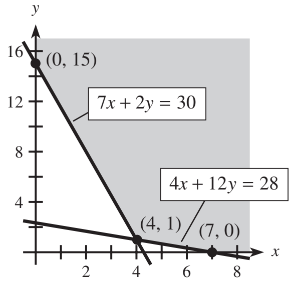
(b)



31. (a) Let $x =$ the number of minutes on finance programs and $y =$ the number of minutes on sports programs.

$7x + 2y \geq 30$
$4x + 12y \geq 28$
$x \geq 0, y \geq 0$

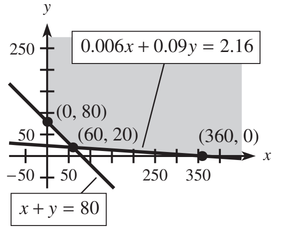
(b)



33. (a) Let $x =$ the number of minutes of radio and $y =$ the number of minutes of television. Constraints are

$x + y \geq 80$
$0.006x + 0.09y \geq 2.16$
$x \geq 0, y \geq 0$

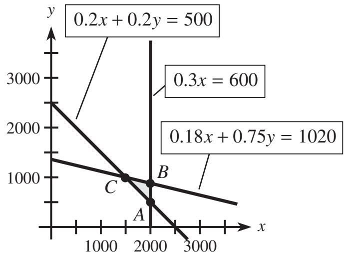
(b)



35. (a) Let $x =$ the number of pounds of regular hot dogs and $y =$ the number of pounds of all-beef hot dogs.

$0.18x + 0.75y \leq 1020$
$0.2x + 0.2y \geq 500$
$0.3x \leq 600$

(b)



$A = (2000, 500)$
$B = (2000, 880)$
$C = (1500, 1000)$

4.2 EXERCISES

1. max = 76 at (4, 4); min = 0 at (0, 0)
3. no max; min = 11 at (1, 3)
5. (0, 0), (0, 20), (10, 18), (15, 10), (20, 0); max = 66 at (10, 18); min = 0 at (0, 0)
7. (0, 60), (10, 30), (20, 20), (70, 0); min = 100 at (20, 20); no max
9. max = 1260 at $x = 12, y = 18$
11. min = 66 at $x = 0, y = 3$
13. max = 22 at (2, 4)
15. max = 30 on line between (0, 5) and (3, 4)
17. min = 32 at (2, 3)
19. min = 9 at (2, 3)
21. max = 10 at (2, 4)
23. min = 3100 at (40, 60)
25. If $x =$ the number of deluxe models and $y =$ the number of economy models, then max = \$132 at (4, 6).
27. If $x =$ the number of cord-type trimmers and $y =$ the number of cordless trimmers, then max = \$9000 at any point with integer coordinates on the segment joining (0, 200) and (200, 100), such as (20, 190).
29. radio = 60, TV = 20, min C = \$16,000
31. inkjet = 45, laser = 25, max P= \$3300
33. 250 fish: 150 bass and 100 trout
35. (a) Max P = \$315,030 when corn = 3749.5 acres and soybeans = 2251.5 acres
(b) Max P = \$315,240 when corn = 3746 acres and soybeans = 2262 acres
(c) \$30/acre
37. 30 days for factory 1 and 20 days for factory 2; minimum cost = \$700,000
39. 60 days for location I and 70 days for location II; minimum cost = \$86,000
41. reg = 2000 lb; all-beef = 880 lb; maximum profit = \$1328
43. From Pittsburgh: 20 to Blairsville, 40 to Youngstown; From Erie: 15 to Blairsville, 0 to Youngstown; minimum cost = \$1540
45. (a) R = \$366,000 with 6 satellite and 17 full-service branches
(b) Branches: used 23 of 25 possible; 2 not used (slack)
New employees: hired 120 of 120 possible; 0 not hired (slack)
Budget: used all \$2.98 million; \$0 not used (slack)
(c) Additional new employees and additional budget. These items are completely used in the current optimal solution; more could change and improve the optimal solution.
(d) Additional branches. The current optimal solution does not use all those allotted; more would just add to the extras.

4.3 EXERCISES

1. $3x + 5y + s_1 = 15, 3x + 6y + s_2 = 20$

3.
$$\left[\begin{array}{ccccc|c} 2 & 5 & 1 & 0 & 0 & 400 \\ 1 & 2 & 0 & 1 & 0 & 175 \\ \hline -3 & -7 & 0 & 0 & 1 & 0 \end{array}\right]$$

5.
$$\left[\begin{array}{ccccccc|c} 2 & 7 & 9 & 1 & 0 & 0 & 0 & 100 \\ 6 & 5 & 1 & 0 & 1 & 0 & 0 & 145 \\ 1 & 2 & 7 & 0 & 0 & 1 & 0 & 90 \\ \hline -2 & -5 & -2 & 0 & 0 & 0 & 1 & 0 \end{array}\right]$$

7. one slack variable for each constraint row (above the last row)

9. (a) $x_1 = 0, x_2 = 0, s_1 = 200, s_2 = 400, s_3 = 350, f = 0$
(b) not complete
(c) $\left[\begin{array}{cccccc|c} \textcircled{10} & 27 & 1 & 0 & 0 & 0 & 200 \\ 4 & 51 & 0 & 1 & 0 & 0 & 400 \\ 15 & 27 & 0 & 0 & 1 & 0 & 350 \\ \hline -8 & -7 & 0 & 0 & 0 & 1 & 0 \end{array}\right]$
$\frac{1}{10}R_1 \to R_1$, then $-4R_1 + R_2 \to R_2$,
$-15R_1 + R_3 \to R_3$, $8R_1 + R_4 \to R_4$

11. (a) $x_1 = 0, x_2 = 45, s_1 = 14, s_2 = 0, f = 75$
(b) not complete
(c) $\left[\begin{array}{ccccc|c} \textcircled{2} & 0 & 1 & -\frac{3}{4} & 0 & 14 \\ 3 & 1 & 0 & \frac{1}{3} & 0 & 45 \\ \hline -6 & 0 & 0 & 3 & 1 & 75 \end{array}\right]$
$\frac{1}{2}R_1 \to R_1$, then $-3R_1 + R_2 \to R_2$, $6R_1 + R_3 \to R_3$

13. (a) $x_1 = 24, x_2 = 0, x_3 = 21,$
$s_1 = 16, s_2 = 0, s_3 = 0, f = 780$
(b) complete (no part (c))

15. (a) $x_1 = 0, x_2 = 0, x_3 = 12,$
$s_1 = 4, s_2 = 6, s_3 = 0, f = 150$
(b) not complete
(c) $\left[\begin{array}{ccccccc|c} 4 & 4 & 1 & 0 & 0 & 2 & 0 & 12 \\ \textcircled{2} & \textcircled{4} & 0 & 1 & 0 & 1 & 0 & 4 \\ -3 & -11 & 0 & 0 & 1 & -1 & 0 & 6 \\ \hline -3 & -3 & 0 & 0 & 0 & 4 & 1 & 150 \end{array}\right]$
Either circled number may act as the next pivot entry, but only one of them. If 4 is used,
$\frac{1}{4}R_2 \to R_2$, then $-4R_2 + R_1 \to R_1$,
$11R_2 + R_3 \to R_3$, $3R_2 + R_4 \to R_4$. If 2 is used,
$\frac{1}{2}R_2 \to R_2$, then $-4R_2 + R_1 \to R_1$,
$3R_2 + R_3 \to R_3$, $3R_2 + R_4 \to R_4$.

17. (a) $x_1 = 0, x_2 = 0, x_3 = 12,$
$s_1 = 5, s_2 = 0, s_3 = 6, f = 120$
(b) no solution (no part (c))

19. $x = 11, y = 9; f = 20$

21. $x = 0, y = 14, z = 11; f = 525$

23. $x = 50, y = 10; f = 100$. Multiple solutions are possible.
Next pivot is circled.
$\left[\begin{array}{cccccc|c} 1 & 0 & 3 & 0 & 6 & 0 & 50 \\ 0 & 0 & 4 & 1 & -4 & 0 & 6 \\ 0 & 1 & -2 & 0 & \textcircled{2} & 0 & 10 \\ \hline 0 & 0 & 9 & 0 & 0 & 1 & 100 \end{array}\right]$

25. $x = 0, y = 5; f = 50$

27. $x = 4, y = 3; f = 17$

29. $x = 4, y = 3; f = 11$

31. $x = 0, y = 2, z = 5; f = 40$

33. $x = 15, y = 15, z = 25; f = 780$

35. $x = 6, y = 2, z = 26; f = 206$

37. $x_1 = 36, x_2 = 24; x_3 = 0, x_4 = 8; f = 1728$

39. $x = 8, y = 16; f = 32$

41. no solution

43. $x = 0, y = 50$ or $x = 40, y = 40; f = 600$

45. (a) $\left[\begin{array}{ccccc|c} 1 & 1 & 1 & 0 & 0 & 60 \\ 1 & 3 & 0 & 1 & 0 & 120 \\ \hline -40 & 60 & 0 & 0 & 1 & 0 \end{array}\right]$
(b) Maximum profit is \$3000 with 30 inkjet and 30 laser printers.

47. 300 style-891, 450 style-917, maximum $P = \$5175$

49. Maximum profit is \$3900 with 11 axles and 2 wheels

51. premium and light = 175 each; maximum $P = \$35{,}000$

53. 21 newspapers, 13 radio; 230,000 exposures

55. medium 1 = 10, medium 2 = 10, medium 3 = 12

57. \$1650 profit with 46 A, 20 B, 6 C

59. 8000 Regular, 0 Special, and 1000 Kitchen Magic; maximum profit = \$32,000

61. (a) 26 one-bedroom; 40 two-bedroom; 48 three-bedroom
(b) \$100,200 per month

63. 20-in. LCDs = 40, 42-in. LCDs =115, 42-in. plasma = 0, 50-in. plasma = 38; max $P = \$12{,}540$

4.4 EXERCISES

1. (a) $\left[\begin{array}{cc|c} 5 & 2 & 16 \\ 1 & 2 & 8 \\ \hline 4 & 5 & g \end{array}\right]$ transpose $= \left[\begin{array}{cc|c} 5 & 1 & 4 \\ 2 & 2 & 5 \\ \hline 16 & 8 & g \end{array}\right]$
(b) maximize $f = 16x_1 + 8x_2$ subject to
$5x_1 + x_2 \le 4, 2x_1 + 2x_2 \le 5, x_1 \ge 0, x_2 \ge 0.$

3. (a) $\left[\begin{array}{cc|c} 1 & 2 & 30 \\ 1 & 4 & 50 \\ \hline 7 & 3 & g \end{array}\right]$ transpose $= \left[\begin{array}{cc|c} 1 & 1 & 7 \\ 2 & 4 & 3 \\ \hline 30 & 50 & g \end{array}\right]$
(b) maximize $f = 30x_1 + 50x_2$ subject to
$x_1 + x_2 \le 7, 2x_1 + 4x_2 \le 3, x_1 \ge 0, x_2 \ge 0$

5. (a) $y_1 = 7, y_2 = 4, y_3 = 0$; min $g = 452$
(b) $x_1 = 15, x_2 = 0, x_3 = 29$; max $f = 452$

7. maximize $f = 11x_1 + 11x_2 + 16x_3$ subject to
$2x_1 + x_2 + x_3 \le 2$
$x_1 + 3x_2 + 4x_3 \le 10$
primal: $y_1 = 16, y_2 = 0; g = 32$ (min)
dual: $x_1 = 0, x_2 = 0, x_3 = 2; f = 32$ (max)

9. maximize $f = 11x_1 + 12x_2 + 6x_3$ subject to
$4x_1 + 3x_2 + 3x_3 \le 3$
$x_1 + 2x_2 + x_3 \le 1$
primal: $y_1 = 2, y_2 = 3; g = 9$ (min)
dual: $x_1 = 3/5, x_2 = 1/5, x_3 = 0; f = 9$ (max)

11. min = 28 at $x = 2, y = 0, z = 1$

13. $y_1 = 2/5, y_2 = 1/5, y_3 = 1/5; g = 16$ (min)

15. (a) minimize $g = 120y_1 + 50y_2$ subject to
$3y_1 + y_2 \ge 40$
$2y_1 + y_2 \ge 20$
(b) primal: $x_1 = 40, x_2 = 0, f = 1600$ (max)
dual: $y_1 = 40/3, y_2 = 0, g = 1600$ (min)

17. min = 480 at $y_1 = 0, y_2 = 0, y_3 = 16$

19. min = 90 at $y_1 = 0, y_2 = 3, y_3 = 1, y_4 = 0$

21. Atlanta = 150 hr, Fort Worth = 50 hr; min C = \$210,000

23. line 1 for 4 hours, line 2 for 1 hour; \$1200

25. A = 12 weeks, B = 0 weeks, C = 0 weeks; cost = $12,000

27. factory 1: 50 days, factory 2: 0 days; min cost $500,000

29. 105 minutes on radio, nothing on TV; min cost $10,500

31. (a) Georgia package = 10
Union package = 20
Pacific package = 5
(b) $4630

33. (a) min cost = $16
(b) Many solutions are possible; two are: 16 oz of food I, 0 oz of food II, 0 oz of food III and 11 oz of food I, 1 oz of food II, 0 oz of food III.

35. Mon. = 8, Tues. = 0, Wed. = 5, Thurs. = 4, Fri. = 5, Sat. = 0, Sun. = 3; min = 25

4.5 EXERCISES

1. $-3x + y \le -5$ **3.** $-6x - y \le -40$

5. (a) maximize $f = 2x + 3y$ subject to
$7x + 4y \le 28$
$3x - y \le -2$
$x \ge 0,\ y \ge 0$

(b) $$\left[\begin{array}{ccccc|c} 7 & 4 & 1 & 0 & 0 & 28 \\ 3 & \textcircled{-1} & 0 & 1 & 0 & -2 \\ \hline -2 & -3 & 0 & 0 & 1 & 0 \end{array}\right]$$

7. (a) Maximize $-g = -3x - 8y$ subject to
$4x - 5y \le 50$
$x + y \le 80$
$x - 2y \le -4$
$x \ge 0,\ y \ge 0$

(b) $$\left[\begin{array}{cccccc|c} 4 & -5 & 1 & 0 & 0 & 0 & 50 \\ 1 & 1 & 0 & 1 & 0 & 0 & 80 \\ 1 & \textcircled{-2} & 0 & 0 & 1 & 0 & -4 \\ \hline 3 & 8 & 0 & 0 & 0 & 1 & 0 \end{array}\right]$$

9. $x = 6,\ y = 8,\ z = 12;\ f = 120$

11. $x = 10,\ y = 17;\ f = 57$

13. $x = 5,\ y = 7;\ f = 31$

15. $x = 5,\ y = 15;\ f = 45$

17. $x = 10,\ y = 20;\ f = 120$

19. $x = 20,\ y = 10,\ z = 0;\ f = 40$

21. $x = 5,\ y = 0,\ z = 3;\ f = 22$

23. $x = 70, y = 0, z = 40; f = 2100$

25. $x_1 = 20, x_2 = 10, x_3 = 20, x_4 = 80, x_5 = 10,\ x_6 = 10;\ f = 3250$

27. regular = 2000 lb; beef = 880 lb; profit = $1328

29. 400 filters, 300 housing units; min cost = $5145

31. Produce 200 of each at Monaca; produce 300 commercial components and 550 domestic furnaces at Hamburg; profit = $355,250

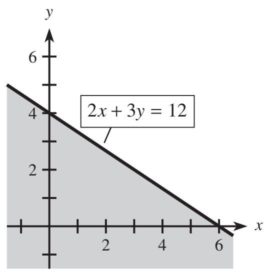
33. Produce 200 of each at Monaca; produce 300 commercial components and 550 domestic furnaces at Hamburg; cost = $337,750

35. I = 3 million, II = 0, III = 3 million; cost = $180,000

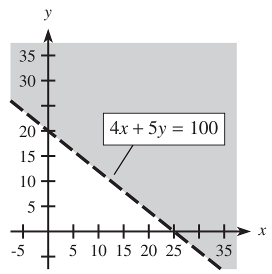
37. 2000 footballs, 0 soccer balls, 0 volleyballs; $60,000

CHAPTER 4 REVIEW EXERCISES

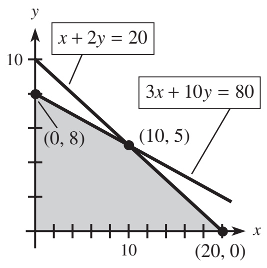
1.



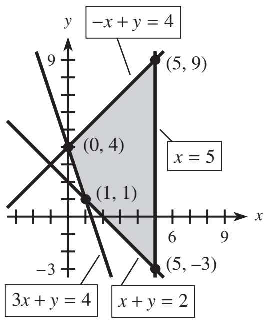
2.



3.



4.



5. max = 25 at (5, 10); min = −12 at (12, 0)

6. max = 194 at (17, 23); min = 104 at (8, 14)

7. max = 140 at (20, 0); min = −52 at (20, 32)

8. min = 115 at (5, 7); no max exists, f can be made arbitrarily large

9. $f = 66$ at (6, 6) **10.** $f = 43$ at (7, 9)

11. $g = 24$ at (3, 3) **12.** $g = 84$ at (64, 4)

13. $f = 76$ at (12, 8) **14.** $f = 75$ at (15, 15)

15. $f = 168$ at (12, 7) **16.** $f = 260$ at (60, 20)

17. $f = 360$ at (40, 30) **18.** $f = 80$ at (20, 10)

19. $f = 640$ on the line between (160, 0) and (90, 70)

20. no solution **21.** $g = 32$ at $y_1 = 2,\ y_2 = 3$

22. $g = 20$ at $y_1 = 4,\ y_2 = 2$

23. $g = 7$ at $y_1 = 1,\ y_2 = 5$

24. $g = 1180$ at $y_1 = 80,\ y_2 = 20$

25. $f = 165$ at $x = 20,\ y = 21$

26. $f = 54$ at $x = 6,\ y = 5$ **27.** $f = 270$ at (5, 3, 2)

28. $g = 140$ at $y_1 = 0,\ y_2 = 20,\ y_3 = 20$

29. $g = 1400$ at $y_1 = 0,\ y_2 = 100,\ y_3 = 100$

30. $f = 156$ at $x = 15,\ y = 2$

31. $f = 31$ at $x = 4,\ y = 5$ **32.** $f = 4380$ at (40, 10, 0, 0)

33. $f = 1000$ at $x_1 = 25, x_2 = 62.5, x_3 = 0, x_4 = 12.5$

34. $g = 2020$ at $x_1 = 0, x_2 = 100, x_3 = 80, x_4 = 20$

35. P = $14,750 when 110 large and 75 small swing sets are made

36. C = $300,000 when factory 1 operates 30 days, factory 2 operates 25 days

37. P = $320; I = 40, II = 20

38. P = $420; Jacob's ladders = 90, locomotive engines = 30

39. (a) Let x_1 = the number of 27-in LCD sets,
x_2 = the number of 32-in LCD sets,
x_3 = the number of 42-in LCD sets,
x_4 = the number of 42-in plasma sets.

(b) Maximize $P = 80x_1 + 120x_2 + 160x_3 + 200x_4$ subject to

$$\begin{aligned} 8x_1 + 10x_2 + 12x_3 + 15x_4 &\le 1870 \\ 2x_1 + 4x_2 + 4x_3 + 4x_4 &\le 530 \\ x_1 + x_2 + x_3 + x_4 &\le 200 \\ x_3 + x_4 &\le 100 \\ x_2 &\le 120 \end{aligned}$$

(c) $x_1 = 15, x_2 = 25, x_3 = 0, x_4 = 100$; max profit $= \$24{,}200$

40. food I = 0 oz, food II = 3 oz; $C = \$0.60$ (min)

41. cost = \$5.60; $A = 40$ lb, $B = 0$ lb

42. cost = \$8500; $A = 20$ days, $B = 15$ days, $C = 0$ days

43. pancake mix = 8000 lb; cake mix = 3000 lb; profit = \$3550

44. Texas: 55 desks, 65 computer tables; Louisiana: 75 desks, 65 computer tables; cost = \$4245

45. Midland: grade 1 = 486.5 tons, grade 2 = 0 tons; Donora: grade 1 = 13.5 tons, grade 2 = 450 tons; Cost = \$90,635

CHAPTER 4 TEST

1. max = 120 at $(0, 24)$

2. (a) C;

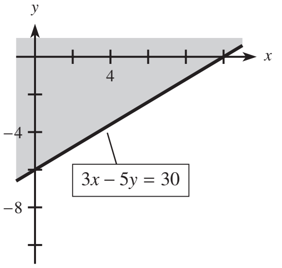
$$\left[\begin{array}{ccccccc|c} 1 & 2 & 0 & 1 & 0 & -3/2 & 0 & 40 \\ 0 & \textcircled{1} & 0 & -2 & 1 & 1/2 & 0 & 15 \\ 0 & 3 & 1 & -1 & 0 & 1/4 & 0 & 60 \\ \hline 0 & 0 & 0 & 4 & 0 & 6 & 1 & 220 \end{array}\right]$$

$-2R_2 + R_1 \to R_1 \quad -3R_2 + R_3 \to R_3$

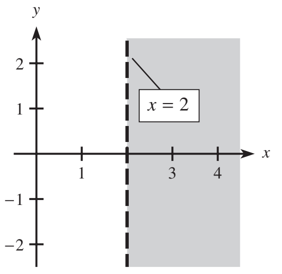
(b) A; pivot column is column 3, but new pivot is undefined.

(c) B; $x_1 = 40, x_2 = 12, x_3 = 0, s_1 = 0, s_2 = 20, s_3 = 0; f = 170$; This solution is not optimal; the next pivot is the 3-6 entry.

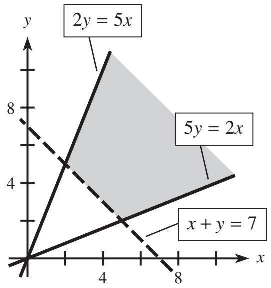
3. (a)



(b)



(c)



4. maximize $f = 100x_1 + 120x_2$ subject to

$$\begin{aligned} 3x_1 + 4x_2 &\le 2 \\ 5x_1 + 6x_2 &\le 3 \\ x_1 + 3x_2 &\le 5 \\ x_1 \ge 0,\ x_2 &\ge 0 \end{aligned}$$

5. min = 21 at $(1, 8)$; no max exists, f can be made arbitrarily large

6. min = 136 at $(28, 52)$

7. maximize $-g = -7x - 3y$ subject to

$$\begin{aligned} x - 4y &\le -4 \\ x - y &\le 5 \\ 2x + 3y &\le 30 \end{aligned}$$

8. max: $x_1 = 17, x_2 = 15, x_3 = 0; f = 658$ (max)
min: $y_1 = 4, y_2 = 18, y_3 = 0; g = 658$ (min)

9. max: = 6300 at $x = 90, y = 0$

10. max = 1200 at $x = 0, y = 16, z = 12$

11. If x = the number of barrels of beer and y = the number of barrels of ale, then maximize $P = 35x + 30y$ subject to

$$\begin{aligned} 3x + 2y &\le 1200 \\ 2x + 2y &\le 1000 \end{aligned}$$

$P = \$16{,}000$ (max) at $x = 200, y = 300$

12. If x = the number of day calls and y = the number of evening calls, then minimize $C = 3x + 4y$ subject to

$$\begin{aligned} 0.3x + 0.3y &\ge 150 \\ 0.1x + 0.3y &\ge 120 \\ x &\le 0.5(x + y) \end{aligned}$$

$C = \$1850$ (min) at $x = 150, y = 350$

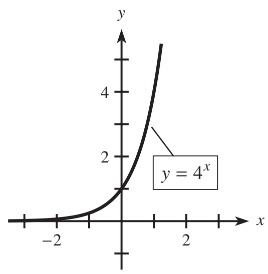
13. max profit = \$5000 when product 1 = 25 tons, product 2 = 62.5 tons, product 3 = 0 tons, product 4 = 12.5 tons

5.1 EXERCISES

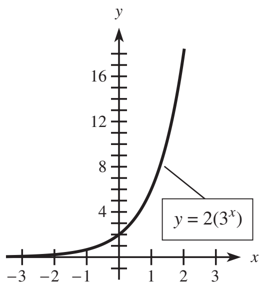
1. (a) 3.162278 (b) 0.01296525

3. (a) 1.44225 (b) 7.3891

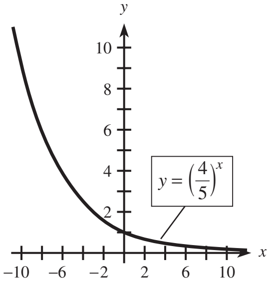
5.



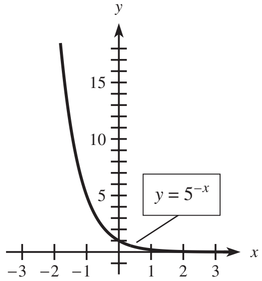
7.



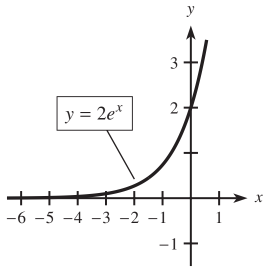
9.



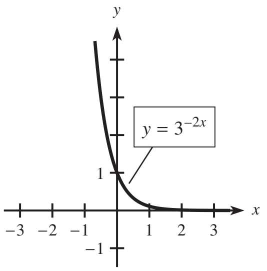
11.



13.

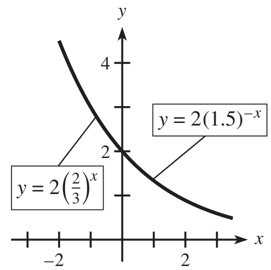


15.



17. (a) $y = 3(2.5)^{-x}$
(b) Decay. They have the form $y = C \cdot b^x$ for $0 < b < 1$ or $y = C \cdot a^{-x}$ for $a > 1$.
(c) The graphs are identical.

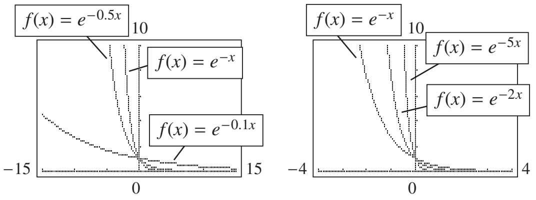
19. (a) and (b)



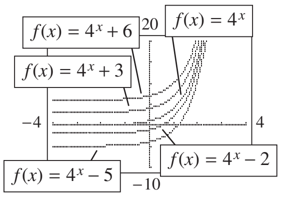
(c) $(1.5)^{-x} = \left(\frac{3}{2}\right)^{-x} = \left(\frac{2}{3}\right)^{x}$

21. $y = \left(\frac{5}{4}\right)^{-x}$

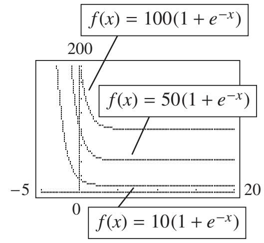
23. All graphs have the same basic shape. For larger positive values of k, the graphs fall more sharply. For positive values of k nearer 0, the graphs fall more slowly.



25. $y = f(x) + C$ is the same graph as $y = f(x)$ but shifted C units on the y-axis.



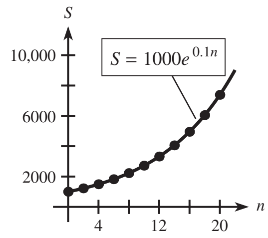
27. (a)



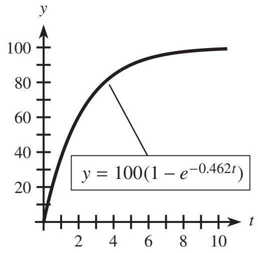
(b) As c changes, the y-intercept and the asymptote change.

29. $1884.54

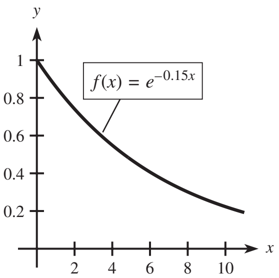
31.



33. At time 0, the concentration is 0. The concentration rises rapidly for the first 4 minutes and then tends toward 100% as time nears 10 minutes.

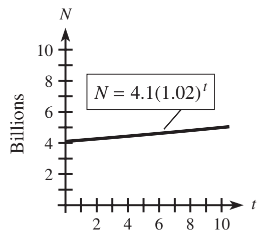


35.

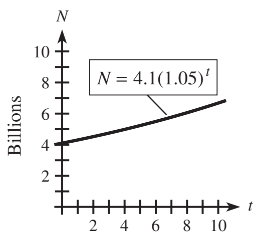


As the TV sets age (x increases), the fraction of sets still in service declines.

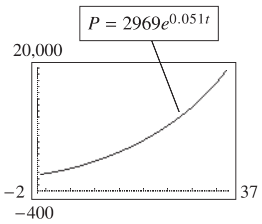
37.



39.



41. (a) Growth; $e > 1$ and the exponent is positive for $t > 0$.
(b)



43. The linear model fails in 2007, giving a negative number of processors.

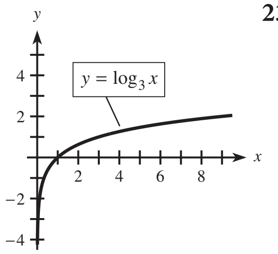
45. (a) $y = 492.4(1.070^x)$
(b) \$24,608 billion is an overestimate (c) 2021

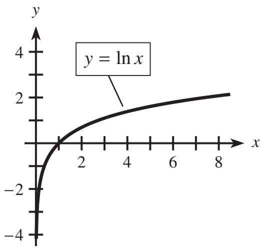
47. (a) $y = 2.74(1.042^x)$
(b) 284.1
(c) 2015

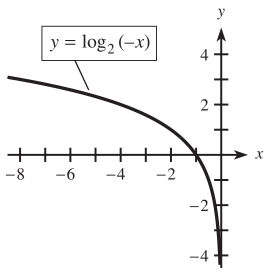
49. (a) $y = 98.221(0.870^x)$
(b) decay; base satisfies $0 < b < 1$
(c) 1.5

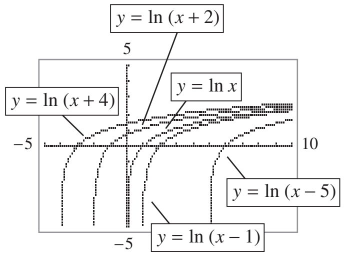
5.2 EXERCISES

1. $2^4 = 16$ **3.** $4^{1/2} = 2$ **5.** $x = 81$ **7.** $x = \frac{1}{4}$
9. $x = 16$ **11.** $x = 26.75$ **13.** $x \approx 2.013$
15. $\log_2 32 = 5$
17. $\log_4(\frac{1}{4}) = -1$ **19.** $3x + 5 = \ln(0.55)$; $x \approx -1.866$

21.


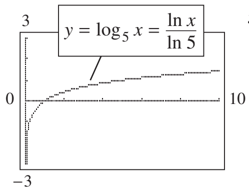
23.


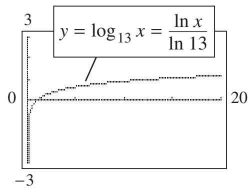
25.


27. (a) 3 (b) −1 **29.** x **31.** 3
33. (a) 4.9 (b) 0.4 (c) 12.4 (d) 0.9
35. $\log x - \log(x + 1)$ **37.** $\log_7 x + \frac{1}{3}\log_7(x + 4)$
39. $\ln(x/y)$ **41.** $\log_5[x^{1/2}(x + 1)]$
43. equivalent; Properties V and III
45. not equivalent; $\log(\sqrt[3]{8}/5)$
47. (a)


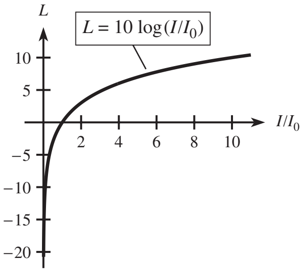
(b) For each c, the domain is $x > c$ and the vertical asymptote is at $x = c$.
(c) Each x-intercept is at $x = c + 1$.
(d) The graph of $y = f(x - c)$ is the graph of $y = f(x)$ shifted c units on the x-axis.

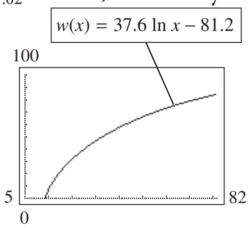
49. (a) 4.0875 (b) −0.1544

51.


53.


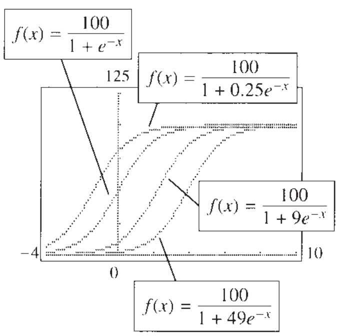
55. If $\log_a M = u$ and $\log_a N = v$, then $a^u = M$ and $a^v = N$. Therefore, $\log_a(M/N) = \log_a(a^u/a^v) = \log_a(a^{u-v}) = u - v = \log_a M - \log_a N$.

57. 63.1 times as severe **59.** 3.2 times as severe
61. 40
63. $L = 10 \log(I/I_0)$


65. 0.1 and 1×10^{-14}
67. $\text{pH} = \log \frac{1}{[H^+]} = \log 1 - \log[H^+] = -\log[H^+]$
69. $\log_{1.02} 2 = 4t$; $t \approx 8.75$ years
71. (a)


(b) 83.6 million (c) 2023
73. $y = -3130.317 + 4056.819 \ln x$; \$11,293
75. (a) $y = 47.725 + 12.785 \ln x$
(b) reasonably good fit (c) 88.9%

5.3 EXERCISES

1. $\frac{5}{3}$ **3.** 2.943 **5.** 9.390 **7.** 18.971 **9.** 151.413
11. 6.679 **13.** $10^5 = 100{,}000$ **15.** 7
17. $5e^{10}$ **19.** $\frac{10^6}{2} = 5 \cdot 10^5$ **21.** 3
23. (a)


(b) Different c-values change the y-intercept and how the graph approaches the asymptote.

25. (a) 2038 (b) 4.9 months

27. (a) 13.86 years (b) $105,850.00

29. 24.5 years **31.** 128,402

33. (a) 2015

(b) The intent of such plan would be to reduce future increases in health care expenditures. A new model might not be exponential, or, if it were, it would be one that rose more gently.

35. (a) $4.98 (b) 8 **37.** $502 **39.** $420.09

41. $2706.71 **43.** (a) $10,100.31 (b) 6.03 years

45. (a) $5469.03

(b) 7 years, 9 months (approximately)

47. (a) $142.5 billion (b) 35.3 years, in 2021

(c) the Financial Crisis of 2008

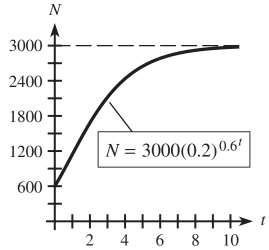
49. (a) 109.99, 307.66; It would have taken $109.99 in 1990, and will take $307.66 in 2015, to buy what cost $100 in 1982.

(b) 2022

51. $x \approx 993.3$; about 993 units

53. (a) 600 (b) 2119 (c) 3000

(d)



55. (a) 10 (b) 2.5 years

57. (a) 37 (b) 1.5 hours

59. (a) 52 (b) the 10th day

61. (a) 2% (b) 20 months ($x = 19.6$)

63. (a) 0.23 km^3 (b) 5.9 years

65. (a) 160.48 million

(b) $t \approx 84.7$; in 2025

67. (a) $y = \dfrac{80.8}{1 + 0.2e^{-0.324x}}$

(b) 80.69%

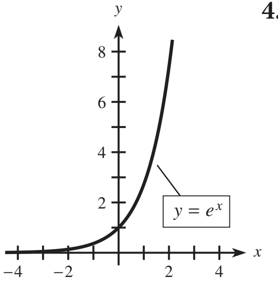
(c) in 2023

CHAPTER 5 REVIEW EXERCISES

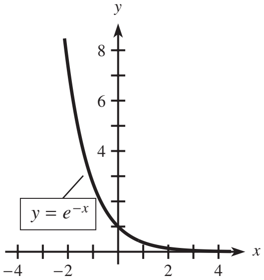
1. (a) $\log_2 y = x$ (b) $\log_3 2x = y$

2. (a) $7^{-2} = \frac{1}{49}$ (b) $4^{-1} = x$

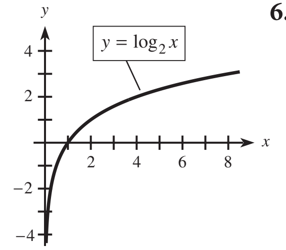
3.



4.



5.



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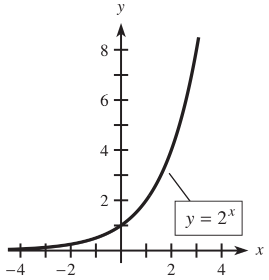
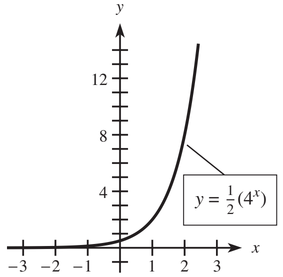


7.



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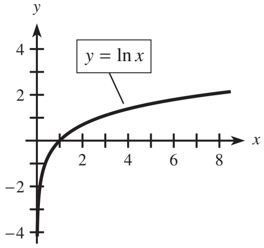

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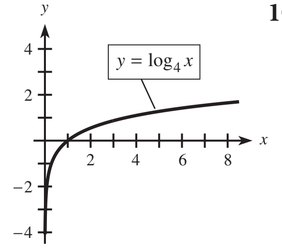

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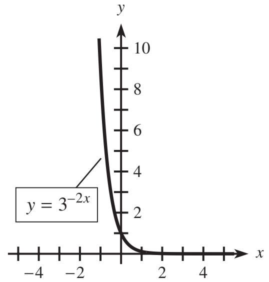

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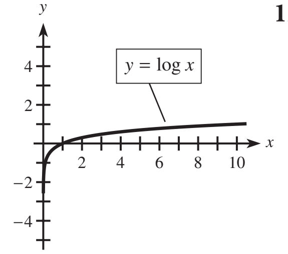

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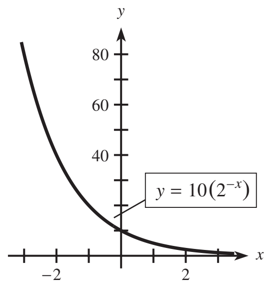


13. 0 **14.** 3 **15.** $\frac{1}{2}$ **16.** -1 **17.** 8

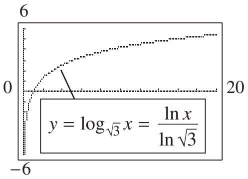
18. 1 **19.** 5 **20.** 3.15 **21.** -2.7

22. 0.6 **23.** 5.1 **24.** 15.6 **25.** $\log y + \log z$

26. $\frac{1}{2}\ln(x+1) - \frac{1}{2}\ln x$ **27.** no **28.** -2

29. 5 **30.** 1 **31.** 0 **32.** 3.4939 **33.** -1.5845

34.



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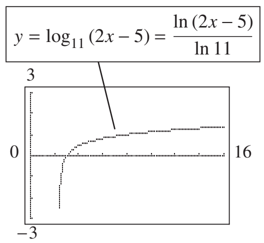


36. $x = 1.5$ **37.** $x \approx 51.224$

38. $x \approx 28.175$ **39.** $x \approx 40.236$

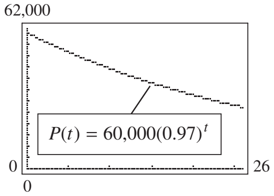
40. $x = 8$ **41.** $x = 14$ **42.** $x = 6$

43. Growth exponential, because the general outline has the same shape as a growth exponential.

44. Decay exponential, because the general shape is similar to the graph of a decay exponential, and the number will diminish toward 0.

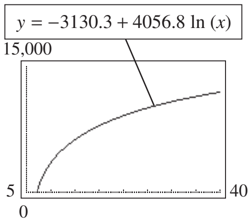
45. (a) \$32,627.66

(b)



46. (a) $y = 266.4(1.074^x)$ (b) \$4598 billion
(c) 2017

47. (a) \$11,293 (b)



48. (a) $y = -11.4 + 19.0 \ln x$
(b) 68.0% (c) 2023

49. (a) -3.9 (b) $0.14B_0$ (c) $0.004B_0$ (d) yes

50. (a) 27,441 (b) 12 weeks

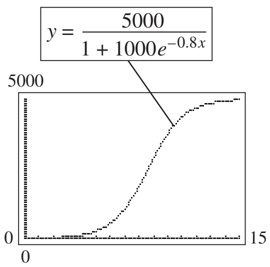
51. 1366 **52.** 5.8 years

53. (a) \$5532.77 (b) 5.13 years

54. logistic, because the graph begins like an exponential function but then grows at a slower rate

55. (a) 3000 (b) 8603 (c) 10,000

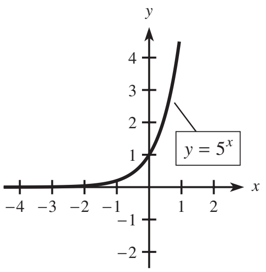
56. (a)



(b) 5 (c) 4970 (d) 10 days

CHAPTER 5 TEST

1.



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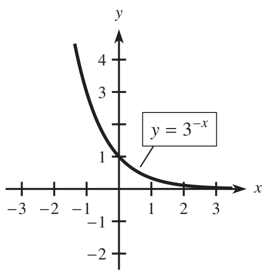

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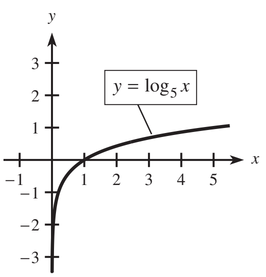

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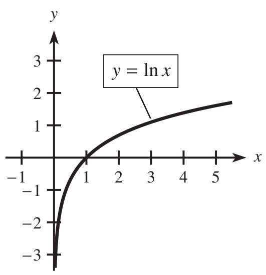

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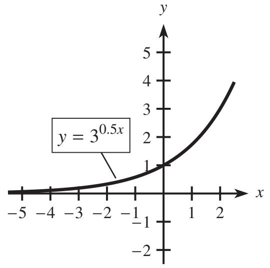

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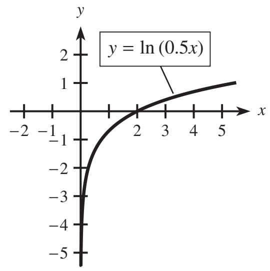

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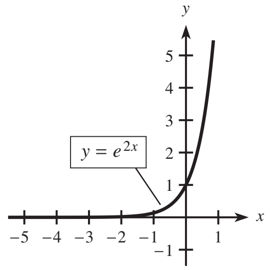

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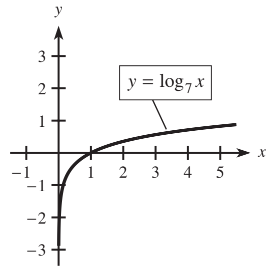


9. 54.598 **10.** 0.100 **11.** 1.386

12. 1.322 **13.** $x = 7^{3.1}, x \approx 416.681$

14. $\log_3(27) = 2x; x = 1.5$ **15.** $x \approx 0.203$

16. $x = 8$ **17.** 3 **18.** x^4 **19.** 3 **20.** x^2

21. $\ln M + \ln N$ **22.** $\ln(x^3 - 1) - \ln(x + 2)$

23. $\frac{\ln(x^3 + 1)}{\ln 4} \approx 0.721 \ln(x^3 + 1)$

24. $x \approx 38.679$

25. a decay exponential

26. With years on the horizontal axis, a growth exponential would probably be the best model.

27. (a) \$3363.3 billion
(b) about 11.2 years

28. about 6.2 months

29. (a) about \$16,716 billion
(b) $x \approx 39.8$; in 2025

30. (a) exponential; data continue to rise rapidly
(b) $y = 165.550(1.055^x)$
(c) \$819.42 billion

6.1 EXERCISES

1. $r = 0.0625, I = 250, P = 1000, t = 4$

3. $P = 8000, S = 9600, I = 1600, r = 0.05, t = 4$

5. (a) \$9600 (b) \$19,600

7. (a) \$30 (b) \$1030

9. \$864 **11.** \$3850 **13.** 13%

15. (a) 5.13% (b) 4.29% **17.** \$1631.07

19. \$12,000 **21.** 10 years **23.** pay on time

25. (a) \$2120 (b) \$2068.29 (nearest cent)

27. 3, 6, 9, 12, 15, 18, 21, 24, 27, 30

29. $-\frac{1}{3}, \frac{1}{5}, -\frac{1}{7}, \frac{1}{9}, -\frac{1}{11}, \frac{1}{13}$ **31.** $-1, -\frac{1}{4}, -\frac{1}{15}, 0; a_{10} = \frac{1}{20}$

33. (a) $d = 3, a_1 = 2$ (b) 11, 14, 17

35. (a) $d = \frac{3}{2}, a_1 = 3$ (b) $\frac{15}{2}, 9, \frac{21}{2}$

37. -35 **39.** 203 **41.** 2185 **43.** 1907.5

45. −15,862.5 **47.** 21, 34, 55 **49.** \$4800
51. the job starting at \$40,000 (\$58,000 versus \$57,600)
53. (a) \$3000 (b) \$4500 (c) plan II, by \$1500
(d) \$10,000 (e) \$13,500 (f) plan II, by \$3500
(g) plan II

6.2 EXERCISES

(Minor differences may occur because of rounding.)
1. $S = 3216.87$ = future value; principal = 2000; rate = 0.02; periods = 24
3. $P = 6049.97$ = principal; future value = 25,000; rate = 0.03; periods = 48
5. (a) 8% (b) 7 (c) 2% = 0.02 (d) 28
7. (a) 9% (b) 5 (c) $\left(\frac{9}{12}\right)\% = 0.0075$ (d) 60
9. \$24,846.79 **11.** $S = \$4755.03$; $I = \$1555.03$
13. \$13,322.92 **15.** \$5583.95
17. \$7309.98 **19.** \$502.47
21. (a) \$12,245.64 (b) \$11,080.32
(c) A $\frac{1}{2}$% increase in the interest rate reduces the amount required by \$1165.32.
23. \$50.26 more at 8% **25.** (a) 7.55% (b) 6.18%
27. 8% compounded monthly, 8% compounded quarterly, 8% compounded annually
29. 8.2% continuously yields 8.55%. 8.4% compounded quarterly yields 8.67% and so is better.
31. The higher graph is for continuous compounding because its yield (its effective annual rate) is higher.
33. 37.02% **35.** 3 years **37.** 4% **39.** \$3996.02
41. (a) \$2,124,876.38 (b) \$480,087.44 more
43. 5.12 years (approximately) **45.** \$13,916.24
47.

	A	B	C
1		Future Value	(Yearly)
2	End of Year	Quarterly	Monthly
3	0	\$5000.00	\$5000.00
4	1	\$5322.52	\$5324.26
5	2	\$5665.84	\$5669.54
6	3	\$6031.31	\$6037.22
7	4	\$6420.36	\$6428.74
8	5	\$6834.50	\$6845.65
9	6	\$7275.35	\$7289.60
10	7	\$7744.64	\$7762.34
11	8	\$8244.20	\$8265.74
12	9	\$8775.99	\$8801.79
13	10	\$9342.07	\$9372.59

(a) from quarterly and monthly spreadsheets: after $6\frac{1}{2}$ years (26 quarters or 78 months)
(b) See the spreadsheet.
49. (a) 24, 48, 96 (b) 24, 16, $\frac{32}{3}$
51. $10(2^{12})$ **53.** $4 \cdot \left(\frac{3}{2}\right)^{15}$ **55.** $\dfrac{6(1-3^{17})}{-2}$
57. $\dfrac{3^{35}-1}{2}$ **59.** $18\left[1-\left(\frac{2}{3}\right)^{18}\right]$
61. \$350,580 (approx.) **63.** 24.4 million (approx.)
65. 35 years **67.** 40.5 ft **69.** \$4096
71. 320,000 **73.** \$7,231,366 **75.** 6; $5^6 = 15{,}625$
77. 305,175,780

6.3 EXERCISES

1. $S = \$285{,}129$ = future value; $R = 2500$; $i = 0.02$; $n = 60$
3. $R = \$1426$ = payment; $S = 80{,}000$; $i = 0.04$; $n = 30$
5. (a) The higher graph is \$1120 per year.
(b) $R = \$1000$: $S = \$73{,}105.94$;
$R = \$1120$: $S = \$81{,}878.65$;
Difference = \$8772.71
7. \$7328.22 **9.** \$1072.97 **11.** \$1482.94
13. $n \approx 27.1$; 28 quarters
15. A sinking fund is a savings plan, so the 10% rate in part (a) is better.
17. \$4651.61 **19.** \$1180.78 **21.** \$4152.32
23. \$3787.92
25. (a) ordinary annuity (b) \$4774.55
27. (a) annuity due (b) \$3974.73
29. (a) ordinary annuity (b) \$1083.40
31. (a) ordinary annuity (b) \$266.10
33. (a) ordinary annuity (b) $n \approx 108.5$; 109 months
35. (a) annuity due (b) \$235.16
37. (a) annuity due (b) \$26,517.13
39. (a) ordinary annuity (b) \$795.75
41. \$53,677.40
43. (a) $n \approx 35$ quarters (b) \$1,062,412 (nearest dollar)
45. The spreadsheet shows the amounts at the end of each of the first 12 months and at the end of the last 12 months. The amount after 10 years is shown.

	A	B	C
1		Future Value	
2	End of Month	Ordinary Ann.	Annuity Due
3	0	0	100
4	1	$100.00	$100.60
5	2	$200.60	$201.80
6	3	$301.80	$303.61
7	4	$403.61	$406.04
8	5	$506.04	$509.07
9	6	$609.07	$612.73
10	7	$712.73	$717.00
11	8	$817.00	$821.91
12	9	$921.91	$927.44
13	10	$1027.44	$1033.60
14	11	$1133.60	$1140.40
15	12	$1240.40	$1247.85
	⋮	⋮	⋮
112	109	$15324.39	$15416.34
113	110	$15516.34	$15609.44
114	111	$15709.44	$15803.70
115	112	$15903.70	$15999.12
116	113	$16099.12	$16195.71
117	114	$16295.71	$16393.49
118	115	$16493.49	$16592.45
119	116	$16692.45	$16792.60
120	117	$16892.60	$16993.96
121	118	$17093.96	$17196.52
122	119	$17296.52	$17400.30
123	120	$17500.30	$17605.30

(a) $12,000
(b) Annuity due. Each payment for an annuity due earns 1 month's interest more than that for an ordinary annuity.

6.4 EXERCISES

1. $A_n = \$22{,}480$ = present value; $R = 1300$; $i = 0.04$; $n = 30$
3. $R = \$809$ = payment; $A_n = 135{,}000$; $i = 0.005$; $n = 360$
5. $69,913.77 **7.** $2,128,391 **9.** $4595.46
11. $n \approx 73.8$; 74 quarters **13.** $1141.81; premium
15. (a) The higher graph corresponds to 8%.
(b) $1500 (approximately)
(c) With an interest rate of 10%, a present value of about $9000 is needed to purchase an annuity of $1000 for 25 years. If the interest rate is 8%, about $10,500 is needed.
17. Ordinary annuity—payments at the end of each period
Annuity due—payments at the beginning of each period
19. $69,632.02 **21.** $445,962.23 **23.** $2145.59
25. (a) ordinary annuity (b) $10,882.46
27. (a) annuity due (b) $316,803.61
29. (a) ordinary annuity
(b) Taking $500,000 and $140,000 payments for the next 10 years has a slightly higher present value: $1,506,436.24.
31. (a) annuity due (b) $146,235.06
33. (a) annuity due (b) $22,663.74
35. (a) ordinary annuity (b) $11,810.24
37. (a) ordinary annuity (b) $27,590.62
39. (a) $8629.16 (b) $9883.48
41. (a) $30,078.99 (b) $16,900 (c) $607.02
(d) $36,421.20
43. (a) $4504.83 (b) $n \approx 21.9$; 22 withdrawals
45. $7957.86 **47.** $74,993.20 **49.** $59,768.91
51. $1317.98 **53.** $257,412.87

55. (a) The spreadsheet below shows the payments for the first 12 months and the last 12 months. Full payments for $13\frac{1}{2}$ years.

	A	B	C	D
1	End of Month	Acct. Value	Payment	New Balance
2	0	$100000.00	$0.00	$100000.00
3	1	$100650.00	$1000.00	$99650.00
4	2	$100297.73	$1000.00	$99297.73
5	3	$99943.16	$1000.00	$98943.16
6	4	$99586.29	$1000.00	$98586.29
7	5	$99227.10	$1000.00	$98227.10
8	6	$98865.58	$1000.00	$97865.58
9	7	$98501.70	$1000.00	$97501.70
10	8	$98135.47	$1000.00	$97135.47
11	9	$97766.85	$1000.00	$96766.85
12	10	$97395.83	$1000.00	$96395.83
13	11	$97022.40	$1000.00	$96022.40
14	12	$96646.55	$1000.00	$95646.55
	⋮	⋮	⋮	⋮
154	152	$10684.71	$1000.00	$9684.71
155	153	$9747.66	$1000.00	$8747.66
156	154	$8804.52	$1000.00	$7804.52
157	155	$7855.25	$1000.00	$6855.25
158	156	$6899.81	$1000.00	$5899.81
159	157	$5938.16	$1000.00	$4938.16
160	158	$4970.25	$1000.00	$3970.25
161	159	$3996.06	$1000.00	$2996.06
162	160	$3015.53	$1000.00	$2015.53
163	161	$2028.64	$1000.00	$1028.64
164	162	$1035.32	$1000.00	$35.32
165	163	$35.55	$35.55	$0.00

(b) The spreadsheet below shows the payments for the first 12 months and the last 12 months. Full payments for almost 4 years.

	A	B	C	D
1	End of Month	Acct. Value	Payment	New Balance
2	0	$100000.00	$0.00	$100000.00
3	1	$100650.00	$2500.00	$98150.00
4	2	$98787.98	$2500.00	$96287.98
5	3	$96913.85	$2500.00	$94413.85
6	4	$95027.54	$2500.00	$92527.54
7	5	$93128.97	$2500.00	$90628.97
8	6	$91218.05	$2500.00	$88718.05
9	7	$89294.72	$2500.00	$86794.72
10	8	$87358.89	$2500.00	$84858.89
11	9	$85410.47	$2500.00	$82910.47
12	10	$83449.39	$2500.00	$80949.39
13	11	$81475.56	$2500.00	$78975.56
14	12	$79488.90	$2500.00	$76988.90
	⋮	⋮	⋮	⋮
38	36	$27734.95	$2500.00	$25234.95
39	37	$25398.98	$2500.00	$22898.98
40	38	$23047.83	$2500.00	$20547.83
41	39	$20681.39	$2500.00	$18181.39
42	40	$18299.57	$2500.00	$15799.57
43	41	$15902.26	$2500.00	$13402.26
44	42	$13489.38	$2500.00	$10989.38
45	43	$11060.81	$2500.00	$8560.81
46	44	$8616.45	$2500.00	$6116.45
47	45	$6156.21	$2500.00	$3656.21
48	46	$3679.98	$2500.00	$1179.98
49	47	$1187.65	$1187.65	$0.00

6.5 EXERCISES

1. (a) the 10-year loan, because the loan must be paid more quickly
(b) the 25-year loan, because the loan is paid more slowly

3. $1288.29

5. $553.42 **7.** $10,345.11

9.

Period	Payment	Interest
1	$39,505.50	$9000.00
2	39,505.50	6254.51
3	39,505.43	3261.92
	118,516.43	18,516.43

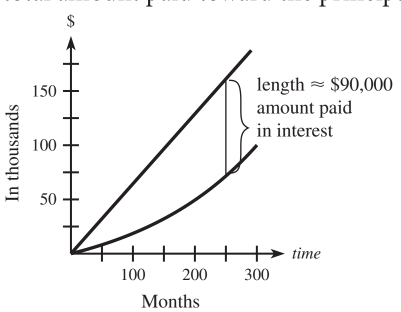
Period	Balance Reduction	Unpaid Balance
		$100,000.00
1	$30,505.50	69,494.50
2	33,250.99	36,243.51
3	36,243.51	0.00
	100,000.00	

11.

Period	Payment	Interest
1	$5380.54	$600.00
2	5380.54	456.58
3	5380.54	308.87
4	5380.54	156.71
	21,522.16	1,522.16

Period	Balance Reduction	Unpaid Balance
		$20,000.00
1	$4780.54	15,219.46
2	4923.96	10,295.50
3	5071.67	5,223.83
4	5223.83	0.00
	20,000.00	

13. $8852.05
15. $5785.83
17. (a) $17,436.92
(b) $348,738.40 + $150,000 = $498,738.40
(c) $148,738.40
19. (a) $1237.78
(b) $9902.24 + $2000 = $11,902.24
(c) $1902.24
21. $8903.25
23. (a) $276,991.32 (b) $263,575.30
25. (a) $89,120.53 (b) $6451.45
27. (a) $368.43; $383.43 (b) $n \approx 57.1$
(c) $211.95
29. (a) The line is the total amount paid ($644.30 per month × the number of months). The curve is the total amount paid toward the principal.
(b)



31.

	Rate	Payment	Total Interest
(a)	8%	$366.19	$2577.12
	8.5%	$369.72	$2746.56
(b)	6.75%	$518.88	$106,796.80
	7.25%	$545.74	$116,466.40

(c) The duration of the loan seems to have the greatest effect. It greatly influences payment size (for a $15,000 loan versus one for $80,000), and it also affects total interest paid.

33.

		Payment	Points	Total Paid
(a)	(i)	$738.99	_	$221,697
	(ii)	$722.81	$1000	$217,843
	(iii)	$706.78	$2000	$214,034

(b) The 7% loan with 2 points.
35. (a) $17,525.20 (b) $508.76
(c) $n \approx 33.2$; 34 quarters (d) $471.57
37. The spreadsheet shows the amortization schedule for the first 12 and the last 12 payments.

	A	B	C	D	E
1	Period	Payment	Interest	Bal. Reduction	Unpaid Bal.
2	0				$16700.00
3	1	$409.27	$114.12	$295.15	$16404.85
4	2	$409.27	$112.10	$297.17	$16107.68
5	3	$409.27	$110.07	$299.20	$15808.48
6	4	$409.27	$108.02	$301.25	$15507.23
7	5	$409.27	$105.97	$303.30	$15203.93
8	6	$409.27	$103.89	$305.38	$14898.55
9	7	$409.27	$101.81	$307.46	$14591.09
10	8	$409.27	$99.71	$309.56	$14281.52
11	9	$409.27	$97.59	$311.68	$13969.84
12	10	$409.27	$95.46	$313.81	$13656.03
13	11	$409.27	$93.32	$315.95	$13340.08
14	12	$409.27	$91.16	$318.11	$13021.97
	⋮	⋮	⋮	⋮	⋮
39	37	$409.27	$32.11	$377.16	$4322.49
40	38	$409.27	$29.54	$379.73	$3942.75
41	39	$409.27	$26.94	$382.33	$3560.43
42	40	$409.27	$24.33	$384.94	$3175.49
43	41	$409.27	$21.70	$387.57	$2787.91
44	42	$409.27	$19.05	$390.22	$2397.70
45	43	$409.27	$16.38	$392.89	$2004.81
46	44	$409.27	$13.70	$395.57	$1609.24
47	45	$409.27	$11.00	$398.27	$1210.97
48	46	$409.27	$8.27	$401.00	$809.97
49	47	$409.27	$5.53	$403.74	$406.24
50	48	$409.27	$2.78	$406.24	$0.00

CHAPTER 6 REVIEW EXERCISES

1. $1, \frac{1}{4}, \frac{1}{9}, \frac{1}{16}$
2. Arithmetic: (a) and (c) (a) $d = -5$ (c) $d = \frac{1}{6}$
3. 235 **4.** 109 **5.** 315
6. Geometric: (a) and (b) (a) $r = 8$ (b) $r = -\frac{3}{4}$
7. 8 **8.** 2, 391, $484\frac{4}{9}$
9. $S = R\left[\frac{(1+i)^n - 1}{i}\right]$ **10.** $I = Prt$
11. $A_n = R\left[\frac{1-(1+i)^{-n}}{i}\right]$ **12.** $S = P(1+i)^n$
13. $A_n = R\left[\frac{1-(1+i)^{-n}}{i}\right]$, solved for R
14. $S = Pe^{rt}$ **15.** $10,880 **16.** $6\frac{2}{3}\%$
17. $2941.18 **18.** $4650
19. the $40,000 job ($490,000 versus $472,500)
20. (a) 40 (b) 2% = 0.02
21. (a) $S = P(1+i)^n$ (b) $S = Pe^{rt}$
22. (b) monthly **23.** $372.79 **24.** $14,510.26
25. $1616.07 **26.** $21,299.21 **27.** 34.3 quarters
28. (a) 13.29% (b) 14.21%
29. (a) 7.40% (b) 7.47%
30. 2^{63} **31.** $2^{32} - 1$ **32.** $29,428.47
33. $6069.44 **34.** $31,194.18 **35.** $10,841.24
36. $n \approx 36$ quarters
37. $130,079.36 **38.** $12,007.09
39. (a) $11.828 million (b) $161.5 million
40. $1726.85 **41.** $5390.77
42. $n \approx 16$ half-years (8 years)
43. $88.85 **44.** $3443.61 **45.** $163,792.21
46.

Payment Number	Payment Amount	Interest	Balance Reduction	Unpaid Balance
57	$699.22	$594.01	$105.21	$94,936.99
58	$699.22	$593.36	$105.86	$94,831.13

47. 16.32% **48.** $213.81
49. (a) $1480 (b) $1601.03
50. $9319.64 **51.** 14.5 years **52.** 79.4%
53. $4053.54; discount
54. (a) $4728.19 (b) $5398.07 (c) $1749.88 (d) 10.78%
55. $21,474.08 **56.** $12,162.06
57. Quarterly APY = 6.68%. This rate is better for the bank; it pays less interest. Continuous APY = 6.69%. This rate is better for the consumer, who earns more interest.
58. $32,834.69 **59.** $3,466.64
60. (a) $1185.51 (b) $355,653 (c) $171,653 (d) $156,366.25
61. (a) $95,164.21 (b) $1300.14 **62.** $994.08
63. Future value of IRA = $172,971.32
Present value needed = $2,321,520.10
Future value needed from deposits = $2,148,548.78
Deposits = $711.60
64. Regular payment = $64,337.43
Unpaid balance = $2,365,237.24
Number of $70,000 payments = $n \approx 46.1$
Savings = $118,546.36

CHAPTER 6 TEST

1. 25.3 years (approximately) **2.** $840.75
3. 6.87% **4.** $158,524.90 **5.** 33.53%
6. (a) $698.00 (b) $112,400
7. $2625 **8.** $7999.41 **9.** 8.73%
10. $119,912.92 **11.** $40,552.00
12. $32,488 (to the nearest dollar) **13.** $6781.17
14. $n \approx 66.8$; 67 half-years
15. $1688.02
16. (a) $279,841.35 (b) $13,124.75
17. $116,909.10 **18.** $29,716.47
19. (a) The difference between successive terms is always −5.5.
(b) 23.8 (c) 8226.3
20. 1000 mg (approximately)
21. $12,975.49; premium
22. (a) $145,585.54 (with the $2000); $147,585.54 (without the $2000)
(b) $n \approx 318.8$ months
Total interest = $243,738.13
(c) $n \approx 317.2$ months
Total interest = $245,378.24
(d) Paying the $2000 is slightly better; it saves about $1640 in interest.

7.1 EXERCISES

1. (a) $\frac{2}{5}$ (b) 0 (c) 1 **3.** $\frac{1}{4}$ **5.** 1
7. (a) $\frac{3}{10}$ (b) $\frac{1}{2}$ (c) $\frac{1}{5}$ (d) $\frac{3}{5}$ (e) $\frac{7}{10}$
9. (a) $\frac{1}{13}$ (b) $\frac{1}{2}$ (c) $\frac{1}{4}$
11. {HH, HT, TH, TT}; (a) $\frac{1}{4}$ (b) $\frac{1}{2}$ (c) $\frac{1}{4}$
13. (a) $\frac{1}{12}$ (b) $\frac{1}{12}$ (c) $\frac{1}{36}$ **15.** (a) $\frac{1}{2}$ (b) $\frac{5}{12}$
17. (a) 431/1200
(b) If fair, $\Pr(6) = \frac{1}{6}$; 431/1200 not close to $\frac{1}{6}$, so not a fair die
19. (a) 2:3 (b) 3:2 **21.** (a) $\frac{1}{21}$ (b) $\frac{20}{21}$
23. 0.46 **25.** (a) 63/425 (b) 32/425
27. (a) R: 0.63 D: 0.41 I: 0.51
(b) Republican
29. (a) 1/3601 (b) 100/3601 (c) 3500/3601 (d) 30%
31. (a) 0.402 (b) 0.491 (c) 100%; yes
33. $S = \{A+, A-, B+, B-, AB+, AB-, O+, O-\}$; No. Type O+ is the most frequently occurring blood type.
35. (a) 0.04 (b) 0.96 **37.** (a) 0.13 (b) 0.87
39. 0.03 **41.** 0.75 **43.** $\frac{1}{3}$ **45.** $\frac{1}{3}$
47. 0.22; yes, 0.39 is much higher than 0.22 **49.** $\frac{3}{8}$
51. (a) no (b) {BB, BG, GB, GG} (c) $\frac{1}{2}$
53. $\frac{1}{8}$ **55.** $\frac{3}{125}$
57. $\Pr(A) = 0.000019554$, or about 1.9 accidents per 100,000

Pr(B) = 0.000035919, or about 3.6 accidents per 100,000
Pr(C) = 0.000037679, or about 3.8 accidents per 100,000
Intersection C is the most dangerous.
59. (a) 557/1200 (b) 11/120
61. (a) boy: 1/5; girl: 4/5
(b) boy: 0.4946; girl: 0.5054 (c) part (b)
63. 3/4 **65.** $3/3995 \approx 0.00075$

7.2 EXERCISES

1. $\frac{1}{6}$ **3.** $\frac{2}{3}$ **5.** $\frac{2}{5}$ **7.** (a) $\frac{1}{7}$ (b) $\frac{5}{7}$
9. $\frac{3}{4}$ **11.** $\frac{2}{3}$ **13.** $\frac{10}{17}$ **15.** $\frac{2}{3}$
17. (a) $\frac{1}{2}$ (b) $\frac{1}{3}$ (c) $\frac{8}{9}$ (d) $\frac{1}{9}$
19. 0.54 **21.** (a) 362/425 (b) $\frac{66}{85}$
23. $\frac{17}{50}$ **25.** (a) $\frac{5}{6}$ (b) $\frac{1}{6}$
27. (a) 0.35 (b) 0.08 (c) 0.83
29. (a) 0.508 (b) 0.633 (c) 0.761
31. (a) 0.267 (b) 0.371 (c) 0.931
33. (a) $\frac{11}{12}$ (b) $\frac{5}{6}$ **35.** (a) $\frac{1}{2}$ (b) $\frac{7}{8}$ (c) $\frac{3}{4}$
37. 0.56 **39.** 0.965
41. (a) 0.72 (b) 0.84 (c) 0.61
43. $\frac{31}{40}$ **45.** 0.13

7.3 EXERCISES

1. (a) $\frac{1}{2}$ (b) $\frac{1}{13}$ **3.** (a) $\frac{1}{3}$ (b) $\frac{1}{3}$ **5.** $\frac{4}{7}$
7. (a) $\frac{2}{3}$ (b) $\frac{4}{9}$ (c) $\frac{3}{5}$ **9.** (a) $\frac{1}{4}$ (b) $\frac{1}{2}$
11. $\frac{1}{36}$ **13.** (a) $\frac{1}{8}$ (b) $\frac{7}{8}$
15. (a) $\frac{3}{50}$ (b) $\frac{1}{15}$
(c) The events in part (a) are independent because the result of the first draw does not affect the probability for the second draw.
17. (a) $\frac{4}{25}$ (b) $\frac{9}{25}$ (c) $\frac{6}{25}$ (d) 0 **19.** $\frac{5}{68}$
21. (a) $\frac{1}{5}$ (b) $\frac{3}{5}$ (c) 0 **23.** (a) $\frac{1}{17}$ (b) 13/204
25. (a) $\frac{13}{17}$ (b) $\frac{4}{17}$ (c) $\frac{8}{51}$ **27.** $\frac{31}{52}$ **29.** $\frac{25}{96}$
31. $\frac{43}{50}$ **33.** $\frac{65}{87}$ **35.** $35/435 = \frac{7}{87}$ **37.** $\frac{1}{10}$
39. 1/144,000,000 **41.** 0.004292 **43.** 0.06
45. 0.045 **47.** $(0.95)^5 = 0.774$ **49.** 0.06
51. (a) 0.366 (b) 0.634
53. (a) 0.4565 (b) 0.5435
55. (a) $(\frac{1}{3})^3(\frac{1}{5})^4 = 1/16{,}875$
(b) $(\frac{2}{3})^3(\frac{4}{5})^4 = 2048/16{,}875$ (c) 14,827/16,875
57. 4/11; 4:7 **59.** (a) 364/365 (b) $\frac{1}{365}$
61. (a) 0.59 (b) 0.41

7.4 EXERCISES

1. $\frac{2}{5}$ **3.** (a) $\frac{2}{21}$ (b) $\frac{4}{21}$ (c) $\frac{23}{35}$
5. (a) $\frac{5}{42}$ (b) $\frac{10}{21}$ (c) $\frac{4}{9}$
7. (a) $\frac{1}{30}$ (b) $\frac{1}{2}$ (c) $\frac{5}{6}$ **9.** $\frac{3}{5}$
11. (a) $\frac{6}{25}$ (b) $\frac{9}{25}$ (c) $\frac{12}{25}$ (d) $\frac{19}{25}$
13. $\frac{2}{3}$ **15.** $\frac{2}{3}$ **17.** 0.3095
19. (a) 81/10,000 (b) 1323/5000
21. (a) $\frac{6}{35}$ (b) $\frac{6}{35}$ (c) $\frac{12}{35}$
23. (a) $\frac{4}{7}$ (b) $\frac{5}{14}$ (c) $\frac{7}{10}$ (d) $\frac{16}{25}$ **25.** $\frac{17}{45}$
27. 0.079 **29.** (a) 49/100 (b) $\frac{12}{49}$

7.5 EXERCISES

1. 360 **3.** 151,200 **5.** 1
7. (a) $6 \cdot 5 \cdot 4 \cdot 3 = 360$ (b) $6^4 = 1296$ **9.** $n!$
11. $n + 1$ **13.** 16 **15.** 4950 **17.** 1 **19.** 1
21. 10 **23.** (a) 8 (b) 240 **25.** 604,800
27. 120 **29.** 24 **31.** 64 **33.** 720
35. $2^{10} = 1024$ **37.** $4({}_{13}C_5) = 5148$
39. 10,816,000 **41.** 30,045,015 **43.** 792
45. 210 **47.** 2,891,999,880 **49.** 3,700,000

7.6 EXERCISES

1. $\frac{1}{120}$ **3.** (a) 120 (b) $\frac{1}{120}$
5. 0.639 **7.** (a) 1/10,000 (b) 1/5040
9. $1/10^6$ **11.** 0.000048 **13.** $1/10! = 1/3{,}628{,}800$
15. (a) $\frac{1}{22}$ (b) $\frac{6}{11}$ (c) $\frac{9}{22}$
17. 0.098 **19.** $\dfrac{{}_{90}C_{28} \cdot {}_{10}C_2}{{}_{100}C_{30}}$
21. (a) 0.119 (b) 0.0476 (c) 0.476
23. 0.0238 **25.** (a) 0.721 (b) 0.262 (c) 0.279
27. (a) $\frac{1}{3}$ (b) $\frac{1}{6}$ **29.** $\dfrac{{}_{20}C_{10}}{{}_{80}C_{10}} = 0.00000011$
31. (a) 0.033 (b) 0.633
33. (a) 0.0005 (b) 0.002 **35.** 0.00198

7.7 EXERCISES

1. can **3.** cannot, sum ≠ 1
5. cannot, not square **7.** can
9. [0.248 0.752] **11.** [0.228 0.236 0.536]
13. [0.25 0.75] **15.** [0.249 0.249 0.502]
17. $[\frac{1}{4} \ \ \frac{3}{4}]$ **19.** $[\frac{1}{4} \ \ \frac{1}{4} \ \ \frac{1}{2}]$
21. [0.5 0.4 0.1]; [0.44 0.43 0.13]; [0.431 0.43 0.139]; [0.4292 0.4291 0.1417]
23. $\begin{array}{c} \\ R \\ N \end{array}\begin{array}{c} \begin{matrix} R & N \end{matrix} \\ \begin{bmatrix} 0.8 & 0.2 \\ 0.3 & 0.7 \end{bmatrix} \end{array}$ **25.** 0.45
27. $\begin{array}{c} \\ A \\ F \\ V \end{array}\begin{array}{c} \begin{matrix} A & F & V \end{matrix} \\ \begin{bmatrix} 0 & 0.7 & 0.3 \\ 0.6 & 0 & 0.4 \\ 0.8 & 0.2 & 0 \end{bmatrix} \end{array}$
29. [0.3928 0.37 0.2372]
31. [46/113 38/113 29/113]
33. $\begin{array}{c} \\ r \\ u \end{array}\begin{array}{c} \begin{matrix} r & u \end{matrix} \\ \begin{bmatrix} 0.7 & 0.3 \\ 0.1 & 0.9 \end{bmatrix} \end{array}$; [1/4 3/4]
35. $[\frac{1}{14} \ \ \frac{3}{14} \ \ \frac{5}{7}]$ **37.** $[\frac{4}{7} \ \ \frac{2}{7} \ \ \frac{1}{7}]$
39. [49/100 42/100 9/100]

CHAPTER 7 REVIEW EXERCISES

1. (a) $\frac{5}{9}$ (b) $\frac{1}{3}$ (c) $\frac{2}{9}$
2. (a) $\frac{3}{4}$ (b) $\frac{1}{2}$ (c) $\frac{1}{6}$ (d) $\frac{2}{3}$
3. (a) 3:4 (b) 4:3 **4.** (a) $\frac{1}{4}$ (b) $\frac{1}{2}$ (c) $\frac{1}{4}$
5. (a) $\frac{3}{8}$ (b) $\frac{1}{8}$ (c) $\frac{3}{8}$ **6.** $\frac{2}{13}$ **7.** 16/169
8. $\frac{3}{4}$ **9.** $\frac{2}{13}$ **10.** $\frac{7}{13}$ **11.** (a) $\frac{2}{9}$ (b) $\frac{2}{3}$ (c) $\frac{7}{9}$
12. $\frac{2}{7}$ **13.** $\frac{1}{2}$ **14.** 7/320
15. 7/342 **16.** 3/14 **17.** $\frac{8}{15}$
18. (a) $\frac{3}{14}$ (b) $\frac{4}{7}$ (c) $\frac{3}{8}$ **19.** 49/89
20. 30 **21.** 35 **22.** 26^3 **23.** 56
24. (a) Not square
(b) The row sums are not 1.
25. [0.76 0.24], [0.496 0.504]
26. [0.2 0.8]
27. $\frac{5}{8}$ **28.** $\frac{1}{4}$ **29.** $\frac{29}{50}$
30. $\frac{5}{56}$ **31.** $\frac{33}{56}$ **32.** $\frac{15}{22}$ **33.** 0.72
34. (a) 63/2000 (b) $\frac{60}{63}$ **35.** 39/116 **36.** $4! = 24$
37. ${}_8P_4 = 1680$ **38.** ${}_{12}C_4 = 495$ **39.** ${}_8C_4 = 70$
40. (a) ${}_{12}C_2 = 66$ (b) ${}_{12}C_3 = 220$ **41.** 62,193,780
42. If her assumption about blood groups is accurate, there would be $4 \cdot 2 \cdot 4 \cdot 8 = 256$, not 288, unique groups.
43. $\frac{1}{24}$ **44.** $\frac{3}{500}$ **45.** $\frac{3}{1250}$
46. (a) 0.3398 (b) 0.1975 **47.** $\frac{1}{10}$
48. (a) $({}_{10}C_5)({}_2C_1)/{}_{12}C_6$
(b) $\dfrac{({}_{10}C_5)({}_2C_1) + ({}_{10}C_4)({}_2C_2)}{{}_{12}C_6}$
49. [0.135 0.51 0.355], [0.09675 0.3305 0.57275], [0.0640875 0.288275 0.6476375]
50. [12/265 68/265 37/53]

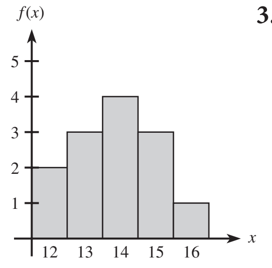
CHAPTER 7 TEST

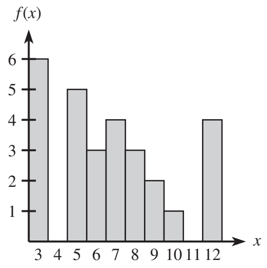
1. (a) $\frac{4}{7}$ (b) $\frac{3}{7}$ **2.** (a) $\frac{2}{7}$ (b) $\frac{5}{7}$
3. (a) 0 (b) 1 **4.** $\frac{1}{7}$ **5.** $\frac{1}{7}$
6. (a) $\frac{2}{7}$ (b) $\frac{4}{7}$ **7.** $\frac{2}{7}$ **8.** $\frac{3}{7}$ **9.** $\frac{2}{3}$
10. 1/17,576 **11.** 0.2389 **12.** (a) $\frac{1}{5}$ (b) $\frac{1}{20}$
13. (a) $\frac{3}{95}$ (b) $\frac{6}{19}$ (c) $\frac{21}{38}$ (d) 0
14. (a) 5,245,786 (b) 1/5,245,786
15. (a) 2,118,760 (b) 1/2,118,760
16. 0.064 **17.** (a) 0.633 (b) 0.962 **18.** 0.229
19. (a) $\frac{1}{5}$ (b) $\frac{1}{14}$ (c) $\frac{13}{14}$ **20.** $\frac{3}{14}$
21. (a) 2^{10} (b) $\dfrac{1}{2^{10}}$ (c) $\frac{1}{3}$ (d) Change the code.
22. (a) $A = \begin{bmatrix} 0.80 & 0.20 \\ 0.07 & 0.93 \end{bmatrix}$ (b) [0.25566 0.74434]; about 25.6% (c) $\frac{7}{27}$; 25.9% of market

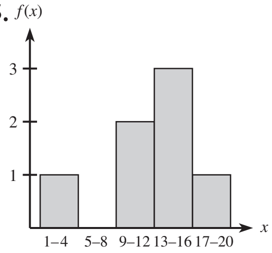
8.1 EXERCISES

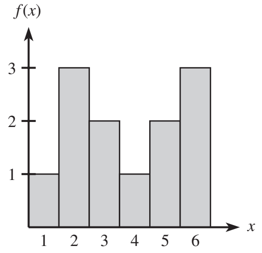
1. 0.0595
3. (a) 1/6 (b) 5/6 (c) 18 (d) 0.045
5. (a) $\frac{1}{64}$ (b) $\frac{5}{16}$ (c) $\frac{15}{64}$ **7.** 0.0284
9. (a) 0.2304 (b) 0.0102 (c) 0.3174
11. 0.0585 **13.** 0.2759
15. (a) 0.375 (b) 0.0625
17. (a) 0.1157 (b) 0.4823
19. (a) $\frac{27}{64}$ (b) $\frac{27}{128}$ (c) $\frac{81}{256}$
21. (a) 0.0729 (b) 0.5905 (c) 0.9914
23. 0.2457 **25.** 0.0007
27. (a) 0.1323 (b) 0.0308
29. (a) 0.9044 (b) 0.0914 (c) 0.0043
31. (a) 0.8683 (b) 0.2099 **33.** 0.740

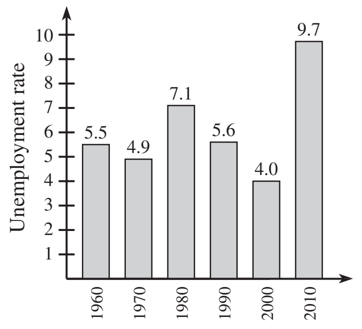
8.2 EXERCISES

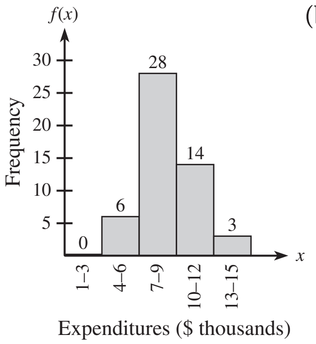
1.


3.


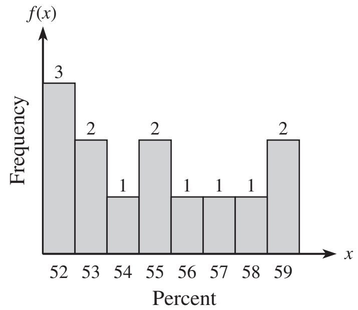
5.


7.


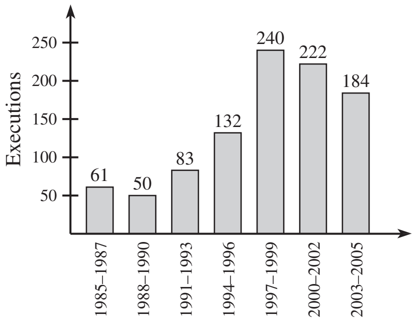
9. 3 **11.** 13 **13.** 5 **15.** 10.5
17. mode = 2, median = 4.5, mean = 6
19. mode = 17, median = 18.5, mean = 23.5
21. mode = 5.3, median = 5.3, mean = 5.32
23. 12.21, 14.5, 14.5 **25.** 9 **27.** 14
29. 4, 8.5714, 2.9277 **31.** 14, 4.6667, 2.1602
33. 2.73, 1.35 **35.** 6.75, 2.96
37. (a)


(b) $\bar{x} = 6.13$, $s = 2.02$
39. (a) (b) $\bar{x} = 8.82$


41. The mean will give the highest measure.
43. The median will give the most representative average.
45. (a) 70.09% (b) 8.79%
47.



49. $\bar{x} = 3.32$ kg; $s = 0.677$ kg
51. (a) \$60,000 (b) \$36,000 (c) \$32,000
53. (a) \$23.325 (b) \$5.139
55. (a) 5.47% (b) 6.41% (c) 1.55%
57. (a)



(b) 46.3 (c) 26.0 (d) no

8.3 EXERCISES

1. no; $\Pr(x) \ngeq 0$ **3.** yes; both conditions satisfied
5. yes; both conditions satisfied
7. no; $\sum \Pr(x) > 1$ **9.** $\frac{15}{8}$ **11.** 5
13. $\mu = \frac{13}{8}, \sigma^2 = 1.48, \sigma = 1.22$
15. $\mu = \frac{13}{3}, \sigma^2 = 2.22, \sigma = 1.49$
17. 3 **19.** 2
21. (a)

x	$\Pr(x)$
0	125/216
1	25/72
2	5/72
3	1/216

(b) $3(\frac{1}{6}) = \frac{1}{2}$ (c) $\sqrt{3(\frac{1}{6})(\frac{5}{6})} = (\frac{1}{6})\sqrt{15}$
23. (a) 42 (b) 3.55
25. (a) 30 (b) 3.464
27. 125
29. $a^6 + 6a^5b + 15a^4b^2 + 20a^3b^3 + 15a^2b^4 + 6ab^5 + b^6$
31. $x^4 + 4x^3h + 6x^2h^2 + 4xh^3 + h^4$
33. 1.85
35. TV: 37,500; personal appearances: 35,300
37. −\$67.33 **39.** Expect to lose \$2 each time.
41. 100
43. E(cost with policy) = \$108,
E(cost without policy) = \$80;
save \$28 by "taking the chance"

45. no; some pipes may be more than 0.01 in. from 2 in. even if average is 2 in.
47. (a) $100(0.10) = 10$ (b) $\sqrt{100(0.10)(0.90)} = 3$
49. (a) 60,000 (b) $\sqrt{24{,}000} = 154.919$
51. 59,690 **53.** (a) 4 (b) 1.79
55. 2, 1.41 **57.** 300

8.4 EXERCISES

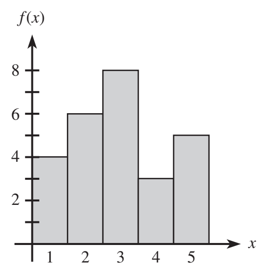
1. 0.4641 **3.** 0.2258 **5.** 0.9153 **7.** 0.1070
9. 0.0166 **11.** 0.0227 **13.** 0.8849 **15.** 0.1915
17. 0.3944 **19.** 0.3830 **21.** 0.7745 **23.** 0.9773
25. 0.0668 **27.** (a) 0.3413 (b) 0.3944
29. 0.9876
31. (a) 0.4192 (b) 0.0227 (c) 0.0581 (d) 0.8965
33. (a) 0.0668 (b) 0.3085 (c) 0.3830
35. (a) 0.0475 (b) 0.2033 (c) 0.5934
37. (a) 0.0227 (b) 0.1587 (c) 0.8186

8.5 EXERCISES

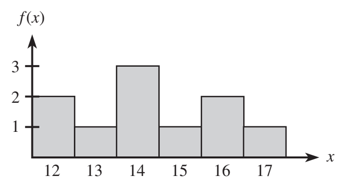
1. yes **3.** no **5.** 0.0668 **7.** 0.0001
9. 0.0521 **11.** 0.0110 **13.** 0.9890
15. 0.7324 **17.** 0.3520 **19.** 0.0443
21. 0.5398 **23.** 0.7852 **25.** 0.0129
27. 0.2514 **29.** 0.0011 **31.** 0.9990
33. 0.1272; 0.4364
35. (a) 0.0038 (b) yes; students were smarter or questions were leaked **37.** 0.0166

CHAPTER 8 REVIEW EXERCISES

1. 0.0774 **2.** (a) 0.3545 (b) 0.5534
3. 0.407
4.

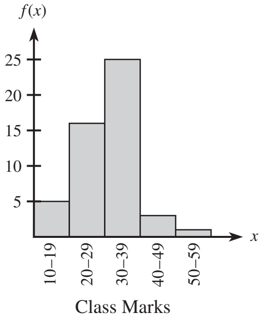


5. 3
6. $\frac{77}{26} \approx 2.96$
7. 3
8.



9. 14
10. 14
11. 14.3
12. $\bar{x} = 3.86; s^2 = 6.81; s = 2.61$
13. $\bar{x} = 2; s^2 = 2.44; s = 1.56$
14. 2.4 **15.** yes **16.** no; $\sum \Pr(x) \neq 1$
17. yes **18.** no; $\Pr(x) \ngeq 0$ **19.** 2
20. (a) 4.125 (b) 2.7344 (c) 1.654
21. (a) $\frac{37}{12}$ (b) 0.9097 (c) 0.9538
22. $\mu = 4, \sigma = (2\sqrt{3})/3$ **23.** 3

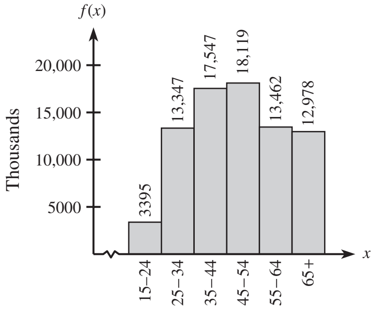
24. $x^5 + 5x^4y + 10x^3y^2 + 10x^2y^3 + 5xy^4 + y^5$
25. 0.9165 **26.** 0.1498 **27.** 0.1039 **28.** 0.3413
29. 0.6826 **30.** 0.1360 **31.** not good **32.** good
33. 0.0151 **34.** 0.9625 **35.** 0.8475 **36.** 0.0119
37. 0.297 **38.** 0.16308 **39.** 0.2048
40. (a) $(99{,}999/100{,}000)^{99{,}999} \approx 0.37$
(b) $1 - (99{,}999/100{,}000)^{100{,}000} \approx 0.63$
41.



42. 30.3%
43. 8.35%
44. 455
45. 3
46. \$18.00
47. –\$0.50
48. (a) 1 (b) $(\frac{4}{5})^4$
49. (a) 0.4773 (b) 0.1360 (c) 0.0227
50. 15% **51.** 0.3821 **52.** 0.4090
53. 0.0262 **54.** 0.1788

CHAPTER 8 TEST

1. (a) $\frac{40}{243}$ (b) $\frac{51}{243} = \frac{17}{81}$
2. (a) 4 (b) $\mu = 4, \sigma^2 = \frac{8}{3}, \sigma = \frac{2}{3}\sqrt{6}$
3. (i) For each x, $0 \le \Pr(x) \le 1$ (ii) $\Sigma\Pr(x) = 1$
4. 5.1
5. $\mu = 16.7, \sigma^2 = 26.61, \sigma = 5.16$
6. $\bar{x} = 21.57$, median $= 21$, mode $= 21$
7. (a) 0.4706 (b) 0.8413 (c) 0.0669
8. (a) 0.3891 (b) 0.5418 (c) 0.1210
9. 0.9554 **10.** 0.6331
11.



12. $\bar{x} = 48.3, s = 15.6$
13. (a) 38.5 (b) under 30; it would be lower
14. (a) 73.8 (b) It might be higher; cell use is spreading.
15. (a) 0.00003 (b) 30
16. 2 (1.8) **17.** 5 (5.4)
18. 0 (0.054) with correct use
19. (a) 0.0158 (b) 0.0901 (c) 0.5383
20. 0.1814

9.1 EXERCISES

1. (a) -8 (b) -8
3. (a) 10 (b) does not exist
5. (a) 0 (b) -6
7. (a) does not exist $(+\infty)$ (b) does not exist $(+\infty)$
(c) does not exist $(+\infty)$ (d) does not exist
9. (a) 3 (b) -6 (c) does not exist
(d) -6
11.

x	$f(x)$
0.9	−2.9
0.99	−2.99
0.999	−2.999
1.001	−3.001
1.01	−3.01
1.1	−3.1

$\lim_{x\to 1} f(x) = -3$

13.

x	$f(x)$
0.9	3.5
0.99	3.95
0.999	3.995
1.001	4.995999
1.01	4.9599
1.1	4.59

$\lim_{x\to 1^-} f(x) = 4$ and $\lim_{x\to 1^+} f(x) = 5$. These limits differ so $\lim_{x\to 1} f(x)$ does not exist

15. -1 **17.** -4 **19.** -2 **21.** 6
23. 3/4 **25.** 3/2 **27.** 0 **29.** does not exist
31. -3 **33.** does not exist **35.** does not exist
37. $3x^2$ **39.** $\frac{1}{30}$ **41.** does not exist
43. -4 **45.** 9
47.

a	$(1 + a)^{1/a}$
0.1	2.5937
0.01	2.7048
0.001	2.7169
0.0001	2.7181
0.00001	2.71827
↓	↓
0	≈ 2.718

49. (a) 2 (b) 6 (c) -8 (d) $-\frac{1}{2}$
51. (a) -6 (b) -85 (c) $-33/17$
53. \$150,000
55. (a) \$32 (thousands) (b) \$55.04 (thousands)
57. (a) \$2800 (b) \$700 (c) \$560
59. (a) 1.52 units/hr (b) 0.85 units/hr (c) lunch
61. (a) 0; $p \to 100^-$ means the water approaches not being treated (containing 100% or all of its impurities); the associated costs of nontreatment approach zero.
(b) ∞ (c) no, because $C(0)$ is undefined
63. (a) \$4,681.25 (b) \$4,681.25 (c) \$4,681.25

65. $C(x) = \begin{cases} 12.76 + 15.96x & 0 \le x \le 10 \\ 172.36 + 13.56(x - 10) & 10 < x \le 120 \\ 1675 & x > 120 \end{cases}$

$\lim_{x\to 10} C(x) = 172.36$

67. 11,228.00. This corresponds to the Dow Jones opening average.

69. (a) 11.5
(b) This predicts the percent of U.S. workers in unions as the year approaches 2015.
(c) Yes. Union membership seems to be dropping, but slowly since 2000.

9.2 EXERCISES

1. (a) continuous
(b) discontinuous; $f(1)$ does not exist
(c) discontinuous; $\lim_{x\to 3} f(x)$ does not exist
(d) discontinuous; $f(0)$ does not exist and $\lim_{x\to 0} f(x)$ does not exist

3. continuous

5. discontinuous; $f(-3)$ does not exist

7. discontinuous; $\lim_{x\to 2} f(x)$ does not exist

9. continuous

11. discontinuity at $x = -2$; $g(-2)$ and $\lim_{x\to -2} g(x)$ do not exist

13. continuous **15.** continuous

17. discontinuity at $x = -1$; $f(-1)$ does not exist

19. discontinuity at $x = 3$; $\lim_{x\to 3} f(x)$ does not exist

21. vertical asymptote: $x = -2$;
$\lim_{x\to +\infty} f(x) = 0$; $\lim_{x\to -\infty} f(x) = 0$; $y = 0$

23. vertical asymptotes: $x = -2$, $x = 3$;
$\lim_{x\to +\infty} f(x) = 2$; $\lim_{x\to -\infty} f(x) = 2$; $y = 2$

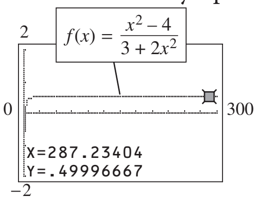
25. (a) 0 (b) $y = 0$ is a horizontal asymptote.

27. (a) 1 (b) $y = 1$ is a horizontal asymptote.

29. (a) 5/3 (b) $y = 5/3$ is a horizontal asymptote.

31. (a) does not exist $(+\infty)$
(b) no horizontal asymptotes

33. (a)



$\lim_{x\to +\infty} f(x) = 0.5$

(b) The table indicates $\lim_{x\to +\infty} f(x) = 0.5$.

35. (a) $x = -1000$ (b) 1000
(c) These values are so large that experimenting with windows may never locate them.

37. $$\lim_{x\to\infty} \frac{a_n + \dfrac{a_{n-1}}{x} + \cdots + \dfrac{a_1}{x^{n-1}} + \dfrac{a_0}{x^n}}{b_n + \dfrac{b_{n-1}}{x} + \cdots + \dfrac{b_1}{x^{n-1}} + \dfrac{b_0}{x^n}} = \frac{a_n + 0 + \cdots + 0 + 0}{b_n + 0 + \cdots + 0 + 0} = \frac{a_n}{b_n}$$

39. (a) no, not at $p = -8$ (b) yes (c) yes
(d) $p > 0$

41. (a) yes, $q = -1$ (b) yes

43. (a) R/i (b) \$10,000 **45.** yes, $0 \le p \le 100$

47. 100%; No, for p to approach 100% (as a limit) requires spending to increase without bound, which is impossible.

49. $R(x)$ is discontinuous at $x = 16{,}750$; $x = 68{,}000$; $x = 137{,}300$; $x = 209{,}250$; and $x = 373{,}650$.

51. (a) \$79.40
(b) $\lim_{x\to 100} C(x) = 19.40$; $\lim_{x\to 500} C(x) = 49.40$
(c) yes

53. (a) $m(x) = 0.59x + 43.20$; $w(x) = 0.79x + 20.86$
(b) $r(x) = \dfrac{0.59x + 43.20}{0.79x + 20.86}$
(c) $\lim_{x\to 0} r(x) \approx 2.07$ means that in 1950 there were about 2.07 men per woman in the U.S. work force. $\lim_{x\to 100} r(x) \approx 1.02$ means that in 2050 it is projected that there will be 1.02 men per woman in the U.S. work force.
(d) $\lim_{x\to\infty} r(x) \approx 0.75 = 3/4$ means that the long term projection is for 3 men per 4 women in the U.S. work force.

9.3 EXERCISES

1. (a) 6 (b) 8 **3.** (a) $\frac{10}{3}$ (b) -5

5. (a) $-3.9, -3.99$ (b) $-4.1, -4.01$ (c) -4

7. (a) 32 (b) 32 (c) (4, 64)

9. (a) verification (b) -8 (c) -8
(d) $(-1, 5)$

11. (a) $P(1, 1)$, $A(3, 0)$ (b) $-\frac{1}{2}$ (c) $-\frac{1}{2}$ (d) $-\frac{1}{2}$

13. (a) $P(1, 3)$, $A(0, 3)$ (b) 0 (c) 0 (d) 0

15. (a) $f'(x) = 10x + 6$ (b) $10x + 6$; -14
(c) -14

17. (a) $p'(q) = 4q + 1$ (b) $4q + 1$; 41 (c) 41

19. (a) 89.000024 (b) 89.0072 (c) ≈ 89

21. (a) 294.000008 (b) 294.0084 (c) ≈ 294

23. -31

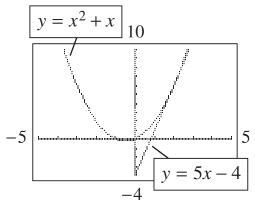
25. (a) At A the slope is positive; at B it is negative.
(b) $-1/3$

27. $f'(4) = 7/3$; $f(4) = -11$ **29.** $y = 5x - 14$

31. (a) a, b, d (b) c (c) A, C, E

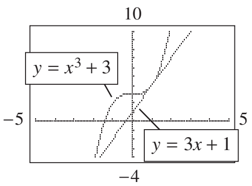
33. (a) A, B, C, D (b) A, D

35. (a) $f'(x) = 2x + 1$ (b) $f'(2) = 5$
(c) $y = 5x - 4$ (d)



37. (a) $f'(x) = 3x^2$ (b) $f'(1) = 3$
(c) $y = 3x + 1$

(d)



39. (a) 43 dollars per unit (b) 95.50 dollars per unit
(c) The average cost per printer when 100 to 300 are produced (a) is \$43 per printer, and the average cost when 300 to 600 are produced (b) is \$95.50 per printer.

41. (a) $-100/3$ (b) $-4/3$

43. AB, AC, BC. Average rate is found from the slope of a segment; AB rises most slowly; BC is steepest.

45. (a) $R'(x) = \overline{MR} = 300 - 2x$
(b) 200; the predicted change in revenue from selling the 51st unit is about \$200.
(c) -100; the predicted change in revenue from the 201st unit is about -100 dollars.
(d) 0 (e) It changes from increasing to decreasing.

47. 200

49. (a) 100; the expected profit from the sale of the 201st car is \$100.
(b) -100; the expected profit from the sale of the 301st car is a loss of \$100.

51. (a) 1.039
(b) If humidity changes by 1%, the heat index will change by about 1.039°F.

53. (a) Marginal revenue is given by the slope of the tangent line, which is steeper at 300 cell phones. Hence marginal revenue is greater for 300 cell phones.
(b) Marginal revenue predicts the revenue from the next unit sold. Hence, the 301st item brings in more revenue because the marginal revenue for 300 cell phones is greater than for 700.

9.4 EXERCISES

1. $y' = 0$ **3.** $f'(t) = 1$

5. $y' = -8 + 4x = 4x - 8$ **7.** $f'(x) = 12x^3 - 6x^5$

9. $y' = 50x^4 - 9x^2 + 5$ **11.** $w'(z) = 7z^6 - 18z^5$

13. $g'(x) = 24x^{11} - 30x^5 + 36x^3 + 1$

15. (a) 30 (b) 30 **17.** (a) 6 (b) 6

19. $y' = -5x^{-6} - 8x^{-9} = \frac{-5}{x^6} - \frac{8}{x^9}$

21. $y' = 11x^{8/3} - \frac{7}{2}x^{3/4} - \frac{1}{2}x^{-1/2}$
$= 11\sqrt[3]{x^8} - \frac{7}{2}\sqrt[4]{x^3} - \dfrac{1}{2\sqrt{x}}$

23. $f'(x) = -4x^{-9/5} - \frac{8}{3}x^{-7/3}$
$= \dfrac{-4}{\sqrt[5]{x^9}} - \dfrac{8}{3\sqrt[3]{x^7}}$

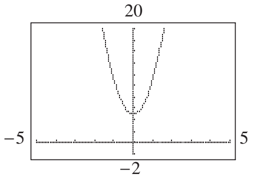
25. $g'(x) = \dfrac{-15}{x^6} - \dfrac{8}{x^5} + \dfrac{2}{\sqrt[3]{x^2}}$

27. $y = -7x + 10$ **29.** $y = 3$

31. $(1, -1)$, $(5, 31)$ **33.** $(0, 9)$, $(3, -18)$

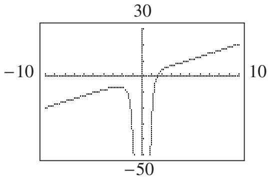
35. (a) $-1/2$ (b) -0.5000 (to four decimal places)

37. (a) $f'(x) = 6x^2 + 5$
(b)



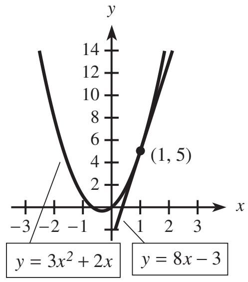
Graph of $f'(x)$ and numerical derivative of $f(x)$

39. (a) $h'(x) = -30x^{-4} + 4x^{-7/5} + 2x$
$= \dfrac{-30}{x^4} + \dfrac{4}{\sqrt[5]{x^7}} + 2x$
(b)



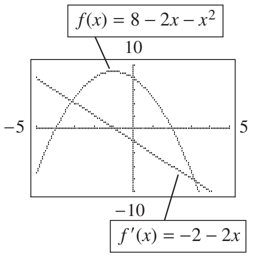
Graph of $h'(x)$ and numerical derivative of $h(x)$

41. (a) $y = 8x - 3$
(b)



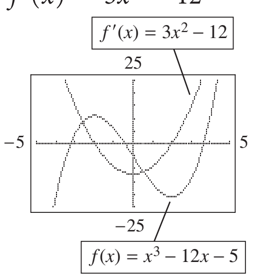
(c) x: $0.7 \to 1.6$; y: $3.0 \to 7.9$

43. (a) $f'(x) = -2 - 2x$
(b)



(c) $f'(x) = 0$ at $x = -1$; $f'(x) > 0$ for $x < -1$; $f'(x) < 0$ for $x > -1$
(d) $f(x)$ has a maximum when $x = -1$.
$f(x)$ rises for $x < -1$.
$f(x)$ falls for $x > -1$.

45. (a) $f'(x) = 3x^2 - 12$
(b)



(c) $f'(x) = 0$ at $x = -2$ and $x = 2$
$f'(x) > 0$ for $x < -2$ and $x > 2$
$f'(x) < 0$ for $-2 < x < 2$

(d) $f(x)$ has a maximum when $x = -2$, a minimum when $x = 2$
$f(x)$ rises when $x < -2$ and when $x > 2$
$f(x)$ falls when $-2 < x < 2$

47. (a) 40; the expected change in revenue from the 301st unit is about \$40
(b) -20; the expected change in revenue from the 601st unit is about -20 dollars

49. (a) 920 (b) 926

51. (a) -4; if the price changes to \$26, the quantity demanded will change by approximately -4 units
(b) $-\frac{1}{2}$; if the price changes to \$101, the quantity demanded will change by approximately $-\frac{1}{2}$ unit

53. (a) $\overline{C}'(x) = (-4000/x^2) + 0.1$ (b) 200
(c) $C'(200) = \overline{C}(200) = 95$

55. (a) $-120{,}000$
(b) If the impurities change from 1% to 2%, then the expected change in cost is $-120{,}000$ (dollars).

57. (a) $WC = 45.0625 - 29.3375s^{0.16}$
(b) -0.31
(c) At 15° F, if the wind speed changes by $+1$ mph (to 26 mph), then the wind chill will change by approximately -0.31°F.

59. (a) $S(x) = 0.105x^{2.53}$
(b) 20.113 million subscriberships per year
(c) $S'(x) = 0.266x^{1.53}$
$S'(23) \approx 32.19$ means that for the next year (2009), the number of subscriberships will change by about 32.19 million.

61. (a) $P(t) = -0.0000729t^3 + 0.0138t^2 + 1.98t + 183$
(b) $P'(t) = -0.0002187t^2 + 0.0276t + 1.98$
(c) 2000: $P'(40) \approx 2.73$ means that for 2001, the U.S. population will rise by about 2.73 million people.
2025: $P'(65) \approx 2.85$ means that for 2026, the U.S. population is expected to rise by about 2.85 million people.

9.5 EXERCISES

1. $y' = 15x^2 - 14x - 6$

3. $f'(x) = (x^{12} + 3x^4 + 4)(6x^2) + (2x^3 - 1)\cdot(12x^{11} + 12x^3) = 30x^{14} - 12x^{11} + 42x^6 - 12x^3 + 24x^2$

5. $y' = (7x^6 - 5x^4 + 2x^2 - 1)(36x^8 + 21x^6 - 10x + 3) + (4x^9 + 3x^7 - 5x^2 + 3x)(42x^5 - 20x^3 + 4x)$

7. $y' = (x^2 + x + 1)(\frac{1}{3}x^{-2/3} - x^{-1/2}) + (x^{1/3} - 2x^{1/2} + 5)(2x + 1)$

9. (a) 40 (b) 40

11. $\dfrac{dp}{dq} = \dfrac{2q^2 - 2q - 6}{(2q - 1)^2}$

13. $\dfrac{dy}{dx} = \dfrac{4x^5 - 4x^3 - 16x}{(x^4 - 2x^2 + 5)^2}$

15. $\dfrac{dz}{dx} = 2x + \dfrac{2x - x^2}{(1 - x - 2x^2)^2}$

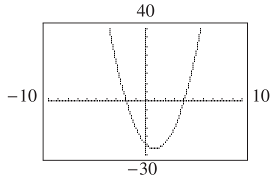
17. $\dfrac{dp}{dq} = \dfrac{2q + 1}{\sqrt[3]{q^2}(1 - q)^2}$ **19.** $y' = \dfrac{2x^3 - 6x^2 - 8}{(x - 2)^2}$

21. (a) $\frac{3}{5}$ (b) $\frac{3}{5}$ **23.** $y = 44x - 32$

25. $y = \frac{10}{3}x - \frac{10}{3}$ **27.** 104

29. 1.3333 (to four decimal places)

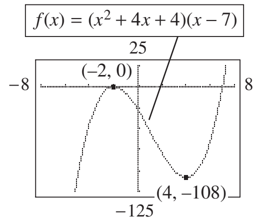
31. (a) $f'(x) = 3x^2 - 6x - 24$



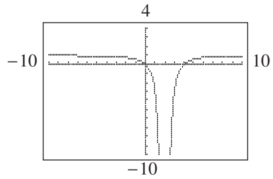
Graph of both $f'(x)$ and numerical derivative of $f(x)$

(b) Horizontal tangents where $f'(x) = 0$; at $x = -2$ and $x = 4$

(c)



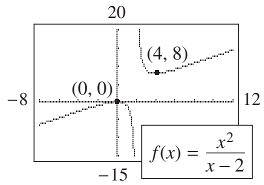
33. (a) $y' = \dfrac{x^2 - 4x}{(x - 2)^2}$



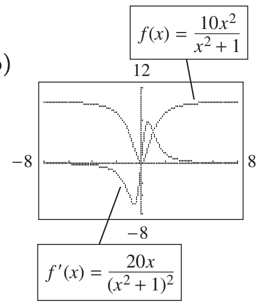
Graph of both y' and the numerical derivative of y

(b) Horizontal tangents where $y' = 0$; at $x = 0$ and $x = 4$

(c)



35. (a) $f'(x) = \dfrac{20x}{(x^2 + 1)^2}$ (b)



(c) $f' = 0$ at $x = 0$
$f' > 0$ for $x > 0$
$f' < 0$ for $x < 0$
(d) f has a minimum at $x = 0$.
f is increasing for $x > 0$.
f is decreasing for $x < 0$.

37. $$f'(x) = \lim_{h\to 0} \frac{\dfrac{u(x+h)}{v(x+h)} - \dfrac{u(x)}{v(x)}}{h}$$
$$= \lim_{h\to 0} \frac{u(x+h)v(x) - u(x)v(x+h)}{h \cdot v(x)v(x+h)}$$
$$= \lim_{h\to 0} \frac{u(x+h)v(x) - u(x)v(x) + u(x)v(x) - u(x)v(x+h)}{h \cdot v(x)v(x+h)}$$
$$= \lim_{h\to 0} \frac{v(x)\left[\dfrac{u(x+h) - u(x)}{h}\right] - u(x)\left[\dfrac{v(x+h) - v(x)}{h}\right]}{v(x)v(x+h)}$$
$$= \frac{v(x)u'(x) - u(x)v'(x)}{[v(x)]^2}$$

39. $C'(p) = 810{,}000/(100 - p)^2$

41. $R'(49) \approx 30.00$ The expected revenue from the sale of the next unit (the 50th) is about \$30.00.

43. $R'(5) = -50$ As the group changes by 1 person (to 31), the revenue will drop by about \$50.

45. $S = 1000x - x^2$

47. $\dfrac{dR}{dn} = \dfrac{r(1-r)}{[1 + (n-1)r]^2}$

49. (a) $P'(6) \approx 0.045$ During the next (7th) month of the campaign, the proportion of voters who recognize the candidate will change by about 0.045, or 4.5%.

(b) $P'(12) \approx -0.010$ During the next (13th) month of the campaign, the proportion of voters who recognize the candidate will drop by about 0.010, or 1%.

(c) It is better for $P'(t)$ to be positive—that is, to have increasing recognition.

51. (a) $f'(20) \approx -0.79$

(b) At 0°F, if the wind speed changes by 1 mph (to 21 mph), the wind chill will change by about -0.79°F.

53. (a) $B'(t) = (0.01t + 3)(0.04766t - 9.79) + (0.01)(0.02383t^2 - 9.79t + 3097.19)$

(b) $B'(60) = 1.00634$ means that in 2010 the number of beneficiaries will be changing at the rate of 1.00634 million per year.

(c) [2000, 2010]: 0.85
[2010, 2020]: 1.55
[2000, 2020]: 1.2
The average rate over [2000, 2010] is best but is still off by almost 0.15 million per year.

55. (a) $p'(t) = \dfrac{2154.06}{(1.38t + 64.1)^2}$

(b) 2005: $p'(55) \approx 0.110$; 2020: $p'(70) \approx 0.083$

(c) $p'(55)$ means that in 2005, the percent of women in the work force was changing about 0.110 percentage points per year. $p'(70)$ predicts the rate in 2020 will be 0.083 percentage points per year.

9.6 EXERCISES

1. $\dfrac{dy}{du} = 3u^2,\ \dfrac{du}{dx} = 2x,\ \dfrac{dy}{dx} = 3u^2 \cdot 2x = 6x(x^2+1)^2$

3. $\dfrac{dy}{du} = 4u^3,\ \dfrac{du}{dx} = 8x - 1,\ \dfrac{dy}{dx} = 4u^3(8x-1)$
$= 4(8x-1)(4x^2 - x + 8)^3$

5. $f'(x) = 20(3x^5 - 2)^{19}(15x^4) = 300x^4(3x^5-2)^{19}$

7. $h'(x) = 6(x^5 - 2x^3 + 5)^7(5x^4 - 6x^2)$
$= 6x^2(5x^2 - 6)(x^5 - 2x^3 + 5)^7$

9. $s'(t) = 5 - 9(2t^4 + 7)^2(8t^3) = 5 - 72t^3(2t^4+7)^2$

11. $g'(x) = -2(x^4 - 5x)^{-3}(4x^3 - 5) = \dfrac{-2(4x^3-5)}{(x^4-5x)^3}$

13. $f'(x) = -12(2x^5+1)^{-5}(10x^4) = \dfrac{-120x^4}{(2x^5+1)^5}$

15. $g'(x) = -\dfrac{3}{4}(2x^3 + 3x + 5)^{-7/4}(6x^2+3)$
$= \dfrac{-3(6x^2+3)}{4(2x^3+3x+5)^{7/4}}$

17. $y' = \dfrac{1}{2}(3x^2 + 4x + 9)^{-1/2}(6x+4)$
$= \dfrac{3x+2}{\sqrt{3x^2+4x+9}}$

19. $y' = \dfrac{66}{9}(x^3-7)^5(3x^2) = 22x^2(x^3-7)^5$

21. $y' = \dfrac{15(3x+1)^4 - 3}{7}$

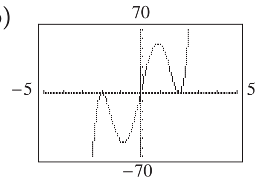
23. (a) and (b) 96,768

25. (a) and (b) 2 **27.** $y = 3x - 5$

29. $9x - 5y = 2$

31. (a) $f'(x) = 6x(x^2-4)^2$

(b)

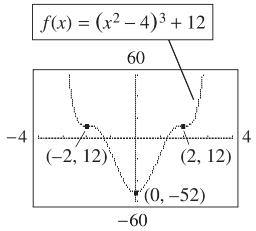


(c) $x = 0, x = 2, x = -2$

(d) $(0, -52)$, $(2, 12)$, $(-2, 12)$

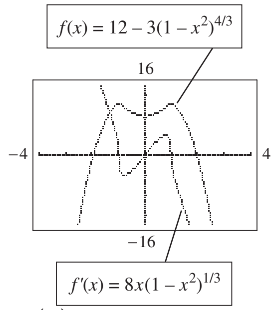
Graph of both $f'(x)$ and numerical derivative of $f(x)$

(e)



33. (a) $f'(x) = 8x(1-x^2)^{1/3}$

(b)



(c) $f'(x) = 0$ at $x = -1, x = 0, x = 1$
$f'(x) > 0$ for $x < -1$ and $0 < x < 1$
$f'(x) < 0$ for $-1 < x < 0$ and $x > 1$

(d) $f(x)$ has a maximum at $x = -1$ and $x = 1$, a minimum at $x = 0$.
$f(x)$ is increasing for $x < -1$ and $0 < x < 1$.
$f(x)$ is decreasing for $-1 < x < 0$ and $x > 1$.

35. (a) $y' = 2x^2$ (b) $y' = -2/x^4$

(c) $y' = 2(2x)^2$ (d) $y' = \dfrac{-18}{(3x)^4}$

37. 120 in./sec

39. \$1499.85 (approximately); if a 101st unit is sold, revenue will change by about \$1499.85

41. (a) -0.114 (approximately)
(b) If the price changes by \$1, to \$22, the weekly sales volume will change by approximately -0.114 thousand unit.

43. (a) $-\$3.20$ per unit
(b) If the quantity demanded changes from 49 to 50 units, the change in price will be about $-\$3.20$.

45. $\frac{dy}{dx} = \left(\frac{8k}{5}\right)(x - x_0)^{3/5}$

47. $\frac{dp}{dq} = -100(2q + 1)^{-3/2} = \frac{-100}{(2q + 1)^{3/2}}$

49. $\frac{dK_c}{dv} = 8(4v + 1)^{-1/2} = \frac{8}{\sqrt{4v + 1}}$

51. (a) \$658.75. If the interest changed from 6% to 7%, the amount of the investment would change by about \$658.75.
(b) \$2156.94. If the interest rate changed from 12% to 13%, the amount of the investment would change by about \$2156.94.

53. (a) 2008: $A'(8) \approx 126.3$; 2015: $A'(15) \approx 231.4$
These mean that the total national expenditures for health are predicted to change by about \$126.3 billion from 2008 to 2009 and about \$231.4 billion from 2015 to 2016.
(b) The average rate for 2014 to 2015 is best: \$228 billion/year.

55. (a) 2005: $G'(5) \approx 1370.64$; 2015: $G'(15) \approx 934.56$
These mean that the GDP was changing at the rate of \$1370.64 billion per year in 2005 and \$934.5 billion per year in 2015.
(b) 912.5 (billion per year)
(c) 2010: $G'(10) \approx 1024.86$. The answer from part (b) is not a good approximation to $G'(10)$.

9.7 EXERCISES

1. 0 **3.** $4(-4x^{-5})$; $-16/x^5$

5. $15x^2 + 4(-x^{-2})$; $15x^2 - 4/x^2$

7. $(x^2 - 2)1 + (x + 4)(2x)$; $3x^2 + 8x - 2$

9. $\frac{x^2(3x^2) - (x^3 + 1)(2x)}{(x^2)^2}$; $(x^3 - 2)/x^3$

11. $(3x^2 - 4)(x^3 - 4x)^9$

13. $\frac{5}{3}x^3[3(4x^5 - 5)^2(20x^4)] + (4x^5 - 5)^3(5x^2)$;
$5x^2(4x^5 - 5)^2(24x^5 - 5)$

15. $(x - 1)^2(2x) + (x^2 + 1)2(x - 1)$;
$2(x - 1)(2x^2 - x + 1)$

17. $\frac{(x^2 + 1)3(x^2 - 4)^2(2x) - (x^2 - 4)^3(2x)}{(x^2 + 1)^2}$;
$\frac{2x(x^2 - 4)^2(2x^2 + 7)}{(x^2 + 1)^2}$

19. $3[(q + 1)(q^3 - 3)]^2[(q + 1)3q^2 + (q^3 - 3)1]$;
$3(4q^3 + 3q^2 - 3)[(q + 1)(q^3 - 3)]^2$

21. $4[x^2(x^2 + 3x)]^3[x^2(2x + 3) + (x^2 + 3x)(2x)]$;
$4x^2(4x + 9)[x^2(x^2 + 3x)]^3 = 4x^{11}(4x + 9)(x + 3)^3$

23. $4\left(\frac{2x - 1}{x^2 + x}\right)^3\left[\frac{(x^2 + x)2 - (2x - 1)(2x + 1)}{(x^2 + x)^2}\right]$;
$\frac{4(-2x^2 + 2x + 1)(2x - 1)^3}{(x^2 + x)^5}$

25. $(8x^4 + 3)^2 3(x^3 - 4x)^2(3x^2 - 4) +$
$(x^3 - 4x)^3 2(8x^4 + 3)(32x^3)$;
$(8x^4 + 3)(x^3 - 4x)^2(136x^6 - 352x^4 + 27x^2 - 36)$

27. $\frac{(4 - x^2)\frac{1}{3}(x^2 + 5)^{-2/3}(2x) - (x^2 + 5)^{1/3}(-2x)}{(4 - x^2)^2}$;
$\frac{2x(2x^2 + 19)}{3\sqrt[3]{(x^2 + 5)^2}(4 - x^2)^2}$

29. $(x^2)\frac{1}{4}(4x - 3)^{-3/4}(4) + (4x - 3)^{1/4}(2x)$;
$(9x^2 - 6x)/\sqrt[4]{(4x - 3)^3}$

31. $(2x)\frac{1}{2}(x^3 + 1)^{-1/2}(3x^2) + (x^3 - 1)^{1/2}(2)$;
$(5x^3 + 2)/\sqrt{x^3 + 1}$

33. (a) $F_1'(x) = 12x^3(x^4 + 1)^4$
(b) $F_2'(x) = \frac{-12x^3}{(x^4 + 1)^6}$
(c) $F_3'(x) = 12x^3(3x^4 + 1)^4$
(d) $F_4'(x) = \frac{-300x^3}{(5x^4 + 1)^6}$

35. $dP/dx = 90(3x + 1)^2$

37. (a) \$59,900
(b) An 11th camper sold would change revenue by about \$59,900.

39. $dC/dy = 1/\sqrt{y + 1} + 0.4$

41. $dV/dx = 144 - 96x + 12x^2$

43. -1.6; This means that from the 9th to the 10th week, sales are expected to change by -1600 dollars (decrease).

45. (a) 2005: \$350/year; 2015: \$549/year
(b) In 2015, the per capita expenditures for health care are predicted to be changing at the rate of \$549 per year.
(c) Average rate = 380; This approximates the instantaneous rate in 2005 quite well.

9.8 EXERCISES

1. $180x^8 - 360x^3 - 72x$ **3.** $6x - 2x^{-3}$

5. $6x + \frac{1}{4}x^{-3/2}$ **7.** $60x^2 - 96$ **9.** $1008x^6 - 720x^3$

11. $-6/x^4$ **13.** $20x^3 + \frac{1}{4}x^{-3/2}$ **15.** $\frac{3}{8}(x + 1)^{-5/2}$

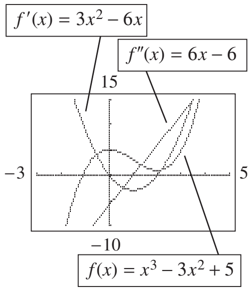
17. 0 **19.** $-15/(16x^{7/2})$ **21.** $24(4x - 1)^{-5/2}$

23. $-2(x + 1)^{-3}$ **25.** 26

27. 16.0000 (to four decimal places)

29. 0.0004261

31. (a) $f'(x) = 3x^2 - 6x$ $f''(x) = 6x - 6$
(b)



(c) $f''(x) = 0$ at $x = 1$
$f''(x) > 0$ for $x > 1$
$f''(x) < 0$ for $x < 1$

(d) $f'(x)$ has a minimum at $x = 1$.
$f'(x)$ is increasing for $x > 1$.
$f'(x)$ is decreasing for $x < 1$.
(e) $f''(x) < 0$ (f) $f''(x) > 0$.

33. $a = 0.12 \text{ m/sec}^2$ **35.** -0.02 \$/unit per unit

37. (a) $\frac{dR}{dm} = mc - m^2$ (b) $\frac{d^2R}{dm^2} = c - 2m$
(c) second

39. (a) 0.0009 (approximately)
(b) When 1 more unit is sold (beyond 25), the marginal revenue will change by about 0.0009 thousand dollars per unit, or \$0.90 per unit.

41. (a) $S' = \frac{-3}{(t+3)^2} + \frac{36}{(t+3)^3}$ (b) $S''(15) = 0$
(c) After 15 weeks, the rate of change of the rate of sales is zero because the rate of sales reaches a minimum value.

43. (a) $y' = 1.175x^{-0.06}$ (b) $y'' = -0.0705x^{-1.06}$
(c) $y'(18{,}000) \approx 0.65$; $y''(18{,}000) \approx 0$
These mean that when the total Starbucks stores number 18,000, the number of U.S. stores is expected to be changing at the rate of 0.65 U.S. stores per total store (or 65 U.S. per 100 total), and this rate is expected to be constant there.

45. (a) $R(x) = -0.0002x^3 + 0.052x^2 - 4.06x + 192$
(b) $R'(x) = -0.0006x^2 + 0.104x - 4.06$
(c) $R''(x) = -0.0012x + 0.104$
(d) $R'(90) \approx 0.44$; $R''(90) \approx -0.004$
In 2040, the economic dependency ratio is expected to be changing at the rate of 0.44 per year, but this rate is expected to be changing at the rate of –0.004 per year per year.

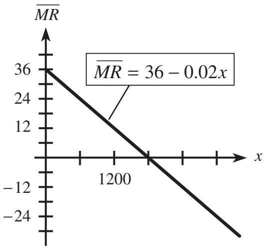
9.9 EXERCISES

1. (a) $\overline{MR} = 4$
(b) The sale of each additional item brings in \$4 revenue at all levels of production.

3. (a) \$3500; this is revenue from the sale of 100 units.
(b) $\overline{MR} = 36 - 0.02x$
(c) \$34; Revenue will increase by about \$34 if a 101st item is sold and by about \$102 if 3 additional units past 100 units are sold.
(d) Actual revenue from the sale of the 101st item is \$33.99.

5. (a) $R(x) = 80x - 0.4x^2$ (in hundreds of dollars)
(b) 7500 subscribers $(x = 75)$; $R = \$375{,}000$
(c) Lower the price per month.
(d) $\overline{MR} = R'(x) = 80 - 0.8x$; when $p = 50$, $x = 75$
$\overline{MR}(75) = 20$ means that if the number of customers increased from 75 to 76 (hundred), the revenue would increase by about 20 (hundred dollars), or \$2000. This means the company should try to increase subscribers by lowering its monthly charge.

7. (a)



(b) $x = 1800$
(c) \$32,400

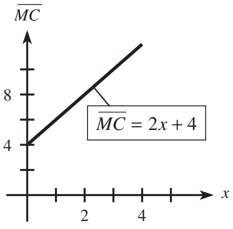
9. $\overline{MC} = 8$ **11.** $\overline{MC} = 13 + 2x$

13. $\overline{MC} = 3x^2 - 12x + 24$ **15.** $\overline{MC} = 27 + 3x^2$

17. (a) \$10; the cost will increase by about \$10.
(b) \$11

19. \$46; the cost will increase by about \$46. For 3 additional units, the cost will increase by about \$138.

21.



23. (a) The 101st item costs more. The tangent line slope is greater at $x = 100$ than at $x = 500$, and the slope of the tangent line gives the marginal cost and predicts the cost of the next item.
(b) More efficient. As x increases, the slopes of the tangents decrease. This means that the costs of additional items decrease as x increases.

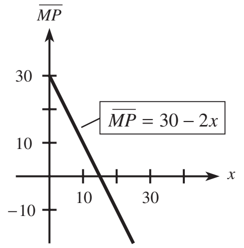
25. $\overline{MP} = 5$; This means that for each additional unit sold, profit changes by \$5.

27. (a) \$5600 (b) $\overline{MP} = 20 - 0.02x$
(c) 10; profit will increase by about \$10 if a 501st unit is sold.
(d) 9.99; the sale of the 501st item results in a profit of \$9.99.

29. (a) $P(x) = R(x) - C(x)$, so profit is the distance between $R(x)$ and $C(x)$ (when $R(x)$ is above $C(x)$). $P(100) < P(700) < P(400)$; $P(100) < 0$, so there is a loss when 100 units are sold.
(b) This asks us to rank $\overline{MP}(100)$, $\overline{MP}(400)$, and $\overline{MP}(700)$. Because $\overline{MP} = \overline{MR} - \overline{MC}$, compare the slopes of the tangents to $R(x)$ and $C(x)$ at the three x-values. Thus $\overline{MP}(700) < \overline{MP}(400) < \overline{MP}(100)$. $\overline{MP}(700) < 0$ because $C(x)$ is steeper than $R(x)$ at $x = 700$. At $x = 100$, $R(x)$ is much steeper than C(x).

31. (a) $A < B < C$. Amount of profit is the height of the graph. There is a loss at A.
(b) $C < B < A$. Marginal profit is the slope of the tangent to the graph. Marginals (slopes) are positive at all three points.

33. (a)

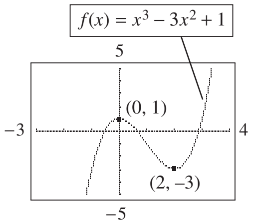


(b) 15 hundred units
(c) 15 hundred units
(d) $25 thousand

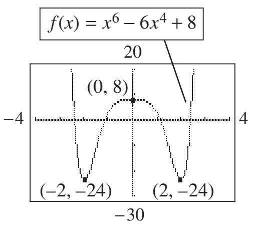
35. 700 **37.** $9000

CHAPTER 9 REVIEW EXERCISES

1. (a) 2 (b) 2 **2.** (a) 0 (b) 0
3. (a) 2 (b) 1 **4.** (a) 2 (b) does not exist
5. (a) does not exist (b) 2
6. (a) does not exist (b) does not exist
7. 55 **8.** 0 **9.** −2 **10.** 4/5 **11.** $\frac{1}{2}$
12. $\frac{1}{5}$ **13.** no limit **14.** 0 **15.** 4
16. no limit **17.** 3 **18.** no limit **19.** $6x$
20. $1 - 4x$ **21.** −14 **22.** 5
23. (a) yes (b) no **24.** (a) yes (b) no
25. 2 **26.** no limit **27.** 1 **28.** no **29.** yes
30. yes **31.** discontinuity at $x = 5$
32. discontinuity at $x = 2$ **33.** continuous
34. discontinuity at $x = 1$
35. (a) $x = 0, x = 1$ (b) 0 (c) 0
36. (a) $x = -1, x = 0$ (b) $\frac{1}{2}$ (c) $\frac{1}{2}$
37. −2; $y = -2$ is a horizontal asymptote.
38. 0; $y = 0$ is a horizontal asymptote.
39. 7 **40.** true **41.** false
42. $f'(x) = 6x + 2$ **43.** $f'(x) = 1 - 2x$
44. $[-1, 0]$; the segment over this interval is steeper.
45. (a) no (b) no **46.** (a) yes (b) no
47. (a) −5.9171 (to four decimal places) (b) −5.9
48. (a) 4.9/3 (b) 7 **49.** about −1/4
50. B, C, A: $B < 0$ and $C < 0$; the tangent line at $x = 6$ falls more steeply than the segment over [2, 10].
51. $20x^4 - 18x^2$ **52.** $90x^8 - 30x^5 + 4$
53. 1 **54.** $1/(2\sqrt{x})$ **55.** 0 **56.** $-4/(3\sqrt[3]{x^4})$
57. $\dfrac{-1}{x^2} + \dfrac{1}{2\sqrt{x^3}}$ **58.** $\dfrac{-3}{x^3} - \dfrac{1}{3\sqrt[3]{x^2}}$
59. $y = 15x - 18$ **60.** $y = 34x - 48$
61. (a) $x = 0, x = 2$ (b) $(0, 1)$ $(2, -3)$
(c)



62. (a) $x = 0, x = 2, x = -2$
(b) $(0, 8)$ $(2, -24)(-2, -24)$
(c)



63. $9x^2 - 26x + 4$ **64.** $21x^6 + 4x^3 + 27x^2$
65. $\dfrac{15q^2}{(2q^3 + 1)^2}$ **66.** $\dfrac{1 - 3t}{[2\sqrt{t}(3t + 1)^2]}$ **67.** $\dfrac{9x + 2}{2\sqrt{x}}$
68. $\dfrac{5x^6 + 2x^4 + 20x^3 - 3x^2 - 4x}{(x^3 + 1)^2}$
69. $(9x^2 - 24x)(x^3 - 4x^2)^2$
70. $6(30x^5 + 24x^3)(5x^6 + 6x^4 + 5)^5$
71. $72x^3(2x^4 - 9)^8$ **72.** $\dfrac{-(3x^2 - 4)}{2\sqrt{(x^3 - 4x)^3}}$
73. $2x(2x^4 + 5)^7(34x^4 + 5)$ **74.** $\dfrac{-2(3x + 1)(x + 12)}{(x^2 - 4)^2}$
75. $36[(3x + 1)(2x^3 - 1)]^{11}(8x^3 + 2x^2 - 1)$
76. $\dfrac{3}{(1 - x)^4}$ **77.** $\dfrac{(2x^2 - 4)}{\sqrt{x^2 - 4}}$ **78.** $\dfrac{2x - 1}{(3x - 1)^{4/3}}$
79. $y'' = \frac{-1}{4}x^{-3/2} - 2$ **80.** $y'' = 12x^2 - 2/x^3$
81. $\dfrac{d^5y}{dx^5} = 0$ **82.** $\dfrac{d^5y}{dx^5} = -30(1 - x)$
83. $\dfrac{d^3y}{dx^3} = -4[(x^2 - 4)^{3/2}]$ **84.** $\dfrac{d^4y}{dx^4} = \dfrac{2x(x^2 - 3)}{(x^2 + 1)^3}$
85. (a) $400,000 (b) $310,000 (c) $90,000
86. (a) $70,000; this is fixed costs.
(b) $0; $x = 1000$ is break-even.
87. (a) $140 per unit
(b) $\overline{C}(x) \to +\infty$; the limit does not exist.
88. (a) $C(x) \to +\infty$; the limit does not exist. As the number of units produced increases without bound, so does the total cost.
(b) $60 per unit. As more units are produced, the average cost of each unit approaches $60.
89. The average annual percent change of (a) elderly men in the work force is 0.29 percentage points per year and of (b) elderly women in the work force is 0.26 percentage points per year.
90. (a) Annual average rate of change of percent of elderly men in the work force:
1950–1960: −1.27 percentage points per year
2000–2008: 0.48 percentage points per year
(b) Annual average rate of change of percent of elderly women in the work force
1950–1960: 0.11 percentage points per year
2000–2008: 0.49 percentage points per year
91. (a) $x'(10) = -1$ means that if price changes from $10 to $11, the number of units demanded will change by about −1.
(b) $x'(20) = -\frac{1}{4}$ means that if price changes from $20 to $21, the number of units demanded will change by about $-\frac{1}{4}$.

92. $h(100) \approx 4.15$; $h'(100) \approx 0.08$ means that when the updraft speed is 100 mph, the hail diameter is about 4.15 inches (softball-sized) and changing at the rate of 0.08 inch per mph of updraft.

93. The slope of the tangent at A gives $\overline{MR}(A)$. The tangent line at A is steeper (so has greater slope) than the tangent line at B. Hence, $\overline{MR}(A) > \overline{MR}(B)$, so the $(A + 1)$st unit will bring more revenue.

94. $R'(10) = 1570$. Raised. An 11th rent increase of \$30 (and hence an 11th vacancy) would change revenue by about \$1570.

95. (a) $P(20) = 23$ means productivity is 23 units per hour after 20 hours of training and experience.
(b) $P'(20) \approx 1.4$ means that the 21st hour of training or experience will change productivity by about 1.4 units per hour.

96. $\dfrac{dq}{dp} = \dfrac{-p}{\sqrt{0.02p^2 + 500}}$

97. $x'(10) = \frac{1}{6}$ means if price changes from \$10 to \$11, the number of units supplied will change by about $\frac{1}{6}$.

98. $s''(t) = a = -2t^{-3/2}$; $s''(4) = -0.25$ ft/sec/sec

99. $P'(x) = 70 - 0.2x$; $P''(x) = -0.2$
$P'(300) = 10$ means that the 301st unit brings in about \$10 in profit.
$P''(300) = -0.2$ means that marginal profit $(P'(x))$ is changing at the rate of -0.2 dollars per unit, per unit.

100. (a) $\overline{MC} = 6x + 6$ (b) 186
(c) If a 31st unit is produced, costs will change by about \$186.

101. $C'(4) = 53$ means that a 5th unit produced would change total costs by about \$53.

102. (a) $\overline{MR} = 40 - 0.04x$ (b) $x = 1000$ units

103. $\overline{MP}(10) = 48$ means that if an 11th unit is sold, profit will change by about \$48.

104. (a) $\overline{MR} = 80 - 0.08x$ (b) 72
(c) If a 101st unit is sold, revenue will change by about \$72.

105. $\dfrac{120x(x + 1)}{(2x + 1)^2}$ **106.** $\overline{MP} = 4500 - 3x^2$

107. $\overline{MP} = 16 - 0.2x$

108. (a) C: Tangent line to $R(x)$ has smallest slope at C, so $\overline{MR}(C)$ is smallest and the next item at C will earn the least revenue.
(b) B: $R(x) > C(x)$ at both B and C. Distance between $R(x)$ and $C(x)$ gives the amount of profit and is greatest at B.
(c) A: $\overline{MR}$ greatest at A and $\overline{MC}$ least at A, as seen from the slopes of the tangents. Hence $\overline{MP}(A)$ is greatest, so the next item at A will give the greatest profit.
(d) C: $\overline{MC}(C) > \overline{MR}(C)$, as seen from the slopes of the tangents. Hence $\overline{MP}(C) < 0$, so the next unit sold reduces profit.

CHAPTER 9 TEST

1. (a) $\frac{3}{4}$ (b) $-8/5$ (c) $9/8$ (d) does not exist

2. (a) $f'(x) = \lim\limits_{h\to 0} \dfrac{f(x + h) - f(x)}{h}$
(b) $f'(x) = 6x - 1$

3. $x = 0, x = 8$

4. (a) $\dfrac{dB}{dW} = 0.523$ (b) $p'(t) = 90t^9 - 42t^6 - 17$
(c) $\dfrac{dy}{dx} = \dfrac{99x^2 - 24x^9}{(2x^7 + 11)^2}$
(d) $f'(x) = (3x^5 - 2x + 3)(40x^9 + 40x^3) + (4x^{10} + 10x^4 - 17)(15x^4 - 2)$
(e) $g'(x) = 9(10x^4 + 21x^2)(2x^5 + 7x^3 - 5)^{11}$
(f) $y' = 2(8x^2 + 5x + 18)(2x + 5)^5$
(g) $f'(x) = \dfrac{6}{\sqrt{x}} + \dfrac{20}{x^3}$

5. $\dfrac{d^3y}{dx^3} = 6 + 60x^{-6}$

6. (a) $y = -15x - 5$ (b) $(4, -90), (-2, 18)$

7. -15 **8.** (a) 2 (b) does not exist (c) -4

9. $g(-2) = 8$; $\lim\limits_{x\to -2^-} g(x) = 8$, $\lim\limits_{x\to -2^+} g(x) = -8$
$\therefore \lim\limits_{x\to -2} g(x)$ does not exist and g(x) is not continuous at $x = -2$.

10. (a) $\overline{MR} = R'(x) = 250 - 0.02x$
(b) $R(72) = 17{,}948.16$ means that when 72 units are sold, revenue is \$17,948.16.
$R'(72) = 248.56$ means that the expected revenue from the 73rd unit is about \$248.56.

11. (a) $P(x) = 50x - 0.01x^2 - 10{,}000$
(b) $\overline{MP} = 50 - 0.02x$
(c) $\overline{MP}(1000) = 30$ means that the predicted profit from the sale of the 1001st unit is approximately \$30.

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13. (a) -5 (b) -1 (c) 4 (d) does not exist
(e) 2 (f) 3/2 (g) $-4, 1, 3, 6$
(h) $-4, 3, 6$
(i) $f'(-2) <$ average rate over $[-2, 2] < f'(2)$

14. (a) 2/3 (b) -4 (c) 2/3

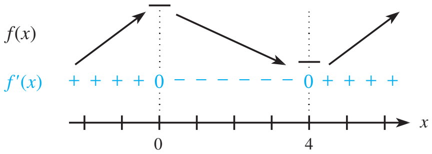
15. (a) B: $R(x) > C(x)$ at B, so there is profit. Distance between $R(x)$ and $C(x)$ gives the amount of profit.
(b) A: $C(x) > R(x)$
(c) A and B: slope of $R(x)$ is greater than the slope of $C(x)$. Hence $\overline{MR} > \overline{MC}$ and $\overline{MP} > 0$.
(d) C: Slope of $C(x)$ is greater than the slope of $R(x)$. Hence $\overline{MC} > \overline{MR}$ and $\overline{MP} < 0$.

10.1 EXERCISES

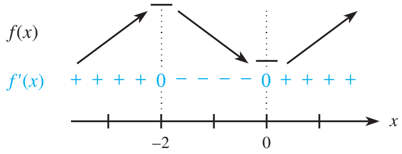
1. (a) $(1, 5)$ (b) $(4, 1)$ (c) $(-1, 2)$

3. (a) $(1, 5)$ (b) $(4, 1)$ (c) $(-1, 2)$

5. (a) 3, 7 (b) $3 < x < 7$ (c) $x < 3, x > 7$
(d) 7 (e) 3

7. (a) $x = 0, x = 4$
(b)


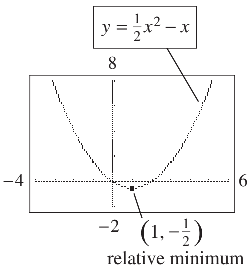
min: $(4, -58)$; max: $(0, 6)$

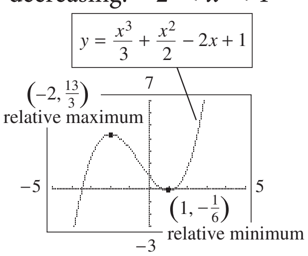
9. (a) $x = -2, x = 0$


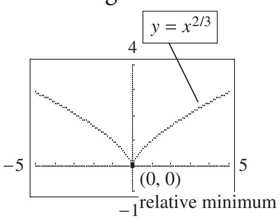
(b) max: $(-2, 5)$; min: $(0, -11)$

11. (a) max: $(-1, 6)$; min: $(1, 2)$
(b) $dy/dx = 3x^2 - 3$; $x = 1, x = -1$
(c) $(1, 2), (-1, 6)$
(d) yes

13. (a) HPI: $(-1, -3)$
(b) $dy/dx = 3x^2 + 6x + 3$; $x = -1$
(c) $(-1, -3)$
(d) yes

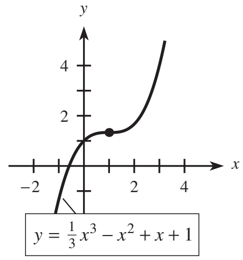
15. (a) $\dfrac{dy}{dx} = x - 1$
(b) $x = 1$
(c) $(1, -\frac{1}{2})$
(d) decreasing: $x < 1$
increasing: $x > 1$
(e)


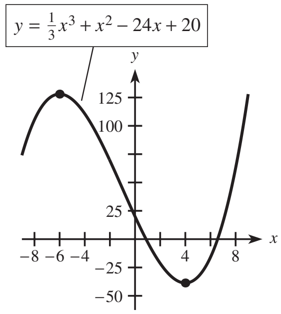
17. (a) $dy/dx = x^2 + x - 2$
(b) $x = -2, x = 1$ (c) $(-2, \frac{13}{3}), (1, -\frac{1}{6})$
(d) increasing: $x < -2$ and $x > 1$
decreasing: $-2 < x < 1$
(e)


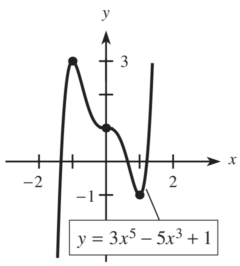
19. (a) $\dfrac{dy}{dx} = \dfrac{2}{3x^{1/3}}$ (b) $x = 0$ (c) $(0, 0)$
(d) decreasing: $x < 0$
increasing: $x > 0$
(e)


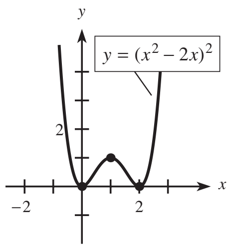
21. (a) $f'(x) = 0$ at $x = -\frac{1}{2}$
$f'(x) > 0$ for $x < -\frac{1}{2}$
$f'(x) < 0$ for $x > -\frac{1}{2}$
(b) $f'(x) = -1 - 2x$ verifies these conclusions.

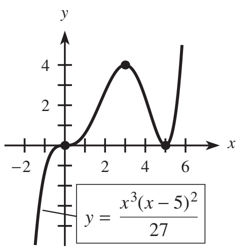
23. (a) $f'(x) = 0$ at $x = 0, x = -3, x = 3$
$f'(x) > 0$ for $-3 < x < 3, x \neq 0$
$f'(x) < 0$ for $x < -3$ and $x > 3$
(b) $f'(x) = \frac{1}{3}x^2(9 - x^2)$ verifies these conclusions.

25. HPI $(1, \frac{4}{3})$
no max or min


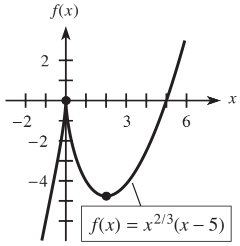
27. $(-6, 128)$ rel max;
$(4, -38\frac{2}{3})$ rel min


29. $(-1, 3)$ rel max;
$(1, -1)$ rel min;
HPI $(0, 1)$


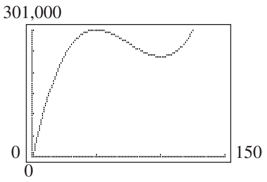
31. $(1, 1)$ rel max;
$(0, 0), (2, 0)$ rel min


33. $(3, 4)$ rel max;
$(5, 0)$ rel min; HPI $(0, 0)$


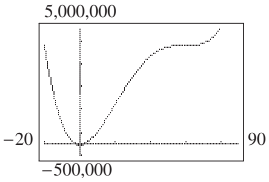
35. (0, 0) rel max;
(2, −4.8) rel min



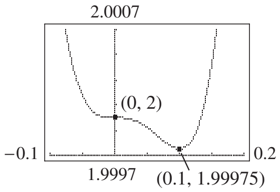
37. (50, 300,500), (100, 238,000)
$0 \le x \le 150$, $0 \le y \le 301{,}000$



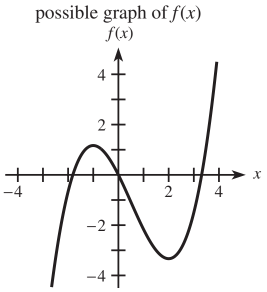
39. (0, −40,000), (60, 4,280,000)
$-20 \le x \le 90$, $-500{,}000 \le y \le 5{,}000{,}000$



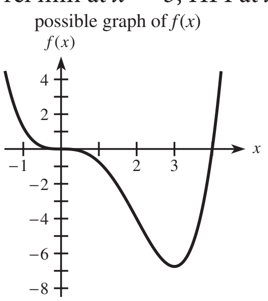
41. (0, 2)
(0.1, 1.99975)
$-0.1 \le x \le 0.2$
$1.9997 \le y \le 2.0007$



43. critical values: $x = -1$, $x = 2$
$f(x)$ increasing for $x < -1$ and $x > 2$
$f(x)$ decreasing for $-1 < x < 2$
rel max at $x = -1$; rel min at $x = 2$



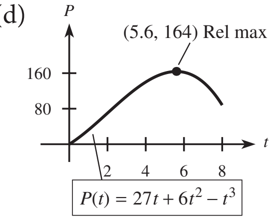
45. critical values: $x = 0$, $x = 3$
$f(x)$ increasing for $x > 3$
$f(x)$ decreasing for $x < 3$, $x \ne 0$
rel min at $x = 3$; HPI at $x = 0$



47. Graph on left is $f(x)$; on right is $f'(x)$ because $f(x)$ is increasing when $f'(x) > 0$ (i.e., above the x-axis) and $f(x)$ is decreasing when $f'(x) < 0$ (i.e., below the x-axis).

49. decreasing for $t \ge 0$

51. (a) $2 \pm \sqrt{13}$
(b) $2 + \sqrt{13} \approx 5.6$
(c) $0 \le t < 2 + \sqrt{13}$
(d)



53. (a) $x = 5$ (b) $0 < x < 5$
(c) increasing for $x > 5$

55. (a) at $x = 150$, increasing; at $x = 250$, changing from increasing to decreasing; at $x = 350$, decreasing
(b) increasing for $x < 250$ (c) 250 units

57. (a) $t = 6$ (b) 6 weeks

59. (a) 10 (b) January 1

61. (a) $x \approx 86.2$ (in 1937). Model achieves its maximum later than the data.
(b) No. For 2010, $R'(160) \approx -3.4$ thousand per year; not approaching 0.

63. (a) $y = 0.000123x^3 - 0.0205x^2 + 0.910x - 2.91$
(b) $x \approx 30.7$; in 1981

65. (a) $y = 0.094t^3 - 25.94t^2 + 2273t - 45{,}828$
(b) $t \approx 72.5$ gives a maximum and $t \approx 110.7$ gives a minimum.
(c) Model's prediction for the year (1973) is fairly close to the data, but its prediction for the thousands of workers is too low.
(d) No, membership is more likely to remain fairly stable.

10.2 EXERCISES

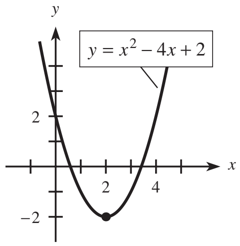
1. (a) concave down (b) concave up

3. (a, c) and (d, e) **5.** (c, d) and (e, f) **7.** c, d, e

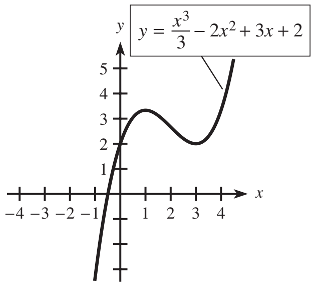
9. concave up when $x > 2$; concave down when $x < 2$; POI at $x = 2$

11. concave up when $x < -2$ and $x > 1$
concave down when $-2 < x < 1$
points of inflection at $x = -2$ and $x = 1$

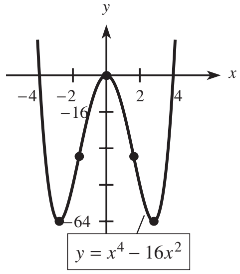
13. no points of inflection;
(2, −2) min



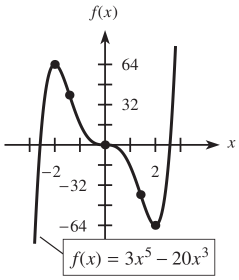
15. $(1, \frac{10}{3})$ max; (3, 2) min; $(2, \frac{8}{3})$ point of inflection



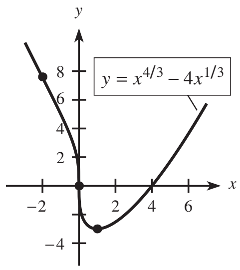
17. (0, 0) rel max; $(2\sqrt{2}, -64)$, $(-2\sqrt{2}, -64)$ min; points of inflection: $(2\sqrt{6}/3, -320/9)$ and $(-2\sqrt{6}/3, -320/9)$



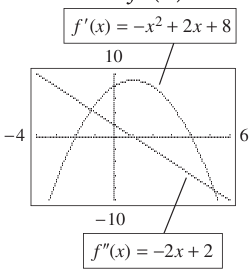
19. $(-2, 64)$ rel max; $(2, -64)$ rel min; points of inflection: $(-\sqrt{2}, 39.6)$, (0, 0), and $(\sqrt{2}, -39.6)$



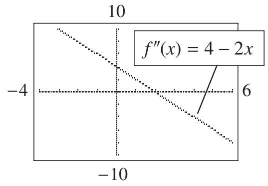
21. $(1, -3)$ min; points of inflection: $(-2, 7.6)$ and (0, 0)



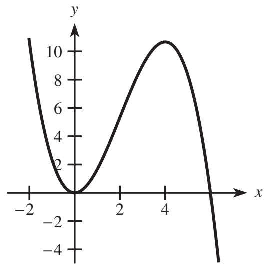
23. (a) $f''(x) = 0$ when $x = 1$
$f''(x) > 0$ when $x < 1$
$f''(x) < 0$ when $x > 1$
(b) rel max for $f'(x)$ at $x = 1$; no rel min
(c)



25. (a) concave up when $x < 2$; concave down when $x > 2$
(b) point of inflection at $x = 2$
(c)



(d) possible graph of $f(x)$



27. (a) G (b) C (c) F (d) H (e) I

29. (a) concave up when $x < 0$
concave down when $x > 0$
point of inflection at $x = 0$
(b) concave up when $-1 < x < 1$
concave down when $x < -1$ and $x > 1$
POI at $x = -1$ and $x = 1$
(c) concave up when $x > 0$
concave down when $x < 0$
point of inflection at $x = 0$

31. (a) $P'(t)$ (b) B (c) C

33. (a) C (b) right (c) yes

35. (a) in an 8-hour shift, max when $t = 8$ (b) 4 hr

37. (a) 9 days (b) 15 days

39. when $x \approx 29.3$, during 1980

41. (a) $y = 0.0000477x^3 - 0.00526x^2 + 0.00509x + 14.4$
(b) (73.0, 5.30)
(c) According to the model, in 1973 the percent foreign-born reached a minimum of 5.3%.

43. (a) $y = -0.405x^3 + 6.02x^2 - 18.0x + 52.0$
(b) $x = 1.83$, during 1914
(c) $x = 8.07$, during 1921

10.3 EXERCISES

1. min -6 at $x = 2$, max $3.\overline{481}$ at $x = -2/3$

3. min -1 at $x = -2$, max 2 at $x = -1$

5. (a) $x = 1800$ units, $R = \$32,400$
(b) $x = 1500$ units, $R = \$31,500$

7. $x = 20$ units, $R = \$24,000$ **9.** 85 people

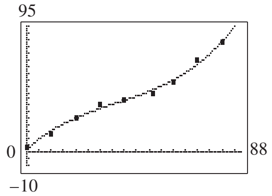
11. $p = \$47.50$, $R = \$225,625$

13. (a) max $= \$2100$ at $x = 10$
(b) $\overline{R}(x) = \overline{MR}$ at $x = 10$

15. $x = 50$ units, $\overline{C} = \$43$

17. $x = 90$ units, $\overline{C} = \$18$

19. 10,000 units ($x = 100$), $\overline{C} = \$216$ per 100 units
21. $\overline{C}(x)$ has its minimum and $\overline{C}(x) = \overline{MC}$ at $x = 5$.
23. (a) A line from (0, 0) to $(x, C(x))$ has slope $C(x)/x = \overline{C}(x)$; this is minimized when the line has the least rise—that is, when the line is tangent to $C(x)$.
(b) $x = 600$ units
25. $x = 80$ units, $P = \$280{,}000$
27. $x = 10\sqrt{15} \approx 39$ units, $P \approx \$71{,}181$ (using $x = 39$)
29. $x = 1000$ units, $P = \$39{,}700$
31. (a) B (b) B (c) B (d) $\overline{MR} = \overline{MC}$
33. \$860 **35.** $x = 600$ units, $P = \$495{,}000$
37. (a) 60 (b) \$570 (c) \$9000
39. (a) 1000 units (b) \$8066.67 (approximately)
41. 2000 units priced at \$90/unit; max profit is \$90,000/wk
43. (a) $R(x) = 2x - 0.0004x^2$
$P(x) = 1.8x - 0.0005x^2 - 800$
(b) $p = \$1.28, x = 1800, P(1800) = \$820$
(c) $p = \$1.25, x = 1875, P(1875) = \$817.19$
Coastal would still provide sodas; profits almost the same.
45. (a) $y = 0.000252x^3 - 0.0279x^2 + 1.63x + 2.16$
(b) (36.9, 37.0)
(c)



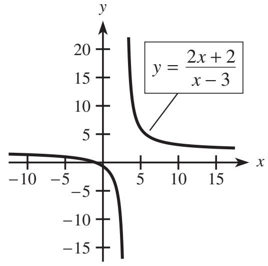
The *rate* of change of the number of beneficiaries was decreasing until 1987, after which the rate has been increasing. Hence, since 1987 the number of beneficiaries has been increasing at an increasing rate.
47. (a) about mid-May
(b) just after September 11, when the terrorists' planes crashed into the World Trade Center and the Pentagon
49. (a) 16.5
(b) 1.9
(c) Rise. As the number of workers per beneficiary drops, either the amount contributed by each worker must rise or support must diminish.

10.4 EXERCISES

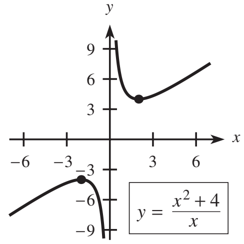
1. (a) $x_1 = \$25$ million, $x_2 = \$13.846$ million
(b) \$38.846 million
3. 100 trees **5.** (a) 5 (b) 237.5 **7.** \$50
9. $m = c$ **11.** 1 week **13.** $t = 8, p = 45\%$
15. 240 ft **17.** 300 ft × 150 ft
19. 20 ft long, $6\frac{2}{3}$ ft across (dividers run across)
21. 4 in. × 8 in. × 8 in. high **23.** 30,000
25. 12,000 **27.** $x = 2$ **29.** 3 weeks from now
31. 25 plates
33. (a) $t \approx 22.8$; in 2018
(b) No. Unless some new technology replaces cell phones, the number will probably level off but not decrease.

10.5 EXERCISES

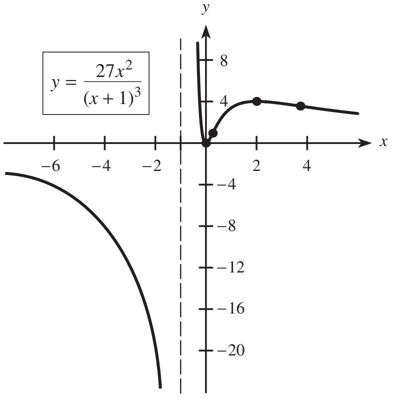
1. (a) $x = 2$ (b) 1 (c) 1 (d) $y = 1$
3. (a) $x = 2, x = -2$ (b) 3 (c) 3 (d) $y = 3$
5. HA: $y = 2$; VA: $x = 3$
7. HA: $y = 0$; VA: $x = -2, x = 2$
9. HA: none; VA: none
11. HA: $y = 2$; VA: $x = 3$
no max, min, or points of inflection



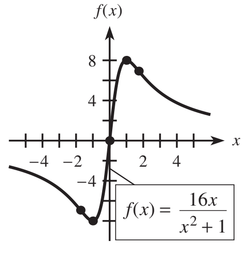
13. VA: $x = 0$;
$(-2, -4)$ rel max;
(2, 4) rel min



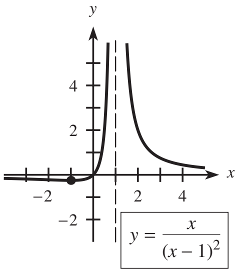
15. VA: $x = -1$; HA: $y = 0$;
(0, 0) rel min; (2, 4) rel max;



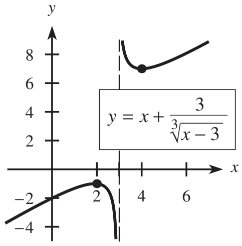
17. HA: $y = 0$; (1, 8) rel max;
$(-1, -8)$ rel min



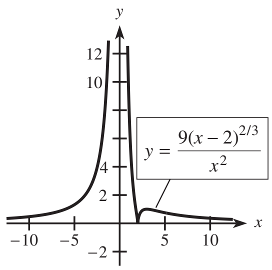
19. HA: $y = 0$; VA: $x = 1$;
$\left(-1, -\frac{1}{4}\right)$ rel min;
point of inflection: $(-2, -2/9)$



21. VA: $x = 3$;
$(2, -1)$ rel max;
$(4, 7)$ rel min



23. HA: $y = 0$;
VA: $x = 0$;
$(2, 0)$ rel min;
$(3, 1)$ rel max;
points of inflection:
$(1.87, 0.66)$,
$(4.13, 0.87)$

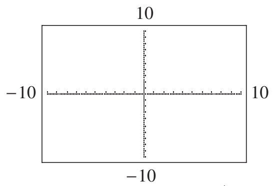


25. (a) HA: approx. $y = -2$; VA: approx. $x = 4$
(b) HA: $y = -\frac{9}{4}$, VA: $x = \frac{17}{4}$

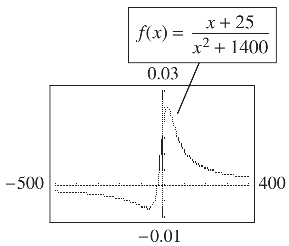
27. (a) HA: approx. $y = 2$;
VA: approx. $x = 2.5$, $x = -2.5$
(b) HA: $y = \frac{20}{9}$, VA: $x = \frac{7}{3}$, $x = -\frac{7}{3}$

29. $f(x) = \dfrac{x + 25}{x^2 + 1400}$

(a)

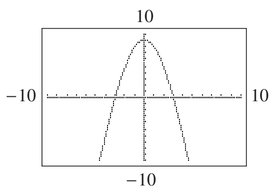


(b) HA: $y = 0$; rel min $(-70, -0.0071)$;
rel max $(20, 0.025)$
(c) x: -500 to 400
y: -0.01 to 0.03

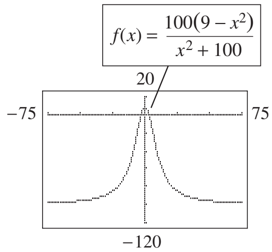


31. $f(x) = \dfrac{100(9 - x^2)}{x^2 + 100}$

(a)

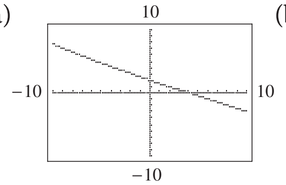


(b) HA: $y = -100$;
rel max $(0, 9)$
(c) x: -75 to 75
y: -120 to 20



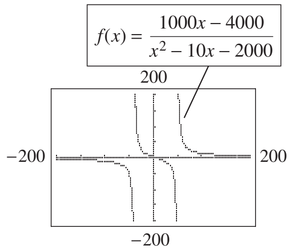
33. $f(x) = \dfrac{1000x - 4000}{x^2 - 10x - 2000}$

(a)



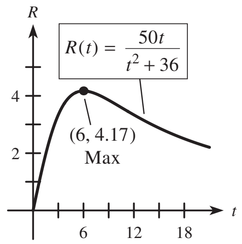
(b) HA: $y = 0$; VA: $x = -40$, $x = 50$; no max or min

(c) x: -200 to 200
y: -200 to 200



35. (a) none (b) $C \geq 0$ (c) $p = 100$ (d) no

37. (a)



(b) 6 weeks
(c) 22 weeks after its release

39. (a) yes, $x = -1$
(b) no; domain is $x \geq 5$
(c) yes, $y = -58.5731$
(d) At 0°F, as the wind speed increases, there is a limiting wind chill of about -58.6°F. This is meaningful because at high wind speeds, additional wind probably has little noticeable effect.

41. (a) $P = C$ (b) C (c) $P' = 0$ (d) 0

43. (a) 57.0
(b) The model predicts that in the long run, 57% of workers will be female.
(c) No. Vertical asymptote is only at $t \approx -46.4$.
(d) $p(t) > 0$ for $t > 0$ and $p(t)$ never exceeds 100, so the model is never inappropriate.

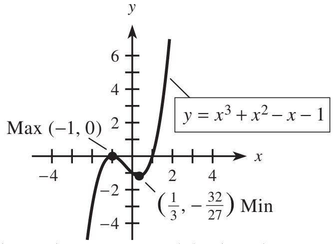
45. (a) No. Barometric pressure can drop off the scale (as shown), but it cannot decrease without bound. In fact, it must always be positive.
(b) See your library with regard to the "storm of the century" in March 1993.

CHAPTER 10 REVIEW EXERCISES

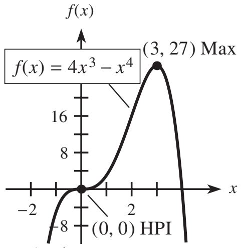
1. (0, 0) max **2.** $(2, -9)$ min **3.** HPI (1, 0)

4. $(1, \frac{3}{2})$ max, $(-1, -\frac{3}{2})$ min

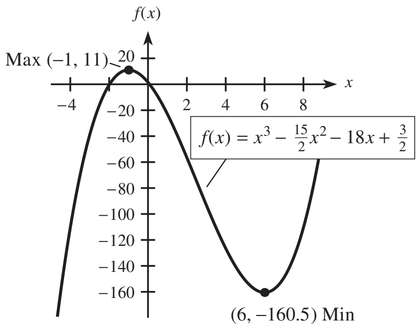
5. (a) $\frac{1}{3}, -1$
(b) $(-1, 0)$ rel max, $(\frac{1}{3}, -\frac{32}{27})$ rel min
(c) none (d)



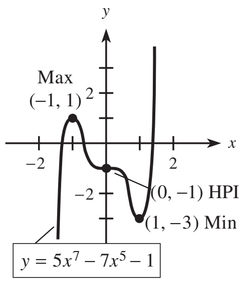
6. (a) 3, 0 (b) (3, 27) max (c) (0, 0)
(d)



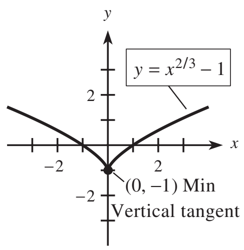
7. (a) $-1, 6$
(b) $(-1, 11)$ rel max, $(6, -160.5)$ rel min
(c) none (d)



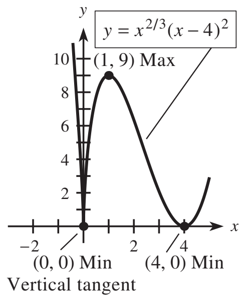
8. (a) $0, \pm 1$ (b) $(-1, 1)$ rel max, $(1, -3)$ rel min
(c) $(0, -1)$ (d)



9. (a) 0 (b) $(0, -1)$ min (c) none
(d)



10. (a) 0, 1, 4
(b) (0, 0) rel min, (1, 9) rel max, (4, 0) rel min
(c) none (d)

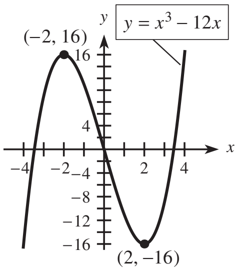


11. concave up

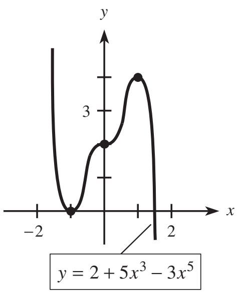
12. concave up when $x < -1$ and $x > 2$; concave down when $-1 < x < 2$; points of inflection at $(-1, -3)$ and $(2, -42)$

13. $(-1, 15)$ rel max; $(3, -17)$ rel min; point of inflection $(1, -1)$

14. $(-2, 16)$ rel max; $(2, -16)$ rel min; point of inflection (0, 0)



15. (1, 4) rel max; $(-1, 0)$ rel min; points of inflection: $\left(\frac{1}{\sqrt{2}}, 2 + \frac{7}{4\sqrt{2}}\right)$, (0, 2), and $\left(-\frac{1}{\sqrt{2}}, 2 - \frac{7}{4\sqrt{2}}\right)$



16. (a) (0, 0) absolute min; (140, 19,600) absolute max
(b) (0, 0) absolute min; (100, 18,000) absolute max

17. (a) (50, 233,333) absolute max; (0, 0) absolute min
(b) (64, 248,491) absolute max; (0, 0) absolute min

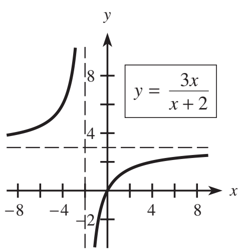
18. (a) $x = 1$ (b) $y = 0$ (c) 0 (d) 0

19. (a) $x = -1$ (b) $y = \frac{1}{2}$ (c) $\frac{1}{2}$ (d) $\frac{1}{2}$

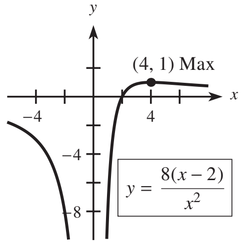
20. HA: $y = \frac{3}{2}$, VA: $x = 2$

21. HA: $y = -1$; VA: $x = 1, x = -1$

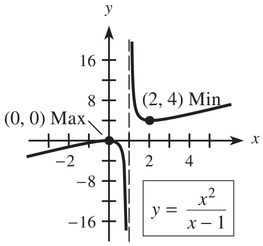
22. (a) HA: $y = 3$; VA: $x = -2$
(b) no max or min (c)



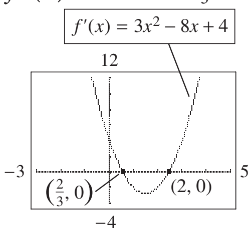
23. (a) HA: $y = 0$; VA: $x = 0$
(b) (4, 1) max (c)



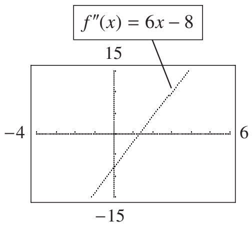
24. (a) HA: none; VA: $x = 1$
(b) (0, 0) rel max; (2, 4) rel min
(c)



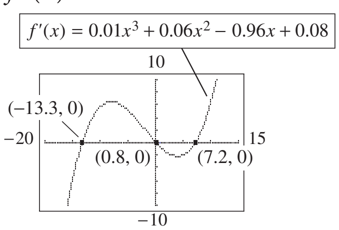
25. (a) $f'(x) > 0$ for $x < \frac{2}{3}$ (approximately) and $x > 2$
$f'(x) < 0$ for about $\frac{2}{3} < x < 2$
$f'(x) = 0$ at about $x = \frac{2}{3}$ and $x = 2$
(b) $f''(x) > 0$ for $x > \frac{4}{3}$
$f''(x) < 0$ for $x < \frac{4}{3}$
$f''(x) = 0$ at $x = \frac{4}{3}$
(c)



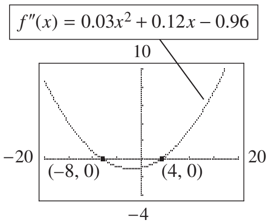
(d)



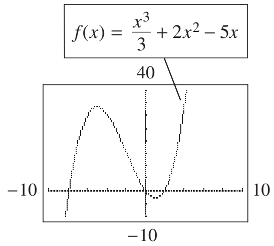
26. (a) $f'(x) > 0$ for about $-13 < x < 0$ and $x > 7$
$f'(x) < 0$ for about $x < -13$ and $0 < x < 7$
$f'(x) = 0$ at about $x = 0$, $x = -13$, $x = 7$
(b) $f''(x) > 0$ for about $x < -8$ and $x > 4$
$f''(x) < 0$ for about $-8 < x < 4$
$f''(x) = 0$ at about $x = -8$ and $x = 4$
(c)



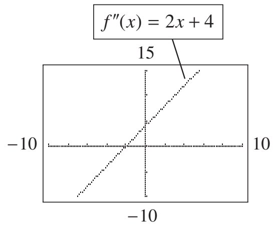
(d)



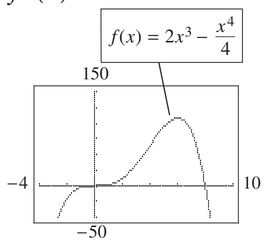
27. (a) $f(x)$ increasing for $x < -5$ and $x > 1$
$f(x)$ decreasing for $-5 < x < 1$
$f(x)$ has rel max at $x = -5$, rel min at $x = 1$
(b) $f''(x) > 0$ for $x > -2$ (where $f'(x)$ increases)
$f''(x) < 0$ for $x < -2$ (where $f'(x)$ decreases)
$f''(x) = 0$ for $x = -2$
(c)



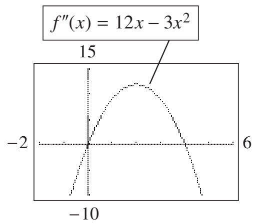
(d)



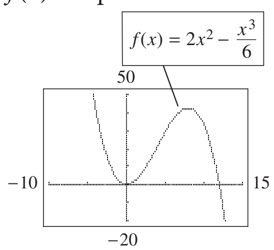
28. (a) $f(x)$ increasing for $x < 6$, $x \neq 0$
$f(x)$ decreasing for $x > 6$
$f(x)$ has rel max at $x = 6$, point of inflection at $x = 0$
(b) $f''(x) > 0$ for $0 < x < 4$
$f''(x) < 0$ for $x < 0$ and $x > 4$
$f''(x) = 0$ at $x = 0$ and $x = 4$
(c)



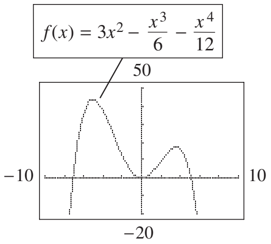
(d)



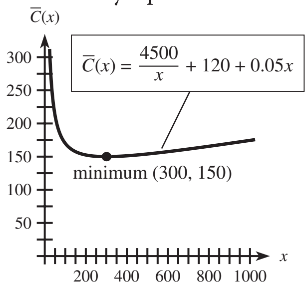
29. (a) $f(x)$ is concave up for $x < 4$.
$f(x)$ is concave down for $x > 4$.
$f(x)$ has point of inflection at $x = 4$.
(b)



30. (a) $f(x)$ is concave up for $-3 < x < 2$.
$f(x)$ is concave down for $x < -3$ and $x > 2$.
$f(x)$ has points of inflection at $x = -3$ and $x = 2$.
(b)



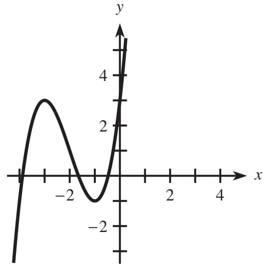
31. $x = 5$ units, $\overline{C} = \$45$ per unit
32. (a) $x = 1600$ units, $R = \$25{,}600$
(b) $x = 1500$ units, $R = \$25{,}500$
33. $P = \$54{,}000$ at $x = 100$ units **34.** $x = 300$ units
35. $x = 150$ units **36.** $x = 7$ units
37. $x = 500$ units, when $\overline{MP} = 0$ and changes from positive to negative.
38. 30 hours
39. (a) $I = 60$. The point of diminishing returns is located at the point of inflection (where bending changes).
(b) $m = f(I)/I =$ the average output
(c) The segment from (0, 0) to $y = f(I)$ has maximum slope when it is tangent to $y = f(I)$, close to $I = 70$.
40. \$260 per bike **41.** \$360 per bike
42. \$93,625 at 325 units
43. (a) 150 (b) \$650
44. \$208,490.67 at 64 units
45. $x = 1000$ mg **46.** 10:00 A.M.
47. 325 in 2015 **48.** 20 mi from A, 10 mi from B
49. 4 ft × 4 ft **50.** $8\frac{3}{4}$ in. × 10. in.
51. 500 mg
52. (a) $x \approx 7.09$; during 2008
(b) point of inflection
53. 24,000
54. (a) vertical asymptote at $x = 0$
(d)



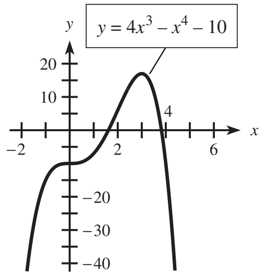
55. (a) 3%
(b) $y = 38$. The long-term market share approaches 38%.

CHAPTER 10 TEST

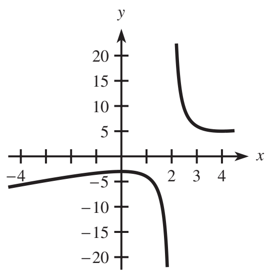
1. max $(-3, 3)$; min $(-1, -1)$;
POI $(-2, 1)$



2. max (3, 17); HPI $(0, -10)$; POI (2, 6)



3. max $(0, -3)$;
min (4, 5);
vertical asymptote $x = 2$



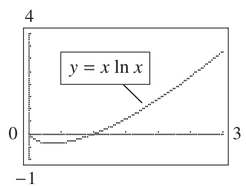
4. $\left(-\frac{1}{\sqrt{2}}, 0\right)$ and $\left(\frac{1}{\sqrt{2}}, \infty\right)$
5. (0, 2), HPI; $\left(-\frac{1}{\sqrt{2}}, 3.237\right)$, $\left(\frac{1}{\sqrt{2}}, 0.763\right)$
6. max $(-1, 4)$; min (1, 0)
7. max 67 at $x = 8$; min -122 at $x = 5$
8. horizontal asymptote $y = 200$; vertical asymptote $x = -300$
9.

Point	f	f'	f''
A	–	+	–
B	+	–	0
C	+	0	+

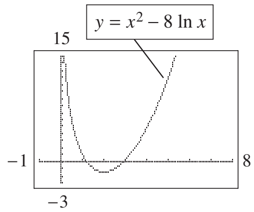
10. (a) 2 (b) $x = -3$
(c) $y = 2$
11. local max at (6, 10)
12. (a) maximum: $x \approx 153.3$ (in 2024)
minimum: $x \approx 27.3$ (in 1898)
(b) The rise of agri-business and the disappearance of the “family farm.”
13. (a) $x = 7200$ (b) \$518,100 **14.** 100 units
15. \$250 **16.** $\frac{10}{3}$ centimeter **17.** 28,000 units
18. (a) $y = -0.0000700x^3 + 0.00567x^2 + 0.863x + 16.0$
(b) $x \approx 27.0$; during 1977
(c) x-coordinate of the point of inflection

11.1 EXERCISES

1. $f'(x) = 4/x$ **3.** $y' = 1/x$
5. $y' = 4/x$ **7.** $f'(x) = \dfrac{4}{4x + 9}$
9. $y' = \dfrac{4x - 1}{2x^2 - x} + 3$ **11.** $dp/dq = 2q/(q^2 + 1)$
13. (a) $y' = \dfrac{1}{x} - \dfrac{1}{x - 1} = \dfrac{-1}{x(x - 1)}$
(b) $y' = \dfrac{-1}{x(x - 1)}$; $\ln\left(\dfrac{x}{x - 1}\right) = \ln(x) - \ln(x - 1)$
15. (a) $y' = \dfrac{2x}{3(x^2 - 1)}$
(b) $y' = \dfrac{2x}{3(x^2 - 1)}$; $\ln(x^2 - 1)^{1/3} = \frac{1}{3}\ln(x^2 - 1)$
17. (a) $y' = \dfrac{4}{4x - 1} - \dfrac{3}{x} = \dfrac{-8x + 3}{x(4x - 1)}$
(b) $y' = \dfrac{-8x + 3}{x(4x - 1)}$;
$\ln\left(\dfrac{4x - 1}{x^3}\right) = \ln(4x - 1) - 3\ln(x)$
19. $\dfrac{dp}{dq} = \dfrac{2q}{q^2 - 1} - \dfrac{1}{q} = \dfrac{q^2 + 1}{q(q^2 - 1)}$
21. $\dfrac{dy}{dt} = \dfrac{2t}{t^2 + 3} - \dfrac{1}{2}\left(\dfrac{-1}{1 - t}\right) = \dfrac{3 + 4t - 3t^2}{2(1 - t)(t^2 + 3)}$
23. $\dfrac{dy}{dx} = \dfrac{3}{x} + \dfrac{1}{2(x + 1)} = \dfrac{7x + 6}{2x(x + 1)}$
25. $y' = 1 - \dfrac{1}{x}$ **27.** $y' = (1 - \ln x)/x^2$
29. $y' = 8x^3/(x^4 + 3)$ **31.** $y' = \dfrac{4(\ln x)^3}{x}$
33. $y' = \dfrac{8x^3 \ln(x^4 + 3)}{x^4 + 3}$ **35.** $y' = \dfrac{1}{x \ln 4}$
37. $y' = \dfrac{4x^3 - 12x^2}{(x^4 - 4x^3 + 1)\ln 6}$
39. rel min $(e^{-1}, -e^{-1})$



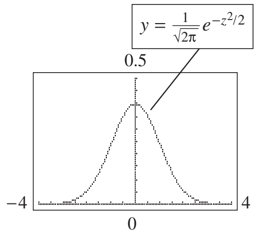
41. rel min $(2, 4 - 8\ln 2)$



43. (a) $\overline{MC} = \dfrac{400}{2x + 1}$
(b) $\overline{MC} = \dfrac{400}{401} \approx 1.0$; the approximate cost of the 201st unit is \$1.00
(c) $\overline{MC} > 0$. Yes
45. (a) $\overline{MR} = \dfrac{2500[(x + 1)\ln(10x + 10) - x]}{(x + 1)\ln^2(10x + 10)}$
(b) 309.67; at 100 units, selling 1 additional unit yields about \$309.67.
47. (a) -5.23 (b) -1.89 (c) increasing
49. A/B **51.** $dR/dI = 1/(I \ln 10)$
53. (a) $y = -2923 + 4013\ln(x)$
(b) \$114.66 per year

11.2 EXERCISES

1. $y' = 5e^x - 1$ **3.** $f'(x) = e^x - ex^{e-1}$
5. $g'(x) = 50e^{-0.1x}$ **7.** $y' = 3x^2e^{x^3}$
9. $y' = 36xe^{3x^2}$
11. $y' = 12x(x^2 + 1)^2e^{(x^2+1)^3}$ **13.** $y' = 3x^2$
15. $y' = e^{-1/x}/x^2$ **17.** $y' = \dfrac{2}{x^3}e^{-1/x^2} - 2xe^{-x^2}$
19. $ds/dt = te^t(t + 2)$ **21.** $y' = 4x^3e^{x^4} - 4e^{4x}$
23. $y' = \dfrac{4e^{4x}}{e^{4x} + 2}$ **25.** $y' = e^{-3x}/x - 3e^{-3x}\ln(2x)$
27. $y' = (2e^{5x} - 3)/e^{3x} = 2e^{2x} - 3e^{-3x}$
29. $y' = 30e^{3x}(e^{3x} + 4)^9$ **31.** $y' = 6^x \ln 6$
33. $y' = 4^{x^2}(2x \ln 4)$
35. (a) $y'(1) = 0$ (b) $y = e^{-1}$
37. (a) $z = 0$ (b)



39. rel min at $x = 1, y = e$
41. rel max at $x = 0, y = -1$
43. (a) $(0.1)Pe^{0.1n}$ (b) $(0.1)Pe^{0.1}$
(c) Yes, because $e^{0.1n} > 1$ for any $n \geq 1$.
45. (a) $\dfrac{dS}{dt} = -50{,}000e^{-0.5t}$
(b) The function is a decay exponential. The derivative is always negative.
47. $40e \approx 108.73$ dollars per unit

49. (a) $\dfrac{dy}{dt} = 46.2e^{-0.462t}$
(b) 29.107 percent per hour

51. $\dfrac{dx}{dt} = -0.0684e^{-0.38t}$

53. 177.1 ($billion/year) **55.** $\dfrac{dI}{dR} = 10^R \ln 10$

57. (a) $d'(t) = 0.138e^{0.0825t}$
(b) 1950: $d'(50) \approx$ \$8.54 billion per year
2015: $d'(115) \approx$ \$1820.7 billion per year

59. $y' = \dfrac{98{,}990{,}100e^{-0.99t}}{(1 + 9999e^{-0.99t})^2}$

61. (a) $P'(t) = \dfrac{1.2595e^{-0.029t}}{(1 + 3.97e^{-0.029t})^2}$
(b) $P'(100) \approx 0.0467$ means that in 2045 the population is expected to change at the rate of 0.0467 billion people per year.
(c) $P''(95) < 0$ means the rate is decreasing.

63. (a) $P'(48) \approx -0.018$
(b) This means that in 2008, the purchasing power of \$1 was changing at the rate of -0.018 dollars per year.
(c) For 2007–2008, average rate $= -0.018$ dollars per year.

65. (a) $y' = 8.864(1.055^x)$
(b) \$33.8 billion

67. (a) $y' = 2.74(1.042^x)$
(b) 14.5
(c) logistic; $y = \dfrac{334}{1 + 390e^{-0.06x}}$
(d) 3.73

11.3 EXERCISES

1. $\frac{1}{2}$ **3.** $-\frac{1}{2}$ **5.** $-\frac{5}{3}$ **7.** $-x/(2y)$
9. $-(2x + 4)/(2y - 3)$ **11.** $y' = -x/y$

13. $y' = \dfrac{-y}{2x - 3y}$ **15.** $\dfrac{dp}{dq} = \dfrac{p^2}{4 - 2pq}$

17. $\dfrac{dy}{dx} = \dfrac{x(3x^3 - 2)}{3y^2(1 + y^2)}$ **19.** $\dfrac{dy}{dx} = \dfrac{4x^3 + 6x^2y^2 - 1}{-4x^3y - 3y^2}$

21. $\dfrac{dy}{dx} = \dfrac{(4x^3 + 9x^2y^2 - 8x - 12y)}{(18y + 12x - 6x^3y + 10y^4)}$ **23.** undefined

25. 1 **27.** $y = \frac{1}{2}x + 1$ **29.** $y = 4x + 5$

31. $\dfrac{dy}{dx} = \dfrac{1}{2xy}$ **33.** $\dfrac{dy}{dx} = \dfrac{-y}{2x \ln x}$ **35.** -15

37. $-1/x$ **39.** $\dfrac{-xy - 1}{x^2}$

41. $ye^x/(1 - e^x)$ **43.** $\frac{1}{3}$ **45.** $y = 3 - x$

47. (a) $(2, \sqrt{2}), (2, -\sqrt{2})$
(b) $(2 + 2\sqrt{2}, 0), (2 - 2\sqrt{2}, 0)$

49. (a) and (b) are verifications
(c) yes, because $x^2 + y^2 = 4$

51. $1/(2x\sqrt{x})$

53. max at (0, 3); min at (0, −3)

55. $\frac{1}{2}$, so an additional 1 (thousand dollars) of advertising yields about $\frac{1}{2}$ (thousand) additional units

57. $-\frac{243}{128}$ hours of skilled labor per hour of unskilled labor

59. At $p = \$80$, $q = 49$ and $dq/dp = -\frac{5}{16}$, which means that if the price is increased to \$81, quantity demanded will decrease by approximately $\frac{5}{16}$ unit.

61. $-0.000436y$ **63.** $\dfrac{dh}{dt} = -\dfrac{3}{44} - \dfrac{h}{12}$

11.4 EXERCISES

1. 36 **3.** $\frac{1}{8}$ **5.** $-\frac{24}{5}$ **7.** $\frac{7}{6}$
9. -5 if $z = 5$, -10 if $z = -5$
11. -80 units/sec **13.** 12π ft²/min
15. $\frac{16}{27}$ in/sec **17.** \$1798/day
19. \$0.42/day
21. 430 units/month **23.** 36π mm³/month

25. $\dfrac{\frac{dW}{dt}}{W} = 3\left(\dfrac{\frac{dL}{dt}}{L}\right)$ **27.** $\dfrac{\frac{dC}{dt}}{C} = 1.54\left(\dfrac{\frac{dW}{dt}}{W}\right)$

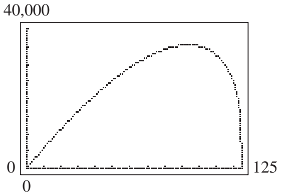
29. $\dfrac{1}{4\pi}$ micrometer/day **31.** $1/(20\pi)$ in/min

33. 0.75 ft/sec **35.** $-120\sqrt{6}$ mph ≈ -294 mph
37. approaching at 61.18 mph **39.** $\frac{1}{25}$ ft/hr

11.5 EXERCISES

1. (a) 1 (b) no change
3. (a) 84 (b) Revenue will decrease.
5. (a) $\frac{100}{99}$ (b) elastic (c) decrease
7. (a) 0.81 (b) inelastic (c) increase
9. (a) $\eta = 11.1$ (approximately) (b) elastic

11. (a) $\eta = \dfrac{375 - 3q}{q}$
(b) unitary: $q = 93.75$; inelastic: $q > 93.75$; elastic: $q < 93.75$
(c) As q increases over $0 < q < 93.75$, p decreases, so elastic demand means R increases. Similarly, R decreases for $q > 93.75$.
(d) maximum for R when $q = 93.75$; yes.



13. (a) $p = 250 - 0.125q$
(b) $\eta = \frac{2000}{q} - 1$
(c) $\eta \approx 2.33$; elastic. no
(d) $q = 1000$; $p = \$125$; max $R = \$125{,}000$

15. \$12/item **17.** $t =$ \$350 **19.** \$115/item
21. \$483 per item; \$40,100 **23.** \$1100/item

CHAPTER 11 REVIEW EXERCISES

1. $dy/dx = (6x - 1)e^{3x^2 - x}$ **2.** $y' = 2x$

3. $\frac{dp}{dq} = \frac{1}{q} - \frac{2q}{q^2 - 1}$

4. $dy/dx = e^{x^2}(2x^2 + 1)$

5. $f'(x) = 10e^{2x} + 4e^{-0.1x}$

6. $g'(x) = 18e^{3x+1}(2e^{3x+1} - 5)^2$

7. $\frac{dy}{dx} = \frac{12x^3 + 14x}{3x^4 + 7x^2 - 12}$

8. $\frac{ds}{dx} = \frac{9x^{11} - 6x^3}{x^{12} - 2x^4 + 5}$ **9.** $dy/dx = 3^{3x-3} \ln 3$

10. $dy/dx = \frac{1}{\ln 8}\left(\frac{10}{x}\right)$ **11.** $\frac{dy}{dx} = \frac{1 - \ln x}{x^2}$

12. $dy/dx = -2e^{-x}/(1 - e^{-x})^2$

13. $y = 12ex - 8e$, or $y \approx 32.62x - 21.75$

14. $y = x - 1$ **15.** $\frac{dy}{dx} = \frac{y}{x(10y - \ln x)}$

16. $dy/dx = ye^{xy}/(1 - xe^{xy})$ **17.** $dy/dx = 2/y$

18. $\frac{dy}{dx} = \frac{2(x + 1)}{3(1 - 2y)}$ **19.** $\frac{dy}{dx} = \frac{6x(1 + xy^2)}{y(5y^3 - 4x^3)}$

20. $d^2y/dx^2 = -(x^2 + y^2)/y^3 = -1/y^3$ **21.** 5/9

22. $\left(-2, \pm\sqrt{\frac{2}{3}}\right)$ **23.** 3/4 **24.** 11 square units/min

25. (a) $y'(t) = \frac{2.62196}{t}$

(b) $y(50) \approx 6.343$ is the predicted number of hectares of deforestation in 2000.
$y'(50) \approx 0.05244$ hectares per year is the predicted rate of deforestation in 2000.

26. (a) 0.328 percentage points per year
(b) increasing, $y'(x) > 0$ for all $x > 0$

27. 135.3 dollars/year

28. (a) 152.5 dollars/year (b) 1.13 times as fast

29. (a) $-0.00001438A_0$ units/year
(b) $-0.00002876A_0$ units/year (c) less

30. $\$1200e \approx \$3261.94$ per unit

31. $-\$603.48$ per year **32.** $-1/(25\pi)$ mm/min

33. $\frac{48}{25}$ ft/min **34.** $\frac{dS/dt}{S} = \frac{1}{3}\left(\frac{dA/dt}{A}\right)$ **35.** yes

36. $t = \$1466.67$, $T \approx \$58,667$

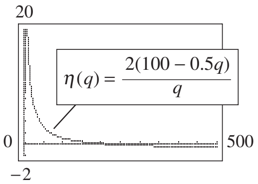
37. $t = \$880$, $T = \$3520$

38. (a) 1 (b) no change

39. (a) $\frac{25}{12}$, elastic (b) revenue decreases

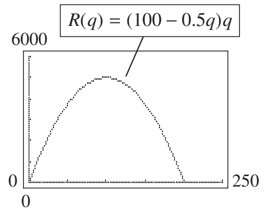
40. (a) 1 (b) no change

41. (a)



(b) $q = 100$

(c) max revenue at $q = 100$



(d) Revenue is maximized where elasticity is unitary.

CHAPTER 11 TEST

1. $y' = 15x^2e^{x^3} + 2x$ **2.** $y' = \frac{12x^2}{x^3 + 1}$

3. $y' = \frac{12x^3}{x^4 + 1}$ **4.** $f'(x) = 20(3^{2x}) \ln 3$

5. $\frac{dS}{dt} = e^{t^4}(4t^4 + 1)$ **6.** $y' = \frac{e^{x^3+1}(3x^3 - 1)}{x^2}$

7. $y' = \frac{1 - \ln x}{x^2}$ **8.** $g'(x) = \frac{8}{(4x + 7) \ln 5}$

9. $y' = \frac{-3x^3}{y}$ **10.** $-\frac{3}{2}$ **11.** $y' = \frac{-e^y}{xe^y - 10}$

12. \$1349.50 per week **13.** $\eta = 3.71$; decreases

14. -0.05 unit per dollar **15.** 586 units per day

16. (a) $y' = 81.778e^{0.062t}$
(b) 2005: $y'(5) \approx 111.5$ (billion dollars per year)
2020: $y'(20) \approx 282.6$ (billion dollars per year)

17. \$540

18. (a) $y = -12.97 + 11.85 \ln x$

(b) $y' = \frac{11.85}{x}$

(c) $y(25) \approx 25.2$ means the model estimates that 25.2% of the U.S. population will have diabetes in 2025.
$y'(25) \approx 0.474$ predicts that in 2025 the percent of the U.S. population with diabetes will be changing by 0.474 percentage points per year.

19. $P'(t) = -0.1548(1.046)^{-t}$; $P'(55) \approx -0.013$ means that in 2015 the purchasing power of a dollar is changing at the rate of $-\$0.013$ per year.

12.1 EXERCISES

1. $x^4 + C$ **3.** $\frac{1}{7}x + C$ **5.** $\frac{1}{8}x^8 + C$

7. $2x^4 + C$ **9.** $27x + \frac{1}{14}x^{14} + C$

11. $3x - \frac{2}{5}x^{5/2} + C$ **13.** $\frac{1}{5}x^5 - 3x^3 + 3x + C$

15. $13x - 3x^2 + 3x^7 + C$

17. $2x + \frac{4}{3}x\sqrt{x} + C$ **19.** $\frac{24}{5}x\sqrt[4]{x} + C$

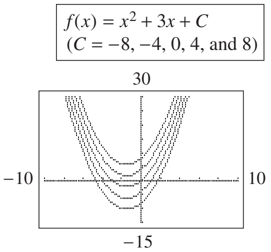
21. $-5/(3x^3) + C$ **23.** $\frac{3}{2}\sqrt[3]{x} + C$

25. $\frac{1}{4}x^4 - 4x - \frac{1}{x^5} + C$

27. $\frac{1}{10}x^{10} + \frac{1}{2x^2} + 3x^{2/3} + C$

29. $2x^8 - \frac{4}{3}x^6 + \frac{1}{4}x^4 + C$ **31.** $-1/x - 1/(2x^2) + C$

33.



35. $f(x) = 18x^8 - 35x^4$ **37.** $\int (5 - \frac{1}{2}x)dx$

39. $\int (3x^2 - 6x)\,dx$ **41.** $R(x) = 30x - 0.2x^2$

43. $R(50) = \$22{,}125$ **45.** $P(t) = \frac{1}{4}t^4 + \frac{4}{3}t^3 + 6t$

47. (a) $x = t^{7/4}/1050$ (b) 0.96 ton

49. (a) $\overline{C}(x) = x/4 + 100/x + 30$ (b) \$56 per unit

51. (a) $H(t) = 5.033t^2 + 100.5t + 1376.8$
(b) \$4016.7 billion

53. (a) The wind chill temperature decreases because
$$\frac{dt}{dw} < 0 \quad \text{for} \quad w > 0.$$
The rate increases because $\frac{d^2t}{dw^2} > 0$ for $w > 0$.
(b) $t = 48.12 - 27.2w^{0.16}$

55. (a) $t \approx 63.1$; in 2024
(b) $P(t) = -0.0000729t^3 + 0.0138t^2 + 1.98t + 181$
(c) 348 million

12.2 EXERCISES

1. $du = 10x^4\,dx$ **3.** $\frac{1}{4}(x^2 + 3)^4 + C$

5. $\frac{1}{5}(5x^3 + 11)^5 + C$ **7.** $\frac{1}{3}(3x - x^3)^3 + C$

9. $\frac{1}{28}(7x^4 + 12)^4 + C$ **11.** $\frac{1}{4}(4x - 1)^7 + C$

13. $-\frac{1}{6}(4x^6 + 15)^{-2} + C$ **15.** $\frac{1}{10}(x^2 - 2x + 5)^5 + C$

17. $-\frac{1}{8}(x^4 - 4x + 3)^{-4} + C$ **19.** $\frac{7}{6}(x^4 + 6)^{3/2} + C$

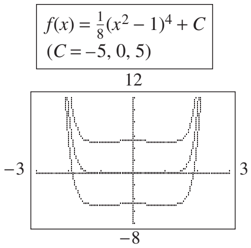
21. $\frac{3}{8}x^8 + \frac{6}{5}x^5 + \frac{3}{2}x^2 + C$ **23.** $10.8x^{10} - 12x^6 + 6x^2 + C$

25. $\frac{2}{9}(x^3 - 3x)^{3/2} + C$ **27.** $\frac{-1}{[10(2x^5 - 5)^3]} + C$

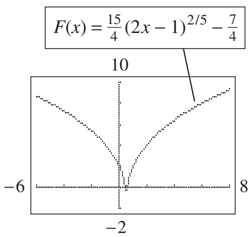
29. $\frac{-1}{[8(x^4 - 4x)^2]} + C$ **31.** $\frac{2}{3}\sqrt{x^3 - 6x^2 + 2} + C$

33. $f(x) = 70(7x - 13)^9$

35. (a) $f(x) = \frac{1}{8}(x^2 - 1)^4 + C$
(b)



37. (a) $F(x) = \frac{15}{4}(2x - 1)^{2/5} + C$
(b)



(c) $x = \frac{1}{2}$
(d) vertical

39. $\int \frac{8x(x^2 - 1)^{1/3}}{3}\,dx$

41. (b) $\frac{-7}{3(x^3 + 4)} + C$
(d) $\int (x^2 + 5)^{-4}\,dx$ (Many answers are possible.)

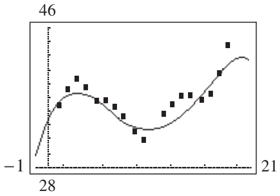
43. $R(x) = \frac{15}{2x + 1} + 30x - 15$

45. 3720 bricks **47.** (a) $s = 10\sqrt{x + 1}$ (b) 50

49. (a) $A(t) = 100/(t + 10) - 1000/(t + 10)^2$
(b) 2.5 million

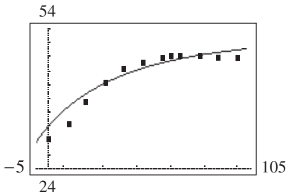
51. 7400

53. (a) $p(t) = -0.0000716(2t + 3)^4 + 0.00742(2t + 3)^3$ $-0.2436(2t + 3)^2 + 5.572t + 35.515$
(b)



(c) The equation fits quite well overall.

55. (a) $p(t) = 56.19 - \frac{1561}{1.38t + 64.1}$
(b)



(c) The model is a good fit to the data.

12.3 EXERCISES

1. $e^{3x} + C$ **3.** $-e^{-x} + C$ **5.** $10{,}000e^{0.1x} + C$

7. $-1200e^{-0.7x} + C$ **9.** $\frac{1}{12}e^{3x^4} + C$

11. $-\frac{3}{2}e^{-2x} + C$ **13.** $\frac{1}{18}e^{3x^6 - 2} + C$

15. $\frac{1}{4}e^{4x} + 6/e^{x/2} + C$

17. $\ln|x^3 + 4| + C$ **19.** $\frac{1}{4}\ln|4z + 1| + C$

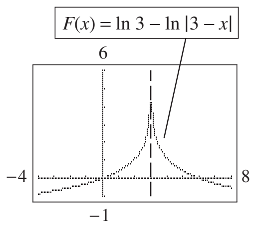
21. $\frac{3}{4}\ln|2x^4 + 1| + C$ **23.** $\frac{2}{5}\ln|5x^2 - 4| + C$

25. $\ln|x^3 - 2x| + C$ **27.** $\frac{1}{3}\ln|z^3 + 3z + 17| + C$

29. $\frac{1}{3}x^3 + \ln|x - 1| + C$ **31.** $x + \frac{1}{2}\ln|x^2 + 3| + C$

33. $f(x) = h(x), \int f(x)\,dx = g(x)$

35. $F(x) = -\ln|3 - x| + C$



37. $f(x) = 1 + \frac{1}{x}; \int \left(1 + \frac{1}{x}\right) dx$

39. $f(x) = 5e^{-x} - 5xe^{-x}; \int (5e^{-x} - 5xe^{-x})\,dx$

41. (c) $\frac{1}{3}\ln|x^3 + 3x^2 + 7| + C$; (d) $\frac{5}{8}e^{2x^4} + C$

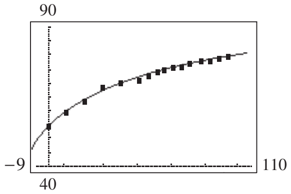
43. \$1030.97 **45.** $n = n_0e^{-Kt}$ **47.** 55

49. (a) $S = Pe^{0.1n}$ (b) $\approx$ 7 years

51. (a) $p = 95e^{-0.491t}$ (b) ≈ 90.45

53. (a) $l(t) = 11.028 + 14.304 \ln(t + 20)$

(b)



(c) The model is a very good fit to the data.

55. (a) Yes. The rate is an exponential that is always positive. Hence the function is always increasing.

(b) $C(t) = 80.39e^{0.0384t} + 0.6635$

(c) $C(35) \approx 308.91$; $C'(35) \approx 11.84$
For 2025, the model predicts that the CPI will be \$308.91 and will be changing at the rate of \$11.84 per year.

12.4 EXERCISES

1. $C(x) = x^2 + 100x + 200$

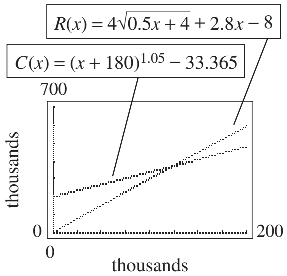
3. $C(x) = 2x^2 + 2x + 80$ **5.** \$3750

7. (a) $x = 3$ units is optimal level
(b) $P(x) = -4x^2 + 24x - 200$ (c) loss of \$164

9. (a) profit of \$3120 (b) 896 units

11. (a) $\overline{C}(x) = \frac{6}{x} + \frac{x}{6} + 8$ (b) \$10.50

13. (a) and (b)



(c) Maximum profit is \$114.743 thousand at $x = 200$ thousand units.

15. $C(y) = 0.80y + 7$

17. $C(y) = 0.3y + 0.4\sqrt{y} + 8$

19. $C(y) = 2\sqrt{y + 1} + 0.4y + 4$

21. $C(y) = 0.7y + 0.5e^{-2y} + 5.15$

23. $C(y) = 0.85y + 5.15$

25. $C(y) = 0.8y + \dfrac{2\sqrt{3y + 7}}{3} + 4.24$

12.5 EXERCISES

1. $4y - 2xy' = 4x^2 - 2x(2x) = 0$ ✔

3. $2y\,dx - x\,dy = 2(3x^2 + 1)\,dx - x(6x\,dx) = 2\,dx$ ✔

5. $y = \frac{1}{2}e^{x^2+1} + C$ **7.** $y^2 = 2x^2 + C$

9. $y^3 = x^2 - x + C$ **11.** $y = e^{x-3} - e^{-3} + 2$

13. $y = \ln|x| - \dfrac{x^2}{2} + \dfrac{1}{2}$ **15.** $\dfrac{y^2}{2} = \dfrac{x^3}{3} + C$

17. $\dfrac{1}{2x^2} + \dfrac{y^2}{2} = C$ **19.** $\dfrac{1}{x} + y + \dfrac{y^3}{3} = C$

21. $\dfrac{1}{y} + \ln|x| = C$ **23.** $x^2 - y^2 = C$

25. $y = C(x + 1)$ **27.** $x^2 + 4\ln|x| + e^{-y^2} = C$

29. $3y^4 = 4x^3 - 1$

31. $2y = 3x + 4xy$ or $y = \dfrac{3x}{2 - 4x}$

33. $e^{2y} = x^2 - \dfrac{2}{x} + 2$ **35.** $y^2 + 1 = 5x$

37. $y = Cx^k$

39. (a) $x = 10{,}000e^{0.06t}$ (b) \$10,618.37; \$13,498.59
(c) 11.55 years

41. $P = 100{,}000e^{0.05t}$; 5% **43.** $\approx$ 8.4 hours

45. $y = \dfrac{32}{(p + 8)^{2/5}}$ **47.** $\approx$ 23,100 years

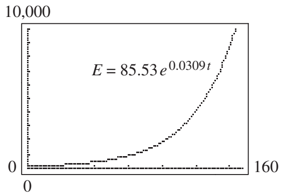
49. $x = 6(1 - e^{-0.05t})$ **51.** $x = 20 - 10e^{-0.025t}$

53. $V = 1.86e^{2-2e^{-0.01t}}$ **55.** $V = \dfrac{k^3t^3}{27}$

57. $t \approx 4.5$ hours

59. (a) $E(t) = 85.53e^{0.0309t}$

(b)



The graph is a similar, but smooth, representation of the data.

61. (a) $P(t) = 80{,}000e^{-0.05t}$ (b) \$37,789.32

CHAPTER 12 REVIEW EXERCISES

1. $\frac{1}{7}x^7 + C$ **2.** $\frac{2}{3}x^{3/2} + C$

3. $3x^4 - x^3 + 2x^2 + 5x + C$

4. $\frac{7}{5}x^5 - \frac{14}{3}x^3 + 7x + C$

5. $\frac{7}{6}(x^2 - 1)^3 + C$ **6.** $\frac{1}{18}(x^3 - 3x^2)^6 + C$

7. $\frac{3}{8}x^8 + \frac{24}{5}x^5 + 24x^2 + C$ **8.** $\frac{5}{63}(3x^3 + 7)^7 + C$

9. $\frac{1}{3}\ln|x^3 + 1| + C$ **10.** $\dfrac{-1}{3(x^3 + 1)} + C$

11. $\frac{1}{2}(x^3 - 4)^{2/3} + C$ **12.** $\frac{1}{3}\ln|x^3 - 4| + C$

13. $\dfrac{1}{2}x^2 - \dfrac{1}{x} + C$

14. $\frac{1}{3}x^3 + \frac{1}{2}x^2 - 2x - \ln|x - 1| + C$

15. $\frac{1}{3}e^{y^3} + C$ **16.** $\frac{1}{39}(3x - 1)^{13} + C$

17. $\frac{1}{2}\ln|2x^3 - 7| + C$ **18.** $\dfrac{-5}{4e^{4x}} + C$

19. $x^4/4 - e^{3x}/3 + C$ **20.** $\frac{1}{2}e^{x^2+1} + C$

21. $\dfrac{-3}{40(5x^8 + 7)^2} + C$ **22.** $-\frac{7}{2}\sqrt{1 - x^4} + C$

23. $\frac{1}{4}e^{2x} - e^{-2x} + C$ **24.** $x^2/2 + 1/(x + 1) + C$

25. (a) $\frac{1}{10}(x^2 - 1)^5 + C$ (b) $\frac{1}{22}(x^2 - 1)^{11} + C$
(c) $\frac{3}{16}(x^2 - 1)^8 + C$ (d) $\frac{3}{2}\ln(x^2 - 1)^{1/3} + C$

26. (a) $\ln|x^2-1|+C$ (b) $\dfrac{-1}{x^2-1}+C$
(c) $3\sqrt{x^2-1}+C$ (d) $\frac{3}{2}\ln|x^2-1|+C$
27. $y=C-92e^{-0.05t}$
28. $y=64x+38x^2-12x^3+C$
29. $(y-3)^2=4x^2+C$ **30.** $(y+1)^2=2\ln|t|+C$
31. $e^y=\dfrac{x^2}{2}+C$ **32.** $y=Ct^4$
33. $3(y+1)^2=2x^3+75$
34. $x^2=y+y^2+4$ **35.** \$28,800 **36.** 472
37. $P(t)=400[1-5/(t+5)+25/(t+5)^2]$
38. $p=1990.099-100{,}000/(t+100)$
39. (a) $y=-60e^{-0.04t}+60$ (b) 23%
40. $R(x)=800\ln(x+2)-554.52$
41. (a) \$1000 (b) $C(x)=3x^2+4x+1000$
42. 80 units, \$440
43. $C(y)=\sqrt{2y+16}+0.6y+4.5$
44. $C(y)=0.8y-0.05e^{-2y}+7.85$ **45.** $W=CL^3$
46. (a) $\ln|P|=kt+C_1$ (b) $P=Ce^{kt}$
(c) $P=50{,}000\,e^{0.1t}$ (d) The interest rate is $k=0.10=10\%$.
47. ≈10.7 million years **48.** $x=360(1-e^{-t/30})$
49. $x=600-500e^{-0.01t}$; ≈ 161 min

CHAPTER 12 TEST

1. $2x^3+4x^2-7x+C$ **2.** $4x+\frac{2}{3}x\sqrt{x}+\frac{1}{x}+C$
3. $\dfrac{(4x^3-7)^{10}}{24}+C$ **4.** $-\frac{1}{6}(3x^2-6x+1)^{-2}+C$
5. $\dfrac{\ln|2s^4-5|}{8}+C$ **6.** $-10{,}000e^{-0.01x}+C$
7. $\frac{5}{8}e^{2y^4-1}+C$ **8.** $e^x+5\ln|x|-x+C$
9. $\dfrac{x^2}{2}-x+\ln|x+1|+C$ **10.** $6x^2-1+5e^x$
11. $y=x^4+x^3+4$ **12.** $y=\frac{1}{4}e^{4x}+\frac{7}{4}$
13. $y=\dfrac{4}{C-x^4}$ **14.** 157,498
15. $P(x)=450x-2x^2-300$
16. $C(y)=0.78y+\sqrt{0.5y+1}+5.6$
17. 332.3 days **18.** $x=16-16e^{-t/40}$

13.1 EXERCISES

1. 7 square units **3.** 7.25 square units
5. 3 square units **7.** 11.25 square units
9. $S_L(10)=4.08$; $S_R(10)=5.28$
11. Both equal 14/3.
13. It would lie between $S_L(10)$ and $S_R(10)$. It would equal 14/3.
15. 3 **17.** 42 **19.** −5 **21.** 180 **23.** 11,315
25. $3-\dfrac{3(n+1)}{n}+\dfrac{(n+1)(2n+1)}{2n^2}=\dfrac{2n^2-3n+1}{2n^2}$
27. (a) $S=(n-1)/n$ (b) 9/10 (c) 99/100
(d) 999/1000 (e) 1
29. (a) $S=\dfrac{(n+1)(2n+1)}{6n^2}$
(b) $77/200=0.385$ (c) $6767/20{,}000\approx0.3384$
(d) $667{,}667/2{,}000{,}000\approx0.3338$ (e) $\frac{1}{3}$
31. $\frac{20}{3}$
33. (a) 7405.7 square units
(b) This represents the total per capita out-of-pocket expenses for health care between 2006 and 2014.
35. There are approximately 90 squares under the curve, each representing 1 second by 10 mph, or
$$1\text{ sec}\times\frac{10\text{ mi}}{1\text{ hr}}\times\frac{1\text{ hr}}{3600\text{ sec}}=\frac{1}{360}\text{ mile.}$$
The area under the curve is approximately $90(\frac{1}{360}\text{ mile})=\frac{1}{4}\text{ mile.}$
37. 1550 square feet
39. 107.734 square units. This represents the total sulphur dioxide emissions (in millions of short tons) from electricity generation from 2010 to 2015.

13.2 EXERCISES

1. 18 **3.** 2 **5.** 60 **7.** $12\sqrt[3]{25}$ **9.** 0 **11.** 98
13. $-\frac{1}{10}$ **15.** 12,960 **17.** 0 **19.** 0 **21.** $\frac{49}{3}$
23. 2 **25.** $e^3/3-1/3$ **27.** 4 **29.** $\frac{8}{3}(1-e^{-8})$
31. (a) $\frac{1}{6}\ln(112/31)\approx0.2140853$ (b) 0.2140853
33. (a) $\frac{3}{2}+3\ln2\approx3.5794415$ (b) 3.5794415
35. (a) A, C (b) B
37. $\int_0^4(2x-\frac{1}{2}x^2)dx$ (b) 16/3
39. (a) $\int_{-1}^0(x^3+1)\,dx$ (b) 3/4
41. $\frac{1}{6}$ **43.** $\frac{1}{2}(e^9-e)$
45. $\int_0^a g(x)\,dx>\int_0^a f(x)\,dx$; more area under $g(x)$
47. same absolute values, opposite signs
49. 6 **51.** 0 **53.** (a) \$450,000 (b) \$450,000
55. (a) \$5390 (b) \$2450
57. \$20,405.39
59. 4146 represents the total million metric tons of CO_2 emissions from 2010 to 2020.
61. 0.04 cm^3 **63.** 1222 (approximately)
65. 0.1808
67. (a) 0.5934 (b) 0.1733
69. (a) $P(t)=-0.0041t^3+0.038t^2+0.052t+4.14$
(b) 24.12; the total amount of oil and petroleum products imported during this time period is 24.12 billion barrels.

13.3 EXERCISES

1. (a) $\int_0^2(4-x^2)\,dx$ (b) $\frac{16}{3}$
3. (a) $\int_1^8[\sqrt[3]{x}-(2-x)]\,dx$ (b) 28.75
5. (a) $\int_1^2[(4-x^2)-(\frac{1}{4}x^3-2)]\,dx$ (b) 131/48
7. (a) $(-1,1),(2,4)$ (b) $\int_{-1}^2[(x+2)-x^2]\,dx$
(c) 9/2
9. (a) $(0,0),(\frac{5}{2},-\frac{15}{4})$

(b) $\int_0^{5/2} [(x - x^2) - (x^2 - 4x)]\,dx$ (c) $\frac{125}{24}$

11. (a) $(-2, -4), (0, 0), (2, 4)$

(b) $\int_{-2}^{0} [(x^3 - 2x) - 2x]\,dx + \int_0^2 [2x - (x^3 - 2x)]\,dx$

(c) 8

13. $\frac{28}{3}$ **15.** $\frac{1}{4}$ **17.** $\frac{16}{3}$ **19.** $\frac{1}{3}$ **21.** $\frac{37}{12}$

23. $4 - 3\ln 3$ **25.** $\frac{8}{3}$ **27.** 6 **29.** 0 **31.** $-\frac{4}{9}$

33. $11.8\overline{3}$

35. average profit $= \dfrac{1}{x_1 - x_0}\displaystyle\int_{x_0}^{x_1} [R(x) - C(x)]\,dx$

37. (a) \$1402 per unit (b) \$535,333.33

39. (a) 102.5 units (b) 100 units

41. (a) 40.05 million/year (b) 69.93 million/year

43. 147 mg

45. 1988: 0.4034; 2000: 0.4264
More equally distributed after Reagan. This is contrary to conventional wisdom.

47. Blacks: 0.4435; Asians: 0.4297
Income was more nearly equally distributed among Asians, although both distributions were similar and not particularly equal.

49. $G = \dfrac{p - 1}{p + 1}$

13.4 EXERCISES

1. \$126,205.10 **3.** \$346,664 (nearest dollar)

5. \$506,000 (nearest thousand)

7. \$18,660 (nearest dollar)

9. \$82,155 (nearest dollar)

11. PV = \$2,657,807 (nearest dollar), FV = \$3,771,608 (nearest dollar)

13. PV = \$190,519 (nearest dollar), FV = \$347,148 (nearest dollar)

15. Gift Shoppe, \$151,024; Wine Boutique, \$141,093. The gift shop is a better buy.

17. \$83.33 **19.** \$161.89 **21.** (5, 56); \$83.33

23. \$11.50 **25.** \$204.17 **27.** \$2766.67

29. \$17,839.58 **31.** \$133.33 **33.** \$2.50

35. \$103.35

13.5 EXERCISES

1. formula 5: $\frac{1}{8}\ln|(4 + x)/(4 - x)| + C$

3. formula 11: $\frac{1}{3}\ln[(3 + \sqrt{10})/2]$

5. formula 14: $w(\ln w - 1) + C$

7. formula 12: $\frac{1}{3} + \frac{1}{4}\ln\left(\frac{3}{7}\right)$

9. formula 13: $\frac{1}{8}\ln\left|\dfrac{v}{3v + 8}\right| + C$

11. formula 7: $\frac{1}{2}[7\sqrt{24} - 25\ln(7 + \sqrt{24}) + 25\ln 5]$

13. formula 16: $\dfrac{(6w - 5)(4w + 5)^{3/2}}{60} + C$

15. formula 3: $\frac{1}{2}(5^{x^2})\log_5 e + C$

17. formula 1: $\frac{1}{3}(13^{3/2} - 8)$

19. formula 9: $-\frac{5}{2}\ln\left|\dfrac{2 + \sqrt{4 - 9x^2}}{3x}\right| + C$

21. formula 10: $\frac{1}{3}\ln|3x + \sqrt{9x^2 - 4}| + C$

23. formula 15: $\frac{3}{4}\left[\ln|2x - 5| - \dfrac{5}{2x - 5}\right] + C$

25. formula 8: $\frac{1}{3}\ln|3x + 1 + \sqrt{(3x + 1)^2 + 1}| + C$

27. formula 6: $\frac{1}{4}[10\sqrt{109} - \sqrt{10} + 9\ln(10 + \sqrt{109}) - 9\ln(1 + \sqrt{10})]$

29. formula 2: $-\frac{1}{6}\ln|7 - 3x^2| + C$

31. formula 8: $\frac{1}{2}\ln|2x + \sqrt{4x^2 + 7}| + C$

33. $2(e^{\sqrt{2}} - e) \approx 2.7899$

35. $\frac{1}{32}[\ln(9/5) - 4/9] \approx 0.004479$ **37.** \$3391.10

39. (a) $C = \frac{1}{2}x\sqrt{x^2 + 9} + \frac{9}{2}\ln|x + \sqrt{x^2 + 9}| + 300 - \frac{9}{2}\ln 3$

(b) \$314.94

41. \$3882.9 thousand

13.6 EXERCISES

1. $\frac{1}{2}xe^{2x} - \frac{1}{4}e^{2x} + C$ **3.** $\frac{1}{3}x^3\ln x - \frac{1}{9}x^3 + C$

5. $\dfrac{104\sqrt{2}}{15}$ **7.** $-(1 + \ln x)/x + C$ **9.** 1

11. $\dfrac{x^2}{2}\ln(2x - 3) - \dfrac{1}{4}x^2 - \dfrac{3}{4}x - \dfrac{9}{8}\ln(2x - 3) + C$

13. $\frac{1}{5}(q^2 - 3)^{3/2}(q^2 + 2) + C$ **15.** 282.4

17. $-e^{-x}(x^2 + 2x + 2) + C$ **19.** $(9e^4 + 3)/2$

21. $\frac{1}{4}x^4\ln^2 x - \frac{1}{8}x^4\ln x + \frac{1}{32}x^4 + C$

23. $\frac{2}{15}(e^x + 1)^{3/2}(3e^x - 2) + C$ **25.** II; $\frac{1}{2}e^{x^2} + C$

27. IV; $\frac{2}{3}(e^x + 1)^{3/2} + C$ **29.** I; $-5e^{-4} + 1$

31. \$2794.46 **33.** \$34,836.73 **35.** 0.264

37. \$5641.3 billion

13.7 EXERCISES

1. 1/5 **3.** 2 **5.** $1/e$ **7.** diverges **9.** diverges

11. 10 **13.** diverges **15.** diverges **17.** 0

19. 0 **21.** 0.5 **23.** $1/(2e)$ **25.** $\frac{3}{2}$

27. $\int_{-\infty}^{\infty} f(x)\,dx = 1$ **29.** $c = 1$ **31.** $c = \frac{1}{4}$

33. 20 **35.** area $= \frac{8}{3}$ **37.** $\int_0^{\infty} Ae^{-rt}\,dt = A/r$

39. \$2,400,000 **41.** \$700,000

43. (a) 0.368 (b) 0.018

45. 0.147

47. (a) $500\left[\dfrac{e^{-0.03b} + 0.03b - 1}{0.0009}\right]$

(b) The amount approaches ∞.

13.8 EXERCISES

1. $h = \frac{1}{2}; x_0 = 0, x_1 = \frac{1}{2}, x_2 = 1, x_3 = \frac{3}{2}, x_4 = 2$

3. $h = \frac{1}{2}; x_0 = 1, x_1 = \frac{3}{2}, x_2 = 2, x_3 = \frac{5}{2}, x_4 = 3, x_5 = \frac{7}{2}, x_6 = 4$

5. $h = 1; x_0 = -1, x_1 = 0, x_2 = 1, x_3 = 2, x_4 = 3, x_5 = 4$
7. (a) 9.13 (b) 9.00 (c) 9 (d) Simpson's
9. (a) 0.51 (b) 0.50 (c) $\frac{1}{2}$ (d) Simpson's
11. (a) 5.27 (b) 5.30 (c) 5.33 (d) Simpson's
13. (a) 3.283 (b) 3.240
15. (a) 0.743 (b) 0.747
17. (a) 7.132 (b) 7.197 **19.** 7.8 **21.** 10.3
23. 119.58 ($119,580) **25.** $32,389.76
27. $14,133.33 **29.** 1222.35 (1222 units)
31. (a)

x	0	0.2	0.4	0.6	0.8	1
$L_a - L_b$	0	0.001	−0.005	−0.002	0.012	0

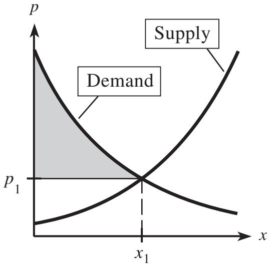
(b) 0.0024
(c) positive; 1990
33. (a) Yes (b) Simpson's (c) 1586.67 ft^2

CHAPTER 13 REVIEW EXERCISES

1. 212 **2.** $\dfrac{3(n+1)}{2n^2}$ **3.** $\frac{91}{72}$ **4.** 1 **5.** 1
6. 14 **7.** $\frac{248}{5}$ **8.** $-\frac{205}{4}$ **9.** $\frac{825}{4}$ **10.** $\frac{4}{13}$
11. -2 **12.** $\frac{1}{6}\ln 47 - \frac{1}{6}\ln 9$ **13.** $\frac{9}{2}$
14. $\ln 4 + \frac{14}{3}$ **15.** 190/3 **16.** $\frac{1}{2}\ln 2$
17. $(1 - e^{-2})/2$ **18.** $(e - 1)/2$ **19.** 95/2
20. 36 **21.** $\frac{1}{4}$ **22.** $\frac{1}{2}$
23. $\frac{1}{2}x\sqrt{x^2 - 4} - 2\ln|x + \sqrt{x^2 - 4}| + C$
24. $2\log_3 e$ **25.** $\frac{1}{2}x^2(\ln x^2 - 1) + C$
26. $\frac{1}{2}\ln|x| - \frac{1}{2}\ln|3x + 2| + C$
27. $\frac{1}{6}x^6 \ln x - \frac{1}{36}x^6 + C$
28. $-e^{-2x}(x^2/2 + x/2 + 1/4) + C$
29. $2x\sqrt{x + 5} - \frac{4}{3}(x + 5)^{3/2} + C$
30. 1 **31.** diverges **32.** -100
33. $\frac{5}{3}$ **34.** $-\frac{1}{2}$
35. (a) $\frac{8}{9} \approx 0.889$ (b) 1.004 (c) 0.909
36. 3.135 **37.** 3.9
38. (a) $n = 5$ (b) $n = 6$ **39.** $28,000
40. $e^{-2.8} \approx 0.061$ **41.** $1297.44 **42.** $76.60
43. 1969: 0.3737; 2000: 0.4264; more equally distributed in 1969
44. (a) (7, 6) (b) $7.33 **45.** $24.50
46. $1,621,803 **47.** (a) $403,609 (b) $602,114
48. $217.42 **49.** $10,066 (nearest dollar)
50. $86,557.41
51. $C(x) = 3x + 30(x + 1)^2 \ln(x + 1) - 15(x + 1)^2 + 2015$
52. $e^{-1.4} \approx 0.247$
53. $4000 thousand, or $4 million
54. $197,365 **55.** $480,000

CHAPTER 13 TEST

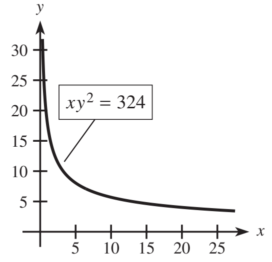
1. 3.496 (approximately)
2. (a) $5 - \dfrac{n+1}{n}$ (b) 4
3. $\int_0^6 (12 + 4x - x^2)\,dx$; 72
4. (a) 4 (b) 3/4 (c) $\frac{5}{4}\ln 5$ (d) 7
(e) 0; limits of integration are the same
(f) $\frac{5}{6}(e^2 - 1)$
5. (a) $3xe^x - 3e^x + C$ (b) $\dfrac{x^2}{2}\ln(2x) - \dfrac{x^2}{4} + C$
6. -8
7. (a) $x[\ln(2x) - 1] + C$
(b) $\dfrac{2(9x + 14)(3x - 7)^{3/2}}{135} + C$
8. 16.089 **9.** (a) $4000 (b) $16,000/3
10. (a) $961.18 thousand (b) $655.68 thousand
(c) $1062.5 thousand
11. 125/6 **12.**



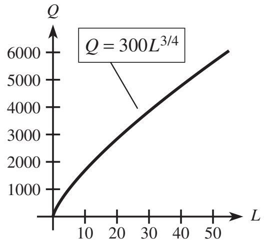
13. Before, 0.446; After, 0.19. The change decreases the difference in income.
14. (a) 20.92 billion barrels
(b) 2.067 billion barrels per year
15. 2.96655 **16.** 6800 ft^2

14.1 EXERCISES

1. $\{(x, y)$: x and y are real numbers$\}$
3. $\{(x, y)$: x and y are real numbers and $y \neq 0\}$
5. $\{(x, y)$: x and y are real numbers and $2x - y \neq 0\}$
7. $\{(p_1, p_2)$: p_1 and p_2 are real numbers and $p_1 \geq 0\}$
9. -2 **11.** $\frac{5}{3}$ **13.** 2500 **15.** 36 **17.** 3
19. $\frac{1}{25}\ln(12)$ **21.** $\frac{13}{3}$
23. $6640.23; the amount that results when $2000 is invested for 20 years
25. 500; if the cost of placing an order is $200, the number of items sold per week is 625, and the weekly holding cost per item is $1, then the most economical order size is 500.
27. Max: $S \approx 112.5$°F; $A \approx 106.3$°F
Min: $S \approx 87.4$°F; $A \approx 77.6$°F
29. (a) $752.80; when $90,000 is borrowed for 20 years at 8%, the monthly payment is $752.80.
(b) $1622.82; when $160,000 is borrowed for 15 years at 9%, the monthly payment is $1622.82.
31. (a) $x = 4$ (b) $y = 2$
(c)



33. (a) 37,500 units
(b) $30(2K)^{1/4}(2L)^{3/4} = 30(2^{1/4})(2^{3/4})K^{1/4}L^{3/4} = 2[30K^{1/4}L^{3/4}]$
(c)



35. (a) 7200 units (b) 5000 units **37.** $284,000

14.2 EXERCISES

1. $\dfrac{\partial z}{\partial x} = 4x^3 - 10x + 6 \quad \dfrac{\partial z}{\partial y} = 9y^2 - 5$

3. $z_x = 3x^2 + 8xy \quad z_y = 4x^2 + 12y$

5. $\dfrac{\partial f}{\partial x} = 9x^2(x^3 + 2y^2)^2 \quad \dfrac{\partial f}{\partial y} = 12y(x^3 + 2y^2)^2$

7. $f_x = 2x(2x^2 - 5y^2)^{-1/2} \quad f_y = -5y(2x^2 - 5y^2)^{-1/2}$

9. $\dfrac{\partial C}{\partial x} = -4y + 20xy \quad \dfrac{\partial C}{\partial y} = -4x + 10x^2$

11. $\dfrac{\partial Q}{\partial s} = \dfrac{2(t^2 + 3st - s^2)}{(s^2 + t^2)^2} \quad \dfrac{\partial Q}{\partial t} = \dfrac{3t^2 - 4st - 3s^2}{(s^2 + t^2)^2}$

13. $z_x = 2e^{2x} + \dfrac{y}{x} \quad z_y = \ln x$

15. $\dfrac{\partial f}{\partial x} = \dfrac{y}{xy + 1} \quad \dfrac{\partial f}{\partial y} = \dfrac{x}{xy + 1}$ **17.** 2

19. 7 **21.** -19

23. (a) 0 (b) $-2xz + 4$ (c) $2y$ (d) $-x^2$

25. (a) $8x_1 + 5x_2$ (b) $5x_1 + 12x_2$ (c) 1

27. (a) 2 (b) 0 (c) 0 (d) $-30y$

29. (a) $2y$ (b) $2x - 8y$ (c) $2x - 8y$ (d) $-8x$

31. (a) $2 + y^2e^{xy}$ (b) $xye^{xy} + e^{xy}$
(c) $xye^{xy} + e^{xy}$ (d) x^2e^{xy}

33. (a) $1/x^2$ (b) 0 (c) 0 (d) $2 + 1/y^2$

35. -6 **37.** (a) $\frac{188}{4913}$ (b) $\frac{-188}{4913}$ **39.** $2 + 2e$

41. 0 **43.** (a) $24x$ (b) $24x$ (c) 0

45. (a) For a mortgage of $100,000 and an 8% interest rate, the monthly payment is $1289.
(b) The rate of change of the payment with respect to the interest rate is $62.51. That is, if the rate goes from 8% to 9% on a $100,000 mortgage, the approximate increase in the monthly payment is $62.51.

47. (a) If the number of items sold per week changes by 1, the most economical order quantity should also increase. $\dfrac{\partial Q}{\partial M} = \sqrt{\dfrac{K}{2Mh}} > 0$
(b) If the weekly storage costs change by 1, the most economical order quantity should decrease.

$$\frac{\partial Q}{\partial h} = -\sqrt{\frac{KM}{2h^3}} < 0$$

49. (a) 23.912; If brand 2 is held constant and brand 1 is increased from 100 to 101 liters, approximately 24,000 additional insects will be killed.

51. (a) $2xy^2$ (b) $2x^2y$

53. $\dfrac{\partial Q}{\partial K} = 100$; If labor hours are held constant at 5832 and K changes by $1 (thousand) to $730,000, Q will change by about 100 units. $\dfrac{\partial Q}{\partial L} = 25$; If capital expenditures are held constant at $729,000 and L changes by 1 hour (to 5833), Q will change by about 25 units.

55. (a) $\dfrac{\partial WC}{\partial s} = 0.16s^{-0.84}(0.4275t - 35.75)$
(b) At $t = 10$, $s = 25$, $\dfrac{\partial WC}{\partial s} \approx -0.34$

This means that if wind speed changes by 1 mph (from 25 mph) while the temperature remains at 10°F, the wind chill temperature will change by about -0.34°F.

14.3 EXERCISES

1. (a) $105 (b) $C_x = 3$ means total costs would change by $3 if labor costs changed by $1 and raw material costs stayed the same.

3. (a) $2 + y/50$ (b) $4 + x/50$

5. (a) $25.78 (b) $74.80

7. (a) If y remains at 10, the expected change in cost for a 9th unit of X is $36.
(b) If x remains at 8, the expected change in cost for an 11th unit of Y is $19.

9. (a) $\sqrt{y^2 + 1}$ dollars per unit
(b) $xy/\sqrt{y^2 + 1}$ dollars per unit

11. (a) $1200y/(xy + 1)$ dollars per unit
(b) $1200x/(xy + 1)$ dollars per unit

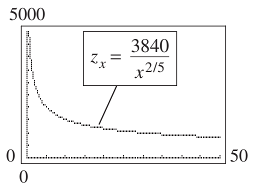
13. (a) $\sqrt{y/x}$ (b) $\sqrt{x/y}$

15. (a) $\ln(y + 1)/(2\sqrt{x})$ (b) $\sqrt{x}/(y + 1)$

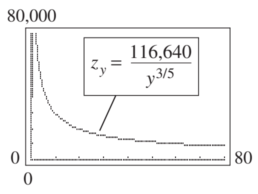
17. $z = 1092$ crates (approximately)

19. $z_x = 3.6$; If 500 acres are planted, the expected change in productivity from a 301st hour of labor is 3.6 crates.

21. (a) $z_x = \dfrac{240y^{2/5}}{x^{2/5}}$ (b)



(c) $z_y = \dfrac{160x^{3/5}}{y^{3/5}}$ (d)



(e) Both z_x and z_y are positive, so increases in both capital investment and work-hours result in increases in productivity. However, both are decreasing, so such increases have a diminishing effect on productivity. Also, z_y decreases more slowly than z_x, so that increases in work-hours have a more significant impact on productivity than increases in capital investment.

23. $q_1 = 188$ units; $q_2 = 270$ units

25. any values for p_1 and p_2 that satisfy $6p_2 - 3p_1 = 100$ and that make q_1 and q_2 nonnegative, such as $p_1 = \$10$, $p_2 = \$21\frac{2}{3}$

27. (a) -3 units per dollar (b) -2 units per dollar
(c) -6 units per dollar (d) -5 units per dollar
(e) complementary

29. (a) -50 units per dollar
(b) $600/(p_B + 1)^2$ units per dollar
(c) $-400/(p_B + 4)^2$ units per dollar
(d) $400/(p_A + 4)^2$ units per dollar
(e) competitive

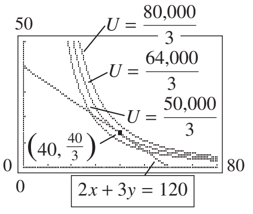
31. (a) Competitive; as the price of one type of car declines, demand for the other declines
(b) (i) $q_{\text{NEW}} = 2600 - p_{\text{NEW}}/30 + p_{\text{USED}}/15$
$q_{\text{USED}} = 750 - 0.25\,p_{\text{USED}} + 0.0125p_{\text{NEW}}$
(ii) Since the mixed partials are both positive (1/15 and 0.0125), the products are competitive.

14.4 EXERCISES

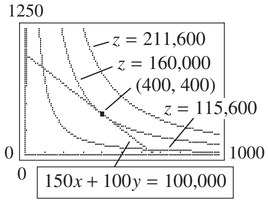
1. max(0, 0, 9) **3.** min(0, 0, 4)
5. saddle$(-2, -3, 16)$ **7.** min$(1, -2, 0)$
9. saddle$(1, -3, 8)$ **11.** max(12, 24, 456)
13. min$(-8, 6, -52)$
15. saddle(0, 0, 0); min$(2, 2, -8)$
17. $\hat{y} = 5.7x - 1.4$
19. $x = 5000$, $y = 128$; $P = \$25{,}409.60$
21. $x = \frac{20}{3}$, $y = \frac{10}{3}$; $W \approx 1926$ lb
23. $x = 28$, $y = 100$; $P = 5987.84$ tons
25. $x = 20$ thousand, $y = 30$ thousand; $P = \$1900$ thousand
27. length $= 100$ in., width $= 100$ in., height $= 50$ in.
29. $x = 15$ thousand, $y = 24$ thousand; $P = \$295$ thousand
31. (a) eat-in = 2400; take-out = 3800
(b) eat-in @ \$3.60; take-out @ \$3.10; max profit = \$12,480
(c) Change pricing; more profitable
33. (a) $\hat{y} = 0.81x - 2400$
(b) $m = 0.81$; means that for every \$1 that males earn, females earn \$0.81.
(c) The slope would probably be smaller. Equal pay for women for equal work is not yet a reality, but much progress has been made since 1965.
35. (a) $\hat{y} = 0.06254x + 6.191$, x in years past 2000, $\hat{y}$ in billions
(b) 6.942 billion
(c) World population is changing at the rate of 0.06254 billion persons per year past 2000.

14.5 EXERCISES

1. 18 at (3, 3) **3.** 35 at (3, 2) **5.** 32 at (4, 2)
7. -28 at $(3, \frac{5}{2})$ **9.** 15 at (5, 3)
11. 3 at (1, 1, 1) **13.** 1 at (0, 1, 0)
15. $x = 2$, $y = 2$
17. $x = 40$, $y = \frac{40}{3}$



19. (a) $x = 400$, $y = 400$
(b) $-\lambda = 1.6$; means that each additional dollar spent on production results in approximately 1.6 additional units produced.
(c)



21. $x = 900$, $y = 300$; 900 units at plant X, 300 units at plant Y
23. $x = \$10{,}003.33$, $y = \$19{,}996.67$
25. length $= 100$ cm, width $= 100$ cm, height $= 50$ cm

CHAPTER 14 REVIEW EXERCISES

1. $\{(x, y)$: x and y are real numbers and $y \neq 2x\}$
2. $\{(x, y)$: x and y are real numbers with $y \geq 0$ and $(x, y) \neq (0, 0)\}$
3. -5 **4.** 896,000
5. $15x^2 + 6y$ **6.** $24y^3 - 42x^3y^2$
7. $z_x = 8xy^3 + 1/y$; $z_y = 12x^2y^2 - x/y^2$
8. $z_x = x/\sqrt{x^2 + 2y^2}$; $z_y = 2y/\sqrt{x^2 + 2y^2}$
9. $z_x = -2y/(xy + 1)^3$; $z_y = -2x/(xy + 1)^3$
10. $z_x = 2xy^3e^{x^2y^3}$; $z_y = 3x^2y^2e^{x^2y^3}$
11. $z_x = ye^{xy} + y/x$; $z_y = xe^{xy} + \ln x$
12. $z_x = y$; $z_y = x$ **13.** -8 **14.** 8
15. (a) $2y$ (b) 0 (c) $2x - 3$ (d) $2x - 3$
16. (a) $18xy^4 - 2/y^2$ (b) $36x^3y^2 - 6x^2/y^4$
(c) $36x^2y^3 + 4x/y^3$ (d) $36x^2y^3 + 4x/y^3$
17. (a) $2e^{y^2}$ (b) $4x^2y^2e^{y^2} + 2x^2e^{y^2}$
(c) $4xye^{y^2}$ (d) $4xye^{y^2}$
18. (a) $-y^2/(xy + 1)^2$ (b) $-x^2/(xy + 1)^2$
(c) $1/(xy + 1)^2$ (d) $1/(xy + 1)^2$
19. max$(-8, 16, 208)$
20. saddles at $(2, -3, 38)$ and $(-2, 3, -38)$; min at $(2, 3, -70)$; max at $(-2, -3, 70)$
21. 80 at (2, 8) **22.** 11,664 at (6, 3)
23. (a) $x^2y = 540$ (b) 3 units