Chapter 3 Matrices

1. What is the order of matrix *E*?

$$E = \begin{bmatrix} 2 & 0 & -1 & 5 \\ 8 & 4 & 2 & -2 \end{bmatrix}$$

A) Matrix *E* is 2 x 4.
B) Matrix *E* is 4 x 2.
C) Matrix *E* is 2 x 2.
D) Matrix *E* is 4 x 4.
E) Matrix *E* is 8 x 2.
Ans: A

2. Write a zero matrix that is the same order as *D*.

$$D = \begin{bmatrix} 3 & 2 \\ 0 & -1 \\ 8 & 2 \end{bmatrix}$$

A) $\begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}$

B) $\begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$

C) $\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$

D) $\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$

E) $\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$

Ans: A

3. Write the matrix that is the negative of matrix *B*.

$$B=\begin{bmatrix}1 & 4 & 3\\ 1 & 2 & 3\\ 3 & 1 & 0\end{bmatrix}$$

A)
$$-B=\begin{bmatrix}-1 & -4 & -3\\ 1 & 2 & 3\\ 3 & 1 & 0\end{bmatrix}$$

B)
$$-B=\begin{bmatrix}-1 & 4 & 3\\ -1 & 2 & 3\\ -3 & 1 & 0\end{bmatrix}$$

C)
$$-B=\begin{bmatrix}-1 & -4 & -3\\ -1 & -2 & -3\\ 3 & 1 & 0\end{bmatrix}$$

D)
$$-B=\begin{bmatrix}-1 & -4 & 3\\ -1 & -2 & 3\\ -3 & -1 & 0\end{bmatrix}$$

E)
$$-B=\begin{bmatrix}-1 & -4 & -3\\ -1 & -2 & -3\\ -3 & -1 & 0\end{bmatrix}$$

Ans: E

4. What is element b_{14}?

$$B=\begin{bmatrix}1 & 1 & 3 & 0\\ 4 & 2 & 1 & 1\\ 3 & 2 & 0 & 1\end{bmatrix}$$

A) $b_{14}=1$
B) $b_{14}=0$
C) $b_{14}=2$
D) $b_{14}=3$
E) $b_{14}=4$
Ans: B

5. Write the transpose of matrix A.

$$A = \begin{bmatrix} 1 & 0 & 3 \\ 3 & 2 & 1 \\ 3 & 0 & 4 \end{bmatrix}$$

A) $$A^T = \begin{bmatrix} 3 & 0 & 4 \\ 3 & 2 & 1 \\ 1 & 0 & 3 \end{bmatrix}$$

B) $$A^T = \begin{bmatrix} 3 & 0 & 1 \\ 1 & 2 & 3 \\ 4 & 0 & 3 \end{bmatrix}$$

C) $$A^T = \begin{bmatrix} -1 & 0 & -3 \\ -3 & -2 & -1 \\ -3 & 0 & -4 \end{bmatrix}$$

D) $$A^T = \begin{bmatrix} 1 & 3 & 3 \\ 0 & 2 & 0 \\ 3 & 1 & 4 \end{bmatrix}$$

E) $$A^T = \begin{bmatrix} 4 & 0 & 3 \\ 1 & 2 & 3 \\ 3 & 0 & 1 \end{bmatrix}$$

Ans: D

6. Write the transpose of matrix F.

$$F = \begin{bmatrix} 1 & 3 & 4 \\ -1 & 1 & 2 \\ 2 & -2 & -4 \end{bmatrix}$$

A) $$F^T = \begin{bmatrix} 1 & 2 & -1 \\ 3 & -2 & 1 \\ 4 & -4 & 2 \end{bmatrix}$$

B) $$F^T = \begin{bmatrix} 4 & 3 & 1 \\ 2 & 1 & -1 \\ -4 & -2 & 2 \end{bmatrix}$$

C) $$F^T = \begin{bmatrix} 1 & -1 & 2 \\ 3 & 1 & -2 \\ 4 & 2 & -4 \end{bmatrix}$$

D) $$F^T = \begin{bmatrix} -4 & -2 & 2 \\ 2 & 1 & -1 \\ 4 & 3 & 1 \end{bmatrix}$$

E) $$F^T = \begin{bmatrix} -1 & -3 & -4 \\ 1 & -1 & -2 \\ -2 & 2 & 4 \end{bmatrix}$$

Ans: C

7. What is the sum of matrix M and its negative?

$$M = \begin{bmatrix} 2 & 0 & 2 \\ 3 & 3 & 1 \\ 4 & 0 & 3 \end{bmatrix}$$

A) $$M + (-M) = \begin{bmatrix} -2 & 0 & -2 \\ -3 & -3 & -1 \\ -4 & 0 & -3 \end{bmatrix}$$

B) $$M + (-M) = \begin{bmatrix} -4 & 0 & -4 \\ -6 & -6 & -2 \\ -8 & 0 & -6 \end{bmatrix}$$

C) $$M + (-M) = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

D) $$M + (-M) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

E) $$M + (-M) = \begin{bmatrix} -2 & -3 & -4 \\ 0 & -3 & 0 \\ -2 & -1 & -3 \end{bmatrix}$$

Ans: C

8. If matrix *M* has element $M_{2j} = 1$, what is *j*?

$$M = \begin{bmatrix} 1 & 0 & 2 \\ 3 & 2 & 1 \\ 4 & 0 & 3 \end{bmatrix}$$

A) $j = 3$

B) $j = 2$

C) $j = 4$

D) $j = 0$

E) $j = 1$

Ans: A

9. Use the following matrices to perform the indicated matrix operations, if possible.

$C + D$

$C = \begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix} \qquad D = \begin{bmatrix} 4 & 1 \\ 3 & 5 \end{bmatrix}$

A) $C + D = \begin{bmatrix} 6 & 6 \\ 2 & 7 \end{bmatrix}$

B) $C + D = \begin{bmatrix} 8 & 3 \\ 3 & 10 \end{bmatrix}$

C) $C + D = \begin{bmatrix} -2 & 2 \\ -2 & -3 \end{bmatrix}$

D) $C + D = \begin{bmatrix} 6 & 3 \\ 4 & 7 \end{bmatrix}$

E) $C + D = \begin{bmatrix} 6 & 4 \\ 4 & 7 \end{bmatrix}$

Ans: E

10. Use the following matrices to perform the indicated matrix operations, if possible.

$A+F$

$$A=\begin{bmatrix} 1 & 0 & 2 \\ 3 & 2 & 1 \\ 4 & 1 & 3 \end{bmatrix} \quad F=\begin{bmatrix} 1 & 3 & 4 \\ -1 & 0 & 1 \\ 3 & 3 & -4 \end{bmatrix}$$

A) $$A+F=\begin{bmatrix} 0 & -3 & -2 \\ 4 & 2 & 0 \\ 1 & -2 & 7 \end{bmatrix}$$

B) $$A+F=\begin{bmatrix} 2 & 3 & 6 \\ 2 & 3 & 2 \\ 7 & 4 & -3 \end{bmatrix}$$

C) $$A+F=\begin{bmatrix} 1 & 0 & 8 \\ -3 & 0 & 1 \\ 12 & 3 & -12 \end{bmatrix}$$

D) $$A+F=\begin{bmatrix} 2 & 0 & 6 \\ 2 & 3 & 2 \\ 7 & 4 & -1 \end{bmatrix}$$

E) $$A+F=\begin{bmatrix} 2 & 3 & 6 \\ 2 & 2 & 2 \\ 7 & 4 & -1 \end{bmatrix}$$

Ans: E

11. Use the following matrices to perform the indicated matrix operations, if possible.

$A - F$

$$A = \begin{bmatrix} 1 & 1 & 3 \\ 3 & 2 & 2 \\ 4 & 0 & 3 \end{bmatrix} \qquad F = \begin{bmatrix} 1 & 2 & 4 \\ -1 & 0 & 0 \\ 3 & -3 & -4 \end{bmatrix}$$

A) $$A - F = \begin{bmatrix} 2 & 3 & 7 \\ 2 & 2 & 2 \\ 7 & -3 & -1 \end{bmatrix}$$

B) $$A - F = \begin{bmatrix} 0 & -1 & -1 \\ 4 & 2 & 2 \\ 1 & 3 & 7 \end{bmatrix}$$

C) $$A - F = \begin{bmatrix} 0 & -1 & 7 \\ 4 & 2 & 2 \\ 7 & 3 & 7 \end{bmatrix}$$

D) $$A - F = \begin{bmatrix} 0 & 1 & -1 \\ 4 & -2 & 2 \\ 1 & 3 & -7 \end{bmatrix}$$

E) $$A - F = \begin{bmatrix} 1 & 2 & 12 \\ -3 & 0 & 0 \\ 12 & 0 & -12 \end{bmatrix}$$

Ans: B

12. Use the following matrices to perform the indicated matrix operations, if possible.

$Z + E^T$

$$Z = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \qquad E = \begin{bmatrix} 4 & 0 & 1 \\ 2 & 3 & 2 \\ 4 & 1 & 2 \end{bmatrix}$$

A) $$Z + E^T = \begin{bmatrix} 4 & 2 & 4 \\ 0 & 3 & 1 \\ 1 & 2 & 2 \end{bmatrix}$$

B) $$Z + E^T = \begin{bmatrix} 1 & 0 & 4 \\ 2 & 3 & 2 \\ 2 & 1 & 4 \end{bmatrix}$$

C) $$Z + E^T = \begin{bmatrix} 4 & 1 & 2 \\ 2 & 3 & 2 \\ 4 & 0 & 1 \end{bmatrix}$$

D) $$Z + E^T = \begin{bmatrix} 0 & 3 & 1 \\ 1 & 2 & 2 \\ 4 & 2 & 4 \end{bmatrix}$$

E) $$Z + E^T = \begin{bmatrix} 1 & 2 & 2 \\ 0 & 3 & 1 \\ 4 & 2 & 4 \end{bmatrix}$$

Ans: A

13. Use the following matrices to perform the indicated matrix operations, if possible.

$A + A^T$

$$A = \begin{bmatrix} 1 & 1 & 3 \\ 3 & 2 & 3 \\ 0 & 2 & 1 \end{bmatrix}$$

A) $A + A^T = \begin{bmatrix} 2 & 2 & 6 \\ 6 & 4 & 6 \\ 0 & 4 & 2 \end{bmatrix}$

B) $A + A^T = \begin{bmatrix} 0 & -2 & 3 \\ 2 & 0 & 1 \\ -3 & -1 & 0 \end{bmatrix}$

C) $A + A^T = \begin{bmatrix} 2 & 4 & 3 \\ 4 & 4 & 5 \\ 3 & 5 & 2 \end{bmatrix}$

D) $A + A^T = \begin{bmatrix} 4 & 4 & 4 \\ 4 & 4 & 5 \\ 1 & 5 & 1 \end{bmatrix}$

E) $A + A^T = \begin{bmatrix} 2 & 1 & 3 \\ 4 & 4 & 5 \\ 3 & 3 & 2 \end{bmatrix}$

Ans: C

14. Use the following matrices to perform the indicated matrix operations, if possible.

$A + F^T$

$$A = \begin{bmatrix} 0 & 0 & 3 \\ 3 & 3 & 1 \\ 3 & 0 & 3 \end{bmatrix} \qquad F = \begin{bmatrix} 1 & 3 & 3 \\ 0 & 0 & 2 \\ 3 & -2 & -3 \end{bmatrix}$$

A) $$A + F^T = \begin{bmatrix} 1 & 3 & 6 \\ 3 & 3 & 3 \\ 6 & -2 & 0 \end{bmatrix}$$

B) $$A + F^T = \begin{bmatrix} 1 & 6 & 6 \\ 0 & 3 & 2 \\ 6 & -1 & 0 \end{bmatrix}$$

C) $$A + F^T = \begin{bmatrix} 1 & 3 & 6 \\ 3 & 3 & -2 \\ 6 & 3 & 0 \end{bmatrix}$$

D) $$A + F^T = \begin{bmatrix} 1 & 0 & 6 \\ 6 & 3 & -1 \\ 6 & 2 & 0 \end{bmatrix}$$

E) $$A + F^T = \begin{bmatrix} 1 & 3 & 3 \\ 6 & 3 & -1 \\ 6 & -3 & 5 \end{bmatrix}$$

Ans: D

15. Use the following matrices to perform the indicated matrix operations, if possible.
$B+F$

$$B=\begin{bmatrix}2 & 1 & 3 & 0\\ 4 & 3 & 1 & 2\\ 3 & 2 & 1 & 2\end{bmatrix} \qquad F=\begin{bmatrix}1 & 3 & 3\\ -1 & 1 & 1\\ 3 & -3 & -4\end{bmatrix}$$

A) $$B+F=\begin{bmatrix}3 & 4 & 6\\ 3 & 4 & 2\\ 6 & -1 & -3\end{bmatrix}$$

B) $$B+F=\begin{bmatrix}3 & 4 & 6 & 0\\ 3 & 4 & 3 & 2\\ 6 & -1 & -3 & 2\end{bmatrix}$$

C) $$B+F=\begin{bmatrix}2 & 2 & 6 & 3\\ 4 & 2 & 2 & 3\\ 3 & 5 & -2 & -2\end{bmatrix}$$

D) $$B+F=\begin{bmatrix}2 & 6 & 3\\ 2 & 2 & 3\\ 5 & -2 & -2\end{bmatrix}$$

E) $B+F$ is undefined

Ans: E

16. Use the following matrices to perform the indicated matrix operations, if possible.
$4D$

$$D=\begin{bmatrix}4 & 2\\ 4 & 5\end{bmatrix}$$

A) $$4D=\begin{bmatrix}16 & 8\\ 16 & 20\end{bmatrix}$$

B) $$4D=\begin{bmatrix}8 & 6\\ 8 & 9\end{bmatrix}$$

C) $$4D=\begin{bmatrix}16 & 8\\ 4 & 5\end{bmatrix}$$

D) $$4D=\begin{bmatrix}16 & 8\\ 16 & 5\end{bmatrix}$$

E) $$4D=\begin{bmatrix}16 & 8\\ 4 & 20\end{bmatrix}$$

Ans: A

17. Use the following matrices to perform the indicated matrix operations, if possible.

$3C+2D$

$C=\begin{bmatrix}5 & 3\\1 & 2\end{bmatrix}$ $\quad D=\begin{bmatrix}4 & 2\\3 & 5\end{bmatrix}$

A) $3C+2D=\begin{bmatrix}22 & 12\\11 & 19\end{bmatrix}$

B) $3C+2D=\begin{bmatrix}23 & 13\\9 & 16\end{bmatrix}$

C) $3C+2D=\begin{bmatrix}23 & 16\\9 & 13\end{bmatrix}$

D) $3C+2D=\begin{bmatrix}13 & 7\\7 & 12\end{bmatrix}$

E) $3C+2D=\begin{bmatrix}120 & 36\\18 & 60\end{bmatrix}$

Ans: B

18. Use the following matrices to perform the indicated matrix operations, if possible.

$8C-6D$

$C=\begin{bmatrix}5 & 3\\1 & 2\end{bmatrix}$ $\quad D=\begin{bmatrix}4 & 2\\3 & 5\end{bmatrix}$

A) $8C-6D=\begin{bmatrix}16 & 12\\26 & 46\end{bmatrix}$

B) $8C-6D=\begin{bmatrix}-2 & 2\\-18 & -28\end{bmatrix}$

C) $8C-6D=\begin{bmatrix}16 & 12\\-10 & -14\end{bmatrix}$

D) $8C-6D=\begin{bmatrix}-16 & -12\\10 & 14\end{bmatrix}$

E) $8C-6D=\begin{bmatrix}36 & 22\\5 & 11\end{bmatrix}$

Ans: C

19. Use the following matrices to perform the indicated matrix operations, if possible.
$6E+9F$

$$E=\begin{bmatrix} 2 & 1 & 1 \\ 1 & 0 & 4 \\ 5 & 1 & 0 \end{bmatrix} \qquad F=\begin{bmatrix} 1 & 2 & 3 \\ -1 & 0 & 1 \\ 2 & -3 & -4 \end{bmatrix}$$

A) $$6E-9F=\begin{bmatrix} 24 & 21 & 27 \\ 3 & 0 & 42 \\ 57 & -9 & -24 \end{bmatrix}$$

B) $$6E-9F=\begin{bmatrix} 3 & -12 & -21 \\ 15 & 0 & 15 \\ 12 & 33 & 36 \end{bmatrix}$$

C) $$6E-9F=\begin{bmatrix} 21 & 24 & 33 \\ -3 & 0 & 33 \\ 48 & -21 & -36 \end{bmatrix}$$

D) $$6E-9F=\begin{bmatrix} 21 & 24 & 33 \\ 15 & 0 & 33 \\ 48 & 33 & 36 \end{bmatrix}$$

E) $$6E-9F=\begin{bmatrix} 108 & 108 & 162 \\ 54 & 0 & 216 \\ 540 & 162 & -36 \end{bmatrix}$$

Ans: C

20. Find x, y, z, and w.

$$\begin{bmatrix} 0 & x & 1 \\ 3 & y & y \\ z & 1 & 3 \end{bmatrix}=\begin{bmatrix} 0 & 5 & 1 \\ 3 & 1 & y \\ 1 & 1 & w \end{bmatrix}$$

A) $x=5, y=3, z=1,$ and $w=1$
B) $x=1, y=3, z=1,$ and $w=3$
C) $x=0, y=1, z=1,$ and $w=3$
D) $x=5, y=1, z=1,$ and $w=3$
E) $x=1, y=5, z=3,$ and $w=1$
Ans: D

21. Find x, y, z, and w.

$$\begin{bmatrix} x & y & (x+4) \\ z & 5 & 3y \end{bmatrix} = \begin{bmatrix} (2x-3) & -2 & w \\ x & (7+y) & -6 \end{bmatrix}$$

A) $x=-3, y=-2, z=3,$ and $w=1$
B) $x=3, y=-0.5, z=-3,$ and $w=-1$
C) $x=3, y=2, z=-3,$ and $w=1$
D) $x=-3, y=-2, z=-6,$ and $w=7$
E) $x=3, y=-2, z=3,$ and $w=7$
Ans: E

22. Solve for x, y, and z.

$$2\begin{bmatrix} x & y \\ y & z \end{bmatrix} + 4\begin{bmatrix} 2x & -y \\ 3y & -4z \end{bmatrix} = \begin{bmatrix} 30 & 6+y \\ -28 & -42 \end{bmatrix}$$

A) $x=3, y=-2,$ and $z=3$
B) $x=3, y=2,$ and $z=-3$
C) $x=-3, y=-2,$ and $z=-3$
D) $x=-3, y=2,$ and $z=3$
E) $x=-3, y=2,$ and $z=-3$
Ans: A

23. Find x, y, z, and w.

$$3\begin{bmatrix} x & 4 \\ 4y & w \end{bmatrix} - 2\begin{bmatrix} 4x & 2z \\ -3 & -2w \end{bmatrix} = \begin{bmatrix} 25 & -8 \\ 42 & 21 \end{bmatrix}$$

A) $x=-5, y=-3, z=-5,$ and $w=-3$
B) $x=5, y=3, z=-5,$ and $w=3$
C) $x=-5, y=3, z=5,$ and $w=3$
D) $x=-5, y=3, z=-5,$ and $w=3$
E) $x=5, y=-3, z=-5,$ and $w=-3$
Ans: C

24. The tables below give the numbers of some species of threatened and endangered wildlife in the United States and in foreign countries in 2003. Write a matrix A that contains the number of each of these species in the United States in 2003 and a matrix B that contains the number of each of these species outside the United States in 2003.

United States

	Mammals	Birds	Reptiles	Amphibians	Fish
Endangered	65	78	14	12	71
Threatened	18	14	32	9	44

Foreign

	Mammals	Birds	Reptiles	Amphibians	Fish
Endangered	262	175	64	8	11
Threatened	17	12	15	5	0

A) $A = \begin{bmatrix} 262 & 175 & 64 & 8 & 11 \\ 17 & 12 & 15 & 5 & 0 \end{bmatrix}$ $B = \begin{bmatrix} 65 & 78 & 14 & 12 & 71 \\ 18 & 14 & 32 & 9 & 44 \end{bmatrix}$

B) $A = \begin{bmatrix} 262 & 64 & 175 & 8 & 11 \\ 17 & 12 & 5 & 15 & 0 \end{bmatrix}$ $B = \begin{bmatrix} 65 & 78 & 12 & 14 & 71 \\ 18 & 32 & 14 & 9 & 44 \end{bmatrix}$

C) $A = \begin{bmatrix} 17 & 12 & 15 & 5 & 0 \\ 262 & 175 & 64 & 8 & 11 \end{bmatrix}$ $B = \begin{bmatrix} 65 & 78 & 14 & 12 & 71 \\ 18 & 14 & 32 & 9 & 44 \end{bmatrix}$

D) $A = \begin{bmatrix} 65 & 78 & 14 & 12 & 71 \\ 18 & 14 & 32 & 9 & 44 \end{bmatrix}$ $B = \begin{bmatrix} 262 & 175 & 64 & 8 & 11 \\ 17 & 12 & 15 & 5 & 0 \end{bmatrix}$

E) $A = \begin{bmatrix} 18 & 14 & 32 & 9 & 44 \\ 65 & 78 & 14 & 12 & 71 \end{bmatrix}$ $B = \begin{bmatrix} 17 & 12 & 15 & 5 & 0 \\ 262 & 175 & 64 & 8 & 11 \end{bmatrix}$

Ans: D

25. The tables below give the numbers of some species of threatened and endangered wildlife in the United States and in foreign countries in 2003. Find a matrix with the total number of these species. Assume that U.S. and foreign species are different.

United States

	Mammals	Birds	Reptiles	Amphibians	Fish
Endangered	65	82	14	12	71
Threatened	9	14	22	9	44

Foreign

	Mammals	Birds	Reptiles	Amphibians	Fish
Endangered	251	180	64	15	11
Threatened	17	12	15	5	0

A) $\begin{bmatrix} 130 & 164 & 28 & 24 & 142 \\ 18 & 28 & 44 & 18 & 88 \end{bmatrix}$

B) $\begin{bmatrix} 316 & 262 & 78 & 27 & 82 \\ 26 & 26 & 37 & 14 & 44 \end{bmatrix}$

C) $\begin{bmatrix} 502 & 360 & 128 & 30 & 22 \\ 34 & 24 & 30 & 10 & 0 \end{bmatrix}$

D) $\begin{bmatrix} 82 & 94 & 29 & 17 & 71 \\ 260 & 194 & 86 & 24 & 55 \end{bmatrix}$

E) $\begin{bmatrix} 316 & 94 & 78 & 17 & 82 \\ 260 & 26 & 86 & 14 & 44 \end{bmatrix}$

Ans: B

26. The tables below give the numbers of some species of threatened and endangered wildlife in the United States and in foreign countries in 2003. Let A represent the matrix that contains the number of each of these species in the United States in 2003 and B represent the matrix that contains the number of each of these species outside the United States in 2003. Find the matrix $B-A$. What do the negative entries in matrix $B-A$ mean?

United States

	Mammals	Birds	Reptiles	Amphibians	Fish
Endangered	65	82	14	12	71
Threatened	18	14	32	9	44

Foreign

	Mammals	Birds	Reptiles	Amphibians	Fish
Endangered	262	175	64	8	11
Threatened	17	12	15	5	0

A) $$B-A=\begin{bmatrix} -197 & -93 & -50 & 4 & 60 \\ 1 & 2 & 17 & 4 & 44 \end{bmatrix}$$

The negative entries tell us that that particular species is extinct.

B) $$B-A=\begin{bmatrix} 197 & 93 & 50 & -4 & -60 \\ -1 & -2 & -17 & -4 & -44 \end{bmatrix}$$

The negative entries tell us how many more endangered or threatened species are in foreign countries.

C) $$B-A=\begin{bmatrix} -197 & -93 & -50 & 4 & 60 \\ 1 & 2 & 17 & 4 & 44 \end{bmatrix}$$

The negative entries tell us that there are more endangered or thereatened species in foreign countries.

D) $$B-A=\begin{bmatrix} 244 & 161 & 32 & -1 & -33 \\ -48 & -70 & 1 & -7 & -71 \end{bmatrix}$$

The negative entries tell us that that particular species is extinct.

E) $$B-A=\begin{bmatrix} 197 & 93 & 50 & -4 & -60 \\ -1 & -2 & -17 & -4 & -44 \end{bmatrix}$$

The negative entries tell us how many more endangered or threatened species are in the United States.

Ans: E

27. The following tables give the rank and number of registered dogs for the top ten breeds for 1995 and their 2002 data. Form a matrix for the 1995 data and a matrix for the 2002 data.

1995

Breed	Rank	Number Registered
Labrador retriever	1	134,051
Rottweiler	2	94,350
German shepherd	3	76,588
Golden retriever	4	64,407
Beagle	5	57,107
Poodle	6	56,284
Cocker spaniel	7	48,465
Dachshund	8	44,880
Pomeranian	9	38,195
Yorkshire terrier	10	36,956

2002

Breed	**Rank**	**Number Registered**
Labrador retriever	1	155,416
Rottweiler	13	22,696
German shepherd	3	47,363
Golden retriever	2	56,224
Beagle	4	44,710
Poodle	8	34,067
Cocker spaniel	15	20,755
Dachshund	5	44,071
Pomeranian	12	24,061
Yorkshire terrier	6	39,077

A) $$A=\begin{bmatrix} 1 & 155,416 \\ 13 & 22,696 \\ 3 & 47,363 \\ 2 & 56,224 \\ 4 & 44,710 \\ 8 & 34,067 \\ 15 & 20,755 \\ 5 & 44,071 \\ 12 & 24,061 \\ 6 & 39,077 \end{bmatrix} \quad B=\begin{bmatrix} 1 & 134,051 \\ 2 & 94,350 \\ 3 & 76,588 \\ 4 & 64,407 \\ 5 & 57,107 \\ 6 & 56,284 \\ 7 & 48,465 \\ 8 & 44,880 \\ 9 & 38,195 \\ 10 & 36,956 \end{bmatrix}$$

B) $A = \begin{bmatrix} 1 & 134,051 \\ 13 & 94,350 \\ 3 & 76,588 \\ 2 & 64,407 \\ 4 & 57,107 \\ 8 & 56,284 \\ 15 & 48,465 \\ 5 & 44,880 \\ 12 & 38,195 \\ 6 & 36,956 \end{bmatrix}$ $B = \begin{bmatrix} 1 & 155,416 \\ 2 & 22,696 \\ 3 & 47,363 \\ 4 & 56,224 \\ 5 & 44,710 \\ 6 & 34,067 \\ 7 & 20,755 \\ 8 & 44,071 \\ 9 & 24,061 \\ 10 & 39,077 \end{bmatrix}$

C) $A = \begin{bmatrix} 1 & 134,051 \\ 2 & 94,350 \\ 3 & 76,588 \\ 4 & 64,407 \\ 5 & 57,107 \\ 6 & 56,284 \\ 7 & 48,465 \\ 8 & 44,880 \\ 9 & 38,195 \\ 10 & 36,956 \end{bmatrix}$ $B = \begin{bmatrix} 1 & 155,416 \\ 13 & 22,696 \\ 3 & 47,363 \\ 2 & 56,224 \\ 4 & 44,710 \\ 8 & 34,067 \\ 15 & 20,755 \\ 5 & 44,071 \\ 12 & 24,061 \\ 6 & 39,077 \end{bmatrix}$

D) $A = \begin{bmatrix} 1 & 1 \\ 2 & 13 \\ 3 & 3 \\ 4 & 2 \\ 5 & 4 \\ 6 & 8 \\ 7 & 15 \\ 8 & 5 \\ 9 & 12 \\ 10 & 6 \end{bmatrix}$ $B = \begin{bmatrix} 134,051 & 155,416 \\ 94,350 & 22,696 \\ 76,588 & 47,363 \\ 64,407 & 56,224 \\ 57,107 & 44,710 \\ 56,284 & 34,067 \\ 48,465 & 20,755 \\ 44,880 & 44,071 \\ 38,195 & 24,061 \\ 36,956 & 39,077 \end{bmatrix}$

E)

$$A=\begin{bmatrix} 1 & 134,051 \\ 2 & 76,588 \\ 3 & 94,350 \\ 4 & 64,407 \\ 5 & 57,107 \\ 6 & 56,284 \\ 7 & 44,880 \\ 8 & 48,465 \\ 9 & 38,195 \\ 10 & 36,956 \end{bmatrix} \quad B=\begin{bmatrix} 1 & 22,696 \\ 13 & 155,416 \\ 3 & 47,363 \\ 2 & 56,224 \\ 4 & 34,067 \\ 8 & 44,710 \\ 15 & 20,755 \\ 5 & 24,061 \\ 12 & 44,071 \\ 6 & 39,077 \end{bmatrix}$$

Ans: C

28. The following tables give the rank and number of registered dogs for the top ten breeds for 1995 and their 2002 data. Use a matrix operation to find the change from 1995 to 2002 in rank for each breed and in registration numbers for each breed.

1995

Breed	Rank	Number Registered
Labrador retriever	1	132,051
Rottweiler	2	95,650
German shepherd	3	77,088
Golden retriever	4	64,107
Beagle	5	57,107
Poodle	6	54,784
Cocker spaniel	7	48,065
Dachshund	8	44,680
Pomeranian	9	37,895
Yorkshire terrier	10	36,881

2002

Breed	**Rank**	**Number Registered**
Labrador retriever	1	154,616
Rottweiler	13	22,196
German shepherd	3	46,963
Golden retriever	2	59,124
Beagle	4	44,610
Poodle	8	33,917
Cocker spaniel	15	20,655
Dachshund	5	42,571
Pomeranian	12	23,061
Yorkshire terrier	6	37,277

A) $$\begin{bmatrix} 0 & 22,565 \\ 11 & -73,454 \\ 0 & -30,125 \\ -2 & -4,983 \\ -1 & -12,497 \\ 2 & -20,867 \\ 8 & -27,410 \\ -3 & -2,109 \\ 3 & -14,834 \\ -4 & 396 \end{bmatrix}$$

B) $\begin{bmatrix} 0 & -22,565 \\ -11 & 73,454 \\ 0 & 30,125 \\ 2 & 4,983 \\ 1 & 12,497 \\ -2 & 20,867 \\ -8 & 27,410 \\ 3 & 2,109 \\ -3 & 14,834 \\ 4 & -396 \end{bmatrix}$

C) $\begin{bmatrix} 2 & 286,667 \\ 15 & 117,846 \\ 6 & 124,051 \\ 6 & 123,231 \\ 9 & 101,717 \\ 14 & 88,701 \\ 22 & 68,720 \\ 13 & 87,251 \\ 21 & 60,956 \\ 16 & 74,158 \end{bmatrix}$

D) $\begin{bmatrix} 0 & 22,565 \\ 11 & 117,846 \\ 0 & -30,125 \\ 2 & -4,983 \\ -1 & 101,717 \\ -2 & -20,867 \\ 8 & -27,410 \\ 3 & 87,251 \\ -3 & -14,834 \\ -4 & 396 \end{bmatrix}$

E) $\begin{bmatrix} 22,565 & 0 \\ -73,454 & 11 \\ -30,125 & 0 \\ -4,983 & -2 \\ -12,497 & -1 \\ -20,867 & 2 \\ -27,410 & 8 \\ -2,109 & -3 \\ -14,834 & 3 \\ 396 & -4 \end{bmatrix}$

Ans: A

29. The following tables give the rank and number of registered dogs for the top ten breeds for 1995 and their 2002 data. Which breed had the greatest improvement in rank?

1995

Breed	Rank	Number Registered
Labrador retriever	1	132,051
Rottweiler	2	95,650
German shepherd	3	77,088
Golden retriever	4	64,107
Beagle	5	57,107
Poodle	6	54,784
Cocker spaniel	7	50,065
Dachshund	8	44,680
Pomeranian	9	37,895
Yorkshire terrier	10	37,001

2002

Breed	**Rank**	**Number Registered**
Labrador retriever	3	37,277
Rottweiler	2	23,061
German shepherd	4	42,571
Golden retriever	8	20,655
Beagle	15	33,917
Poodle	5	44,610
Cocker spaniel	12	56,124
Dachshund	6	35,243
Pomeranian	1	22,196
Yorkshire terrier	13	154,616

A) Yorkshire terrier
B) Beagle
C) Rottweiler
D) Labrador retriever
E) German shepherd
Ans: B

30. The following tables give the rank and number of registered dogs for the top ten breeds for 1995 and their 2002 data. Which breed had the largest increase in the number of registrations?

1995

Breed	Rank	Number Registered
Labrador retriever	1	132,051
Rottweiler	2	93,650
German shepherd	3	76,088
Golden retriever	4	64,107
Beagle	5	57,107
Poodle	6	54,784
Cocker spaniel	7	48,065
Dachshund	8	44,680
Pomeranian	9	37,895
Yorkshire terrier	10	36,881

2002

Breed	**Rank**	**Number Registered**
Labrador retriever	6	37,277
Rottweiler	12	23,061
German shepherd	5	42,571
Golden retriever	15	20,655
Beagle	8	33,917
Poodle	4	44,610
Cocker spaniel	2	56,124
Dachshund	7	35,243
Pomeranian	13	22,196
Yorkshire terrier	1	154,616

A) Labrador retriever
B) Dachshund
C) Pomeranian
D) Rottweiler
E) Yorkshire terrier
Ans: E

31. The following tables give the capital expenditures and gross operating costs of manufacturing establishments for pollution abatement, in millions of dollars. Use a calculator or spreadsheet to find the sum of the matrices containing these data. This will yield the total costs of manufacturing establishments for air, water, and solid contained waste pollution abatement for the years 2002–2005.

Pollution Abatement Capital Expenditures

	Air	Water	Solid Contained Waste
2002	6230.8	2562.0	917.5
2003	3706.3	2914.6	869.1
2004	4603.1	2509.8	953.9
2005	4122.0	2294.9	760.9

Pollution Abatement Gross Operating Costs

	Air	Water	Solid Contained Waste
2002	5010.9	6616.4	5643.5
2003	5033.5	6345.0	6208.2
2004	5395.0	6776.9	5494.5
2005	5774.6	6631.8	5548.6

A) $\begin{bmatrix} 1219.9 & -4054.4 & -4726.0 \\ -1327.2 & -3430.4 & -5339.1 \\ -791.9 & -4267.1 & -4540.6 \\ -1652.6 & -4336.9 & -4787.7 \end{bmatrix}$

B) $\begin{bmatrix} 11241.7 & 7572.9 & 6561.0 \\ 8717.2 & 9259.6 & 6512.6 \\ 9517.0 & 9141.6 & 6363.6 \\ 9896.6 & 8926.7 & 6309.5 \end{bmatrix}$

C) $\begin{bmatrix} -1219.9 & 4054.4 & 4726.0 \\ 1327.2 & 3430.4 & 5339.1 \\ 791.9 & 4267.1 & 4540.6 \\ 1652.6 & 4336.9 & 4787.7 \end{bmatrix}$

D) $\begin{bmatrix} 11241.7 & 9178.4 & 6561.0 \\ 8739.8 & 9259.6 & 7077.3 \\ 9998.1 & 9286.7 & 6448.4 \\ 9896.6 & 8926.7 & 6309.5 \end{bmatrix}$

E) $\begin{bmatrix} 11241.7 & 9178.4 & 6561.0 \\ 10051.3 & 9259.6 & 7077.3 \\ 9998.1 & 8004.3 & 7730.8 \\ 10377.7 & 7843.5 & 6309.5 \end{bmatrix}$

Ans: D

32. The following tables give the capital expenditures and gross operating costs of manufacturing establishments for pollution abatement, in millions of dollars. Which type of pollution abatement was most expensive in 2005?

Pollution Abatement Capital Expenditures

	Air	Water	Solid Contained Waste
2002	6030.8	2712.0	917.5
2003	3806.3	2864.6	969.1
2004	4503.1	2634.8	953.9
2005	4122.0	2294.9	760.9

Pollution Abatement Gross Operating Costs

	Air	Water	Solid Contained Waste
2002	5060.9	6616.4	5693.5
2003	5133.5	6495.0	6008.2
2004	5395.0	6626.9	5494.5
2005	5574.6	6631.8	5348.6

A) Air
B) Solid Contained Waste
C) Water
Ans: A

33. The following tables give the death rates, per 100,000 population, by age for selected years for males and females. If matrix M gives the male data and matrix F gives the female data, use matrix operations to find the death rate per 100,000 for all people in the age categories given and for the given years.

Males

	Under 1	**1–4**	**5–14**	**15–24**	**25–34**	**35–44**	**45–54**
1970	2410	93	51	189	215	403	959
1980	1439	73	47	172	196	309	767
1990	1083	62	29	147	204	310	610
1994	899	52	26	161	207	337	585
2001	765	37	20	117	143	259	544

Females

	Under 1	**1–4**	**5–14**	**15–24**	**25–34**	**35–44**	**45–54**
1970	1864	75	32	78	102	231	517
1980	1142	55	34	58	86	159	413
1990	856	51	19	49	74	148	343
1994	729	37	19	47	76	144	336
2001	620	39	15	43	76	148	317

A) $\begin{bmatrix} 546 & 18 & 19 & 111 & 113 & 172 & 442 \\ 297 & 18 & 13 & 114 & 110 & 150 & 354 \\ 227 & 11 & 10 & 98 & 130 & 162 & 267 \\ 170 & 15 & 7 & 114 & 131 & 193 & 249 \\ 145 & -2 & 5 & 74 & 67 & 111 & 227 \end{bmatrix}$

B) $\begin{bmatrix} -546 & -18 & -19 & -111 & -113 & -172 & -442 \\ -297 & -18 & -13 & -114 & -110 & -150 & -354 \\ -227 & -11 & -10 & -98 & -130 & -162 & -267 \\ -170 & -15 & -7 & -114 & -131 & -193 & -249 \\ -145 & 2 & -5 & -74 & -67 & -111 & -227 \end{bmatrix}$

C) $\begin{bmatrix} 4274 & 168 & 83 & 267 & 317 & 634 & 1476 \\ 2581 & 128 & 81 & 230 & 282 & 468 & 1180 \\ 1939 & 113 & 48 & 196 & 278 & 458 & 953 \\ 1628 & 89 & 45 & 208 & 283 & 481 & 921 \\ 1385 & 76 & 35 & 160 & 219 & 407 & 861 \end{bmatrix}$

D) $\begin{bmatrix} 4274 & 168 & 83 & 267 & 113 & 634 & 1476 \\ 2581 & 128 & 81 & 230 & 282 & 150 & 1180 \\ 1939 & 113 & 48 & 98 & 278 & 458 & 953 \\ 1628 & 89 & 7 & 208 & 283 & 481 & 921 \\ 1385 & 76 & 35 & 160 & 219 & 111 & 861 \end{bmatrix}$

E) $$\begin{bmatrix} 4274 & 148 & 109 & 267 & 317 & 634 & 1476 \\ 2581 & 128 & 66 & 250 & 282 & 468 & 1180 \\ 1939 & 113 & 48 & 194 & 281 & 454 & 953 \\ 1519 & 91 & 45 & 208 & 283 & 481 & 928 \\ 1385 & 76 & 41 & 160 & 219 & 407 & 861 \end{bmatrix}$$

Ans: C

34. The following tables give the death rates, per 100,000 population, by age for selected years for males and females. If matrix M gives the male data and matrix F gives the female data, find matrix M - F and describe what it means.

Males

	Under 1	**1–4**	**5–14**	**15–24**	**25–34**	**35–44**	**45–54**
1970	2410	93	51	189	215	403	959
1980	1439	73	47	172	196	309	767
1990	1083	62	39	147	204	310	610
1994	899	52	26	161	207	347	585
2001	755	37	20	127	143	259	544

Females

	Under 1	**1–4**	**5–14**	**15–24**	**25–34**	**35–44**	**45–54**
1970	1864	75	32	68	102	231	527
1980	1142	55	34	58	86	159	413
1990	856	51	19	49	74	138	343
1994	729	37	19	47	76	144	326
2001	620	39	25	43	66	148	317

A)

$$M-F=\begin{bmatrix} -546 & -18 & -19 & -121 & -113 & -172 & -432 \\ -297 & -18 & -13 & -114 & -110 & -150 & -354 \\ -227 & -11 & -20 & -98 & -130 & -172 & -267 \\ -170 & -15 & -7 & -114 & -131 & -203 & -259 \\ -135 & 2 & 5 & -84 & -77 & -111 & -227 \end{bmatrix}$$

$M-F$ represents how much greater the numbers are for males in each category per

B)

$$M-F=\begin{bmatrix} 546 & 18 & 19 & 121 & 113 & 172 & 432 \\ 297 & 18 & 13 & 114 & 110 & 150 & 354 \\ 227 & 11 & 20 & 98 & 130 & 172 & 267 \\ 170 & 15 & 7 & 114 & 131 & 203 & 259 \\ 135 & -2 & -5 & 84 & 77 & 111 & 227 \end{bmatrix}$$

$M-F$ represents how much greater the numbers are for males in each category per

C)

$$M-F=\begin{bmatrix} -546 & -18 & -19 & -121 & -113 & -172 & -432 \\ -297 & -18 & -13 & -114 & -110 & -150 & -354 \\ -227 & -11 & -20 & -98 & -130 & -172 & -267 \\ -170 & -15 & -7 & -114 & -131 & -203 & -259 \\ -135 & 2 & 5 & -84 & -77 & -111 & -227 \end{bmatrix}$$

$M-F$ represents how much greater the numbers are for females in each category p

D)
$$M - F = \begin{bmatrix} 546 & 18 & 19 & 121 & 113 & 172 & 432 \\ 297 & 18 & 13 & 114 & 110 & 150 & 354 \\ 227 & 11 & 20 & 98 & 130 & 172 & 267 \\ 170 & 15 & 7 & 114 & 131 & 203 & 259 \\ 135 & -2 & -5 & 84 & 77 & 111 & 227 \end{bmatrix}$$

$M - F$ represents how much greater the numbers are for females in each category p

E)
$$M - F = \begin{bmatrix} 546 & 18 & -19 & -121 & -113 & -244 & -432 \\ -297 & 20 & -13 & -89 & -110 & -150 & -354 \\ -43 & -43 & -20 & -112 & -130 & -172 & -472 \\ -26 & -15 & -1 & -114 & -131 & -203 & -441 \\ -135 & 2 & 45 & -84 & -193 & -111 & -227 \end{bmatrix}$$

$M - F$ represents only the male death rates, per 100,000 population by age.

Ans: B

35. Let matrix A represent the sales (in thousands of dollars) for the Walbash Company in 2004 in various cities, and let matrix B represent the sales (in thousands of dollars) for the same company in 2005 in the same cities. Write the matrix that represents the total sales by type (wholesale and retail) and city (Chicago, Atlanta, and Memphis) for both years.

$$\begin{array}{cc} & \begin{array}{ccc} \text{Chicago} & \text{Atlanta} & \text{Memphis} \end{array} \\ A = & \begin{bmatrix} 450 & 330 & 550 \\ 400 & 400 & 200 \end{bmatrix} \begin{array}{c} \text{Wholesale} \\ \text{Retail} \end{array} \end{array}$$

$$\begin{array}{cc} & \begin{array}{ccc} \text{Chicago} & \text{Atlanta} & \text{Memphis} \end{array} \\ B = & \begin{bmatrix} 375 & 350 & 710 \\ 410 & 350 & 200 \end{bmatrix} \begin{array}{c} \text{Wholesale} \\ \text{Retail} \end{array} \end{array}$$

A) $\begin{bmatrix} 75 & -20 & -160 \\ -10 & 50 & 0 \end{bmatrix}$

B) $\begin{bmatrix} -75 & 20 & 160 \\ 10 & -50 & 0 \end{bmatrix}$

C) $\begin{bmatrix} 825 & -20 & -160 \\ 810 & 50 & 400 \end{bmatrix}$

D) $\begin{bmatrix} 825 & 680 & 1260 \\ 810 & 750 & 400 \end{bmatrix}$

E) $\begin{bmatrix} 825 & 680 & 1260 \\ 810 & 730 & 910 \end{bmatrix}$

Ans: D

36. Let matrix A represent the sales (in thousands of dollars) for the Walbash Company in 2004 in various cities, and let matrix B represent the sales (in thousands of dollars) for the same company in 2005 in the same cities. Write the matrix that represents the change in sales by type (wholesale and retail) and city (Chicago, Atlanta, and Memphis) from 2004 to 2005.

$$A = \begin{array}{c} \begin{array}{ccc} \text{Chicago} & \text{Atlanta} & \text{Memphis} \end{array} \\ \begin{bmatrix} 500 & 280 & 600 \\ 450 & 350 & 200 \end{bmatrix} \end{array} \begin{array}{l} \\ \text{Wholesale} \\ \text{Retail} \end{array}$$

$$B = \begin{array}{c} \begin{array}{ccc} \text{Chicago} & \text{Atlanta} & \text{Memphis} \end{array} \\ \begin{bmatrix} 375 & 300 & 760 \\ 410 & 350 & 250 \end{bmatrix} \end{array} \begin{array}{l} \\ \text{Wholesale} \\ \text{Retail} \end{array}$$

A) $\begin{bmatrix} 125 & -20 & -160 \\ 40 & 0 & -50 \end{bmatrix}$

B) $\begin{bmatrix} -125 & 20 & 160 \\ -40 & 0 & 50 \end{bmatrix}$

C) $\begin{bmatrix} 875 & -20 & -160 \\ 860 & 0 & 450 \end{bmatrix}$

D) $\begin{bmatrix} 875 & 580 & 1360 \\ 860 & 700 & 450 \end{bmatrix}$

E) $\begin{bmatrix} -125 & 70 & 160 \\ -40 & -70 & 560 \end{bmatrix}$

Ans: B

37. A poll of 3420 people revealed that of the respondents who were registered Republicans, 893 approved of the president's job performance, 307 did not, and 135 had no opinion. Of the registered Democrats, 92 approved of the president's job performance, 426 did not, and 451 had no opinion. Of those registered as Independents, 127 approved, 64 did not approve, and 751 had no opinion. Of the remaining respondents, who were not registered, 92 approved, 38 did not approve, and 44 had no opinion. Represent these data in a 3×4 matrix.

A) $\begin{bmatrix} 893 & 426 & 751 \\ 307 & 451 & 92 \\ 135 & 127 & 38 \\ 92 & 64 & 44 \end{bmatrix}$

B) $\begin{bmatrix} 3420 & 893 & 307 & 135 \\ 3420 & 92 & 426 & 451 \\ 3420 & 127 & 64 & 751 \\ 3420 & 92 & 38 & 44 \end{bmatrix}$

C) $\begin{bmatrix} 893 & 307 & 135 & 92 \\ 426 & 451 & 127 & 64 \\ 751 & 92 & 38 & 44 \end{bmatrix}$

D) $\begin{bmatrix} 893 & 307 & 135 \\ 92 & 426 & 451 \\ 127 & 64 & 751 \\ 92 & 38 & 44 \end{bmatrix}$

E) $\begin{bmatrix} 893 & 92 & 127 & 92 \\ 307 & 426 & 64 & 38 \\ 135 & 451 & 751 & 44 \end{bmatrix}$

Ans: E

38. From the data in the following table, make a matrix A that gives the value (in billions of dollars) of U.S. imports from the various country groupings in the years 2000 and 2002.

Imports to U.S. from Country Groupings ($billions)

	2000	2002
North America	386.8	363.0
Western Europe	260.7	266.0
South/Central America	73.3	69.5
Pacific Rim	438.0	393.5

Exports from U.S. to Country Groupings ($billions)

	2000	2002
North America	310.3	258.4
Western Europe	181.5	157.0
South/Central America	59.3	51.6
Pacific Rim	222.6	178.6

A) $$A = \begin{bmatrix} 386.8 & 363.0 \\ 260.7 & 266.0 \\ 73.3 & 69.5 \\ 438.0 & 393.5 \end{bmatrix}$$

B) $$A = \begin{bmatrix} 310.3 & 258.4 \\ 181.5 & 157.0 \\ 59.3 & 51.6 \\ 222.6 & 178.6 \end{bmatrix}$$

C) $$A = \begin{bmatrix} 386.8 & 363.0 & 310.3 & 258.4 \\ 260.7 & 266.0 & 181.5 & 157.0 \\ 73.3 & 69.5 & 59.3 & 51.6 \\ 438.0 & 393.5 & 222.6 & 178.6 \end{bmatrix}$$

D) $$A = \begin{bmatrix} 386.8 & 363.0 & 260.7 & 266.0 \\ 73.3 & 69.5 & 438.0 & 393.5 \\ 310.3 & 258.4 & 181.5 & 157.0 \\ 59.3 & 51.6 & 222.6 & 178.6 \end{bmatrix}$$

E) $$A = \begin{bmatrix} 386.8 & 260.7 & 73.3 & 438.0 \\ 363.0 & 266.0 & 69.5 & 393.5 \end{bmatrix}$$

Ans: A

39. Using the data in the following tables, Let matrix A represent imports (in billions of dollars) to U.S. from country groupings, and let matrix B represent exports (in billions of dollars) from U.S. to country groupings. Find the U.S. trade balance with each country grouping in each year by finding matrix $B - A$.

Imports to U.S. from Country Groupings ($billions)

	2000	2002
North America	386.8	363.0
Western Europe	240.7	246.0
South/Central America	73.3	69.5
Pacific Rim	418.0	393.5

Exports from U.S. to Country Groupings ($billions)

	2000	2002
North America	290.3	258.4
Western Europe	201.5	157.0
South/Central America	79.3	71.6
Pacific Rim	222.6	198.6

A)
$$B - A = \begin{bmatrix} 677.1 & 621.4 \\ 442.2 & 403.0 \\ 152.6 & 141.1 \\ 640.6 & 592.1 \end{bmatrix}$$

B)
$$B - A = \begin{bmatrix} 96.5 & 104.6 \\ 39.2 & 89.0 \\ -6.0 & -2.1 \\ 195.4 & 194.9 \end{bmatrix}$$

C)
$$B - A = \begin{bmatrix} -96.5 & 621.4 \\ -39.2 & -89.0 \\ 152.6 & 2.1 \\ -195.4 & 592.1 \end{bmatrix}$$

D)
$$B - A = \begin{bmatrix} -96.5 & -104.6 \\ -39.2 & -89.0 \\ 6.0 & 2.1 \\ -195.4 & -194.9 \end{bmatrix}$$

E)
$$B - A = \begin{bmatrix} -96.5 & -104.6 \\ 39.2 & -83.7 \\ 6.0 & -2.1 \\ 149.3 & -194.9 \end{bmatrix}$$

Ans: D

40. Using the data in the following tables, Let matrix A represent imports (in billions of dollars) to U.S. from country groupings, and let matrix B represent exports (in billions of dollars) from U.S. to country groupings. Find the total U.S. trade with these groupings by finding $A + B$.

Imports to U.S. from Country Groupings ($billions)

	2000	**2002**
North America	366.8	343.0
Western Europe	240.7	246.0
South/Central America	93.3	69.5
Pacific Rim	418.0	393.5

Exports from U.S. to Country Groupings ($billions)

	2000	**2002**
North America	310.3	258.4
Western Europe	181.5	157.0
South/Central America	79.3	71.6
Pacific Rim	202.6	198.6

A) $$A+B=\begin{bmatrix} 677.1 & 601.4 \\ 422.2 & 403.0 \\ 172.6 & 141.1 \\ 620.6 & 592.1 \end{bmatrix}$$

B) $$A+B=\begin{bmatrix} 56.5 & 84.6 \\ 59.2 & 89.0 \\ 14.0 & -2.1 \\ 215.4 & 194.9 \end{bmatrix}$$

C) $$A+B=\begin{bmatrix} 677.1 & 601.4 \\ -59.2 & 403.0 \\ 172.6 & 2.1 \\ -215.4 & 592.1 \end{bmatrix}$$

D) $$A+B=\begin{bmatrix} -56.5 & -84.6 \\ -59.2 & -89.0 \\ -14.0 & 2.1 \\ -215.4 & -194.9 \end{bmatrix}$$

E) $$A+B=\begin{bmatrix} 677.1 & 601.4 \\ 320.0 & 397.7 \\ 172.6 & 141.1 \\ 295.9 & 592.1 \end{bmatrix}$$

Ans: A

41. Ace, Baker, and Champ are being purchased by ALCO, Inc., and their outstanding debts must be paid by the purchaser. The matrix below gives the amounts of debt and the terms for the companies being purchased. If ALCO pays 35% of the amount owed on each debt, write the matrix giving the remaining debts.

$$\begin{array}{r c} & \begin{array}{cc} \text{Due in 30 days} & \text{Due in 60 days} \end{array} \\ \begin{array}{r} \text{Ace} \\ \text{Baker} \\ \text{Champ} \end{array} & \begin{bmatrix} \$40,000 & \$65,000 \\ \$25,000 & \$15,000 \\ \$35,000 & \$58,000 \end{bmatrix} \end{array}$$

A) $\begin{bmatrix} 14,000 & 22,750 \\ 8,750 & 5,250 \\ 12,250 & 20,300 \end{bmatrix}$

B) $\begin{bmatrix} 54,000 & 87,750 \\ 33,750 & 20,250 \\ 47,250 & 78,300 \end{bmatrix}$

C) $\begin{bmatrix} 26,000 & 87,750 \\ 16,250 & 9,750 \\ 47,250 & 37,700 \end{bmatrix}$

D) $\begin{bmatrix} 26,000 & 42,250 \\ 16,250 & 5,250 \\ 12,250 & 37,700 \end{bmatrix}$

E) $\begin{bmatrix} 26,000 & 42,250 \\ 16,250 & 9,750 \\ 22,750 & 37,700 \end{bmatrix}$

Ans: E

42. Ace, Baker, and Champ are being purchased by ALCO, Inc., and their outstanding debts must be paid by the purchaser. The matrix below gives the amounts of debt and the terms for the companies being purchased. Suppose ALCO decides to pay 80% of all debts due in 30 days and to increase the debts due in 60 days by 20%. Write a matrix that gives the debts after these transactions are made.

$$\begin{array}{r c} & \begin{array}{cc} \text{Due in 30 days} & \text{Due in 60 days} \end{array} \\ \begin{array}{r} \text{Ace} \\ \text{Baker} \\ \text{Champ} \end{array} & \begin{bmatrix} \$40,000 & \$65,000 \\ \$30,000 & \$15,000 \\ \$35,000 & \$63,000 \end{bmatrix} \end{array}$$

A) $\begin{bmatrix} 72,000 & 78,000 \\ 54,000 & 18,000 \\ 63,000 & 75,600 \end{bmatrix}$

B) $\begin{bmatrix} 8,000 & 52,000 \\ 6,000 & 12,000 \\ 7,000 & 50,400 \end{bmatrix}$

C) $\begin{bmatrix} 8,000 & 78,000 \\ 6,000 & 18,000 \\ 7,000 & 75,600 \end{bmatrix}$

D) $\begin{bmatrix} 72,000 & 52,000 \\ 54,000 & 12,000 \\ 63,000 & 50,400 \end{bmatrix}$

E) $\begin{bmatrix} 32,000 & 13,000 \\ 24,000 & 3,000 \\ 28,000 & 12,600 \end{bmatrix}$

Ans: C

43. When a firm buys another company, the company frequently has some outstanding debt that the purchaser must pay. Consider three companies, A, B, and C, that are purchased by Maxx Industries. The following table gives the amount of each company's debt, classified by the number of days remaining until the debt must be paid. Suppose Maxx Industries pays 20% of the amount owed on each loan. Write the matrix that would give the remaining debt in each category.

Company	**30 Days**	**60 Days**	**More than 60 Days**
A	$29,000	$26,000	$12,000
B	19,000	52,000	9,000
C	12,000	20,000	124,000

A) $\begin{bmatrix} 5,800 & 5,200 & 2,400 \\ 3,800 & 10,400 & 1,800 \\ 2,400 & 4,000 & 24,800 \end{bmatrix}$

B) $\begin{bmatrix} 23,200 & 20,800 & 9,600 \\ 15,200 & 41,600 & 7,200 \\ 9,600 & 16,000 & 99,200 \end{bmatrix}$

C) $\begin{bmatrix} 34,800 & 31,200 & 14,400 \\ 22,800 & 62,400 & 10,800 \\ 14,400 & 24,000 & 148,800 \end{bmatrix}$

D) $\begin{bmatrix} 23,200 & 5,200 & 9,600 \\ 3,800 & 41,600 & 7,200 \\ 9,600 & 16,000 & 24,800 \end{bmatrix}$

E) $\begin{bmatrix} 23,200 & 20,800 & 9,600 \\ 22,800 & 41,600 & 10,800 \\ 9,600 & 24,000 & 99,200 \end{bmatrix}$

Ans: B

44. When a firm buys another company, the company frequently has some outstanding debt that the purchaser must pay. Consider three companies, A, B, and C, that are purchased by Maxx Industries. The following table gives the amount of each company's debt, classified by the number of days remaining until the debt must be paid. What payment plan did Maxx Industries use if the outstanding debt is given by the following matrix?

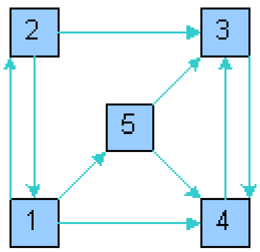
Company	**30 Days**	**60 Days**	**More than 60 Days**
A	$29,000	$30,000	$16,000
B	15,000	56,000	9,000
C	8,000	24,000	124,000

$$-0.3\begin{bmatrix} 0 & 30,000 & 0 \\ 0 & 56,000 & 0 \\ 0 & 24,000 & 0 \end{bmatrix} - \begin{bmatrix} 29,000 & 0 & 0 \\ 15,000 & 0 & 0 \\ 8,000 & 0 & 0 \end{bmatrix} - \begin{bmatrix} 0 & 0 & 16,000 \\ 0 & 0 & 9,000 \\ 0 & 0 & 124,000 \end{bmatrix}$$

A) They paid 30% of the debts due in 30 days, 100% of the debts due in 60 days, and 100% of the debts due in more than 60 days.
B) They paid 30% of the debts due in 30 days, 30% of the debts due in 60 days, and 100% of the debts due in more than 60 days.
C) They paid 100% of the debts due in 30 days, 100% of the debts due in 60 days, and 30% of the debts due in more than 60 days.
D) They paid 100% of the debts due in 30 days, 30% of the debts due in 60 days, and 100% of the debts due in more than 60 days.
E) They paid 100% of the debts due in 30 days, 30% of the debts due in 60 days, and 30% of the debts due in more than 60 days.

Ans: D

45. In order to rank the five members of its chess team for play against another school, the coach draws the following diagram. An arrow from 1 to 2 means player 1 has defeated player 2.



Construct matrix A with elements

$$a_{ij} = \begin{cases} 1 & \text{if } i \text{ defeated } j. \\ 0 & \text{otherwise.} \end{cases}$$

A)

$$\begin{bmatrix} 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 & 1 \\ 1 & 0 & 1 & 0 & 1 \\ 1 & 0 & 0 & 0 & 0 \end{bmatrix}$$

B)

$$\begin{bmatrix} 1 & 1 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 \end{bmatrix}$$

C)

$$\begin{bmatrix} 0 & 1 & 0 & 1 & 1 \\ 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 \end{bmatrix}$$

D)

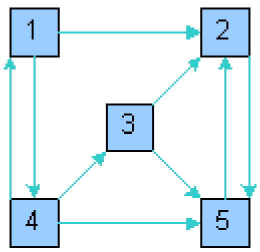
$$\begin{bmatrix} 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 1 & 1 \end{bmatrix}$$

E)

$$\begin{bmatrix} 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 1 & 1 \\ 1 & 1 & 1 & 0 & 1 \\ 1 & 1 & 0 & 1 & 1 \\ 1 & 1 & 0 & 0 & 1 \end{bmatrix}$$

Ans: C

46. In order to rank the five members of its chess team for play against another school, the coach draws the following diagram. An arrow from 1 to 2 means player 1 has defeated player 2.



Construct matrix B with elements

$$b_{ij} = \begin{cases} 1 & \text{if } i \text{ defeated someone who} \\ & \text{defeated } j \text{ and } i \neq j. \\ 0 & \text{otherwise.} \end{cases}$$

A)

$$\begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 1 & 1 & 0 \end{bmatrix}$$

B)

$$\begin{bmatrix} 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

C)

$$\begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 1 \end{bmatrix}$$

D)

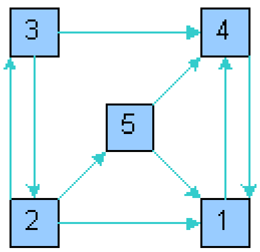
$$\begin{bmatrix} 1 & 1 & 0 & 1 & 0 \\ 1 & 1 & 1 & 1 & 1 \\ 1 & 0 & 1 & 1 & 0 \\ 1 & 0 & 1 & 1 & 0 \\ 1 & 1 & 1 & 1 & 1 \end{bmatrix}$$

E)

$$\begin{bmatrix} 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

Ans: E

47. In order to rank the five members of its chess team for play against another school, the coach draws the following diagram. An arrow from 1 to 2 means player 1 has defeated player 2.



Let matrix A represent the matrix with the following elements:

$$a_{ij} = \begin{cases} 1 & \text{if } i \text{ defeated } j. \\ 0 & \text{otherwise.} \end{cases}$$

Let matrix B represent the matrix with the following elements:

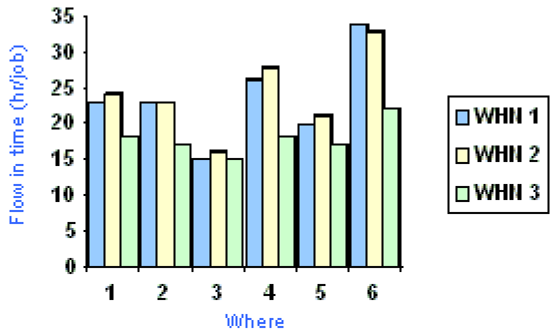
$$b_{ij} = \begin{cases} 1 & \text{if } i \text{ defeated someone who} \\ & \text{defeated } j \text{ and } i \neq j. \\ 0 & \text{otherwise.} \end{cases}$$

The player i is the top-ranked player if the sum of row i in the matrix $A + B$ is the largest. What is the number of this player?

A) player 2
B) player 1
C) player 5
D) player 4
E) player 3

Ans: A

48. The figure that follows depicts the mean (average) flow time for a job when the critical-ratio rule is used to dispatch workers at several machines required to complete the job. In the figure, WHN 1, WHN 2, and WHN 3 represent three different rules for determining when a worker should be transferred to another machine, and Where represents six rules for determining the machine to which the worker is transferred. Construct a 3×6 matrix A to represent these data, with entries rounded to the nearest integer.



A) $\begin{bmatrix} 23 & 24 & 18 \\ 23 & 23 & 17 \\ 15 & 16 & 15 \\ 26 & 28 & 18 \\ 20 & 21 & 17 \\ 34 & 33 & 22 \end{bmatrix}$

B) $\begin{bmatrix} 23 & 23 & 15 & 26 & 20 & 34 \\ 24 & 23 & 16 & 28 & 21 & 33 \\ 18 & 17 & 15 & 18 & 17 & 22 \end{bmatrix}$

C) $\begin{bmatrix} 25 & 25 & 15 & 25 & 20 & 35 \\ 25 & 25 & 15 & 30 & 20 & 35 \\ 20 & 15 & 15 & 20 & 15 & 20 \end{bmatrix}$

D) $\begin{bmatrix} 25 & 25 & 20 \\ 25 & 25 & 15 \\ 15 & 15 & 15 \\ 25 & 30 & 20 \\ 20 & 20 & 15 \\ 35 & 35 & 20 \end{bmatrix}$

E) $\begin{bmatrix} 23 & 24 & 18 & 23 & 23 & 17 \\ 15 & 16 & 15 & 26 & 28 & 18 \\ 20 & 21 & 17 & 34 & 33 & 22 \end{bmatrix}$

Ans: B

49. The matrix that follows represents work efficiency for workers at nine different work centers. Suppose that the work centers changes in rules cause efficiency to decrease by 0.03 at work centers 2 and 5 and to decrease by 0.01 at work center 7. Write the matrix that describes the new efficiencies.

$$\begin{array}{cc} \text{Work Centers} & \\ \begin{array}{ccccccccc} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 \end{array} & \\ \begin{bmatrix} 1.00 & 0.95 & 0.95 & 0.95 & 0.95 & 0.85 & 0.85 & 0.85 & 0.85 \\ 0.85 & 1.00 & 0.95 & 0.95 & 0.95 & 0.95 & 0.85 & 0.85 & 0.85 \\ 0.85 & 0.85 & 1.00 & 0.95 & 0.95 & 0.95 & 0.95 & 0.85 & 0.85 \\ 0.85 & 0.85 & 0.85 & 1.00 & 0.95 & 0.95 & 0.95 & 0.95 & 0.85 \\ 0.85 & 0.85 & 0.85 & 0.85 & 1.00 & 0.95 & 0.95 & 0.95 & 0.95 \\ 0.95 & 0.85 & 0.85 & 0.85 & 0.85 & 1.00 & 0.95 & 0.95 & 0.95 \\ 0.95 & 0.95 & 0.85 & 0.85 & 0.85 & 0.85 & 1.00 & 0.95 & 0.95 \\ 0.95 & 0.95 & 0.95 & 0.85 & 0.85 & 0.85 & 0.85 & 1.00 & 0.95 \\ 0.95 & 0.95 & 0.95 & 0.95 & 0.85 & 0.85 & 0.85 & 0.85 & 1.00 \end{bmatrix} & \begin{array}{c} \\ W \\ o \\ r \\ k \\ e \\ r \\ s \\ \\ \end{array} \end{array}$$

A) $\begin{bmatrix} 1.00 & 0.92 & 0.95 & 0.95 & 0.92 & 0.85 & 0.82 & 0.85 & 0.85 \\ 0.85 & 0.97 & 0.95 & 0.95 & 0.92 & 0.95 & 0.82 & 0.85 & 0.85 \\ 0.85 & 0.82 & 1.00 & 0.95 & 0.92 & 0.95 & 0.92 & 0.85 & 0.85 \\ 0.85 & 0.82 & 0.85 & 1.00 & 0.92 & 0.95 & 0.92 & 0.95 & 0.85 \\ 0.85 & 0.82 & 0.85 & 0.85 & 0.97 & 0.95 & 0.92 & 0.95 & 0.95 \\ 0.95 & 0.82 & 0.85 & 0.85 & 0.82 & 1.00 & 0.92 & 0.95 & 0.95 \\ 0.95 & 0.92 & 0.85 & 0.85 & 0.82 & 0.85 & 0.97 & 0.95 & 0.95 \\ 0.95 & 0.92 & 0.95 & 0.85 & 0.82 & 0.85 & 0.82 & 1.00 & 0.95 \\ 0.95 & 0.92 & 0.95 & 0.95 & 0.82 & 0.85 & 0.82 & 0.85 & 1.00 \end{bmatrix}$

B) $\begin{bmatrix} 1.00 & 0.94 & 0.95 & 0.95 & 0.94 & 0.85 & 0.84 & 0.85 & 0.85 \\ 0.85 & 0.99 & 0.95 & 0.95 & 0.94 & 0.95 & 0.84 & 0.85 & 0.85 \\ 0.85 & 0.84 & 1.00 & 0.95 & 0.94 & 0.95 & 0.94 & 0.85 & 0.85 \\ 0.85 & 0.84 & 0.85 & 1.00 & 0.94 & 0.95 & 0.94 & 0.95 & 0.85 \\ 0.85 & 0.84 & 0.85 & 0.85 & 0.99 & 0.95 & 0.94 & 0.95 & 0.95 \\ 0.95 & 0.84 & 0.85 & 0.85 & 0.84 & 1.00 & 0.94 & 0.95 & 0.95 \\ 0.95 & 0.94 & 0.85 & 0.85 & 0.84 & 0.85 & 0.99 & 0.95 & 0.95 \\ 0.95 & 0.94 & 0.95 & 0.85 & 0.84 & 0.85 & 0.84 & 1.00 & 0.95 \\ 0.95 & 0.94 & 0.95 & 0.95 & 0.84 & 0.85 & 0.84 & 0.85 & 1.00 \end{bmatrix}$

C) $$\begin{bmatrix} 1.00 & 0.94 & 0.95 & 0.95 & 0.94 & 0.85 & 0.82 & 0.85 & 0.85 \\ 0.85 & 0.99 & 0.95 & 0.95 & 0.94 & 0.95 & 0.82 & 0.85 & 0.85 \\ 0.85 & 0.84 & 1.00 & 0.95 & 0.94 & 0.95 & 0.92 & 0.85 & 0.85 \\ 0.85 & 0.84 & 0.85 & 1.00 & 0.94 & 0.95 & 0.92 & 0.95 & 0.85 \\ 0.85 & 0.84 & 0.85 & 0.85 & 0.99 & 0.95 & 0.92 & 0.95 & 0.95 \\ 0.95 & 0.84 & 0.85 & 0.85 & 0.84 & 1.00 & 0.92 & 0.95 & 0.95 \\ 0.95 & 0.94 & 0.85 & 0.85 & 0.84 & 0.85 & 0.97 & 0.95 & 0.95 \\ 0.95 & 0.94 & 0.95 & 0.85 & 0.84 & 0.85 & 0.82 & 1.00 & 0.95 \\ 0.95 & 0.94 & 0.95 & 0.95 & 0.84 & 0.85 & 0.82 & 0.85 & 1.00 \end{bmatrix}$$

D) $$\begin{bmatrix} 1.00 & 0.92 & 0.95 & 0.95 & 0.92 & 0.85 & 0.84 & 0.85 & 0.85 \\ 0.85 & 0.97 & 0.95 & 0.95 & 0.92 & 0.95 & 0.84 & 0.85 & 0.85 \\ 0.85 & 0.82 & 1.00 & 0.95 & 0.92 & 0.95 & 0.94 & 0.85 & 0.85 \\ 0.85 & 0.82 & 0.85 & 1.00 & 0.92 & 0.95 & 0.94 & 0.95 & 0.85 \\ 0.85 & 0.82 & 0.85 & 0.85 & 0.97 & 0.95 & 0.94 & 0.95 & 0.95 \\ 0.95 & 0.82 & 0.85 & 0.85 & 0.82 & 1.00 & 0.94 & 0.95 & 0.95 \\ 0.95 & 0.92 & 0.85 & 0.85 & 0.82 & 0.85 & 0.99 & 0.95 & 0.95 \\ 0.95 & 0.92 & 0.95 & 0.85 & 0.82 & 0.85 & 0.84 & 1.00 & 0.95 \\ 0.95 & 0.92 & 0.95 & 0.95 & 0.82 & 0.85 & 0.84 & 0.85 & 1.00 \end{bmatrix}$$

E) $$\begin{bmatrix} 1.00 & 0.95 & 0.95 & 0.95 & 0.95 & 0.85 & 0.85 & 0.85 & 0.85 \\ 0.82 & 0.97 & 0.92 & 0.92 & 0.92 & 0.92 & 0.82 & 0.82 & 0.82 \\ 0.85 & 0.85 & 1.00 & 0.95 & 0.95 & 0.95 & 0.95 & 0.85 & 0.85 \\ 0.85 & 0.85 & 0.85 & 1.00 & 0.95 & 0.95 & 0.95 & 0.95 & 0.85 \\ 0.82 & 0.82 & 0.82 & 0.82 & 0.97 & 0.92 & 0.92 & 0.92 & 0.92 \\ 0.95 & 0.85 & 0.85 & 0.85 & 0.85 & 1.00 & 0.95 & 0.95 & 0.95 \\ 0.94 & 0.94 & 0.84 & 0.84 & 0.84 & 0.84 & 0.99 & 0.94 & 0.94 \\ 0.95 & 0.95 & 0.95 & 0.85 & 0.85 & 0.85 & 0.85 & 1.00 & 0.95 \\ 0.95 & 0.95 & 0.95 & 0.95 & 0.85 & 0.85 & 0.85 & 0.85 & 1.00 \end{bmatrix}$$

Ans: D

50. Use a computer spreadsheet to find the average efficiency for each work center over the first five workers. Which work center is the most efficient?

$$\begin{array}{c}\text{Work Centers}\\ \begin{bmatrix} 0.95 & 0.95 & 0.95 & 0.95 & 1.00 & 0.85 & 0.85 & 0.85 & 0.85 \\ 0.95 & 1.00 & 0.95 & 0.95 & 0.85 & 0.95 & 0.85 & 0.85 & 0.85 \\ 0.95 & 0.85 & 0.95 & 1.00 & 0.85 & 0.95 & 0.95 & 0.85 & 0.85 \\ 1.00 & 0.85 & 0.95 & 0.85 & 0.85 & 0.95 & 0.95 & 0.95 & 0.85 \\ 0.85 & 0.85 & 1.00 & 0.85 & 0.85 & 0.95 & 0.95 & 0.95 & 0.95 \\ 0.95 & 0.85 & 0.85 & 0.85 & 0.85 & 1.00 & 0.95 & 0.95 & 0.95 \\ 0.95 & 0.95 & 0.85 & 0.85 & 0.85 & 0.85 & 1.00 & 0.95 & 0.95 \\ 0.95 & 0.95 & 0.95 & 0.85 & 0.85 & 0.85 & 0.85 & 1.00 & 0.95 \\ 0.95 & 0.95 & 0.95 & 0.95 & 0.85 & 0.85 & 0.85 & 0.85 & 1.00 \end{bmatrix} \end{array} \begin{array}{c} \text{W}\\ \text{o}\\ \text{r}\\ \text{k}\\ \text{e}\\ \text{r}\\ \text{s} \end{array}$$

A) Number 8
B) Number 5
C) Number 3
D) Number 1
E) Number 4
Ans: C

51. Multiply the matrices.

$$\begin{bmatrix} 2 & 2 & 3 \end{bmatrix} \begin{bmatrix} 6 \\ 6 \\ 7 \end{bmatrix}$$

A) $\begin{bmatrix} 12 & 12 & 21 \end{bmatrix}$
B) $\begin{bmatrix} 45 \end{bmatrix}$
C) $\begin{bmatrix} 39 \end{bmatrix}$
D) $\begin{bmatrix} 3024 \end{bmatrix}$
E) $\begin{bmatrix} 12 \\ 12 \\ 21 \end{bmatrix}$
Ans: B

52. Multiply the matrices.

$$\begin{bmatrix} 3 & 1 & 3 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \\ -3 \end{bmatrix}$$

A) $\begin{bmatrix} 0 & 1 & -9 \end{bmatrix}$

B) $\begin{bmatrix} 3 \end{bmatrix}$

C) $\begin{bmatrix} 0 \end{bmatrix}$

D) $\begin{bmatrix} 0 \\ 1 \\ -9 \end{bmatrix}$

E) $\begin{bmatrix} -8 \end{bmatrix}$

Ans: E

53. Multiply the matrices.

$$\begin{bmatrix} 3 & 1 \end{bmatrix} \begin{bmatrix} 2 & 3 \\ 5 & 6 \end{bmatrix}$$

A) $\begin{bmatrix} 11 & 15 \end{bmatrix}$

B) $\begin{bmatrix} 21 & 9 \end{bmatrix}$

C) $\begin{bmatrix} 9 \\ 21 \end{bmatrix}$

D) $\begin{bmatrix} 9 & 21 \end{bmatrix}$

E) $\begin{bmatrix} 15 \\ 11 \end{bmatrix}$

Ans: A

54. Use the matrices to find CD, if possible.

$$C = \begin{bmatrix} 5 & 4 \\ 2 & 2 \end{bmatrix} \qquad D = \begin{bmatrix} 5 & 2 \\ 3 & 6 \end{bmatrix}$$

A) $\begin{bmatrix} 40 & 32 \\ 16 & 16 \end{bmatrix}$

B) $\begin{bmatrix} 33 \\ 18 \end{bmatrix}$

C) $\begin{bmatrix} 37 & 16 \\ 34 & 16 \end{bmatrix}$

D) $\begin{bmatrix} 37 & 34 \\ 16 & 16 \end{bmatrix}$

E) $\begin{bmatrix} 33 & 18 \end{bmatrix}$

Ans: D

55. Use the matrices to find *DE*, if possible.

$$D = \begin{bmatrix} 5 & 1 \\ 3 & 5 \end{bmatrix} \quad E = \begin{bmatrix} 1 & 0 & 5 \\ 6 & 1 & 0 \end{bmatrix}$$

A) $\begin{bmatrix} 5 & 5 \\ 23 & 3 \end{bmatrix}$

B) $\begin{bmatrix} 23 & 3 & 25 \\ 31 & 5 & 5 \end{bmatrix}$

C) $\begin{bmatrix} 11 & 1 & 25 \\ 33 & 5 & 15 \end{bmatrix}$

D) $\begin{bmatrix} 21 & 15 & 15 \\ 11 & 5 & 5 \end{bmatrix}$

E) $\begin{bmatrix} 5 & 0 \\ 18 & 5 \end{bmatrix}$

Ans: C

56. Use the matrices to find *CF*, if possible.

$$C = \begin{bmatrix} 5 & 3 \\ 1 & 2 \end{bmatrix} \qquad F = \begin{bmatrix} 1 & 0 & -1 & 4 \\ 2 & 0 & 3 & -4 \end{bmatrix}$$

A) $\begin{bmatrix} 5 & 0 \\ 2 & 0 \end{bmatrix}$

B) $\begin{bmatrix} 11 & 0 & 4 & 8 \\ 5 & 0 & 5 & -4 \end{bmatrix}$

C) $\begin{bmatrix} 5 & 0 & -5 & 12 \\ 2 & 0 & 3 & -8 \end{bmatrix}$

D) $\begin{bmatrix} 7 & 0 & -2 & 16 \\ 7 & 0 & 3 & 4 \end{bmatrix}$

E) *CF* is undefined.

Ans: B

57. Use the matrices to find *EC*, if possible.

$$C = \begin{bmatrix} 5 & 4 \\ 2 & 2 \end{bmatrix} \qquad E = \begin{bmatrix} 2 & 1 & 5 \\ 6 & 1 & 0 \end{bmatrix}$$

A) $\begin{bmatrix} 10 & 4 \\ 12 & 2 \end{bmatrix}$

B) $\begin{bmatrix} 34 & 9 & 25 \\ 16 & 4 & 10 \end{bmatrix}$

C) $\begin{bmatrix} 14 & 25 \\ 16 & 2 \end{bmatrix}$

D) $\begin{bmatrix} 22 & 7 & 25 \\ 20 & 6 & 20 \end{bmatrix}$

E) *EC* is undefined.

Ans: E

58. Use the matrices to find BA, if possible.

$$A = \begin{bmatrix} 3 & 0 & 2 \\ 3 & 2 & 1 \\ 4 & 1 & 3 \end{bmatrix} \qquad B = \begin{bmatrix} 1 & 2 & 3 & 0 \\ 4 & 3 & 1 & 2 \\ 3 & 2 & 0 & 1 \end{bmatrix}$$

A) $\begin{bmatrix} 3 & 0 & 6 \\ 12 & 6 & 1 \\ 12 & 2 & 0 \end{bmatrix}$

B) $\begin{bmatrix} 9 & 10 & 9 & 2 \\ 14 & 14 & 11 & 5 \\ 17 & 17 & 13 & 5 \end{bmatrix}$

C) $\begin{bmatrix} 1 & 0 & 6 \\ 12 & 6 & 1 \\ 12 & 0 & 0 \end{bmatrix}$

D) $\begin{bmatrix} 1 & 0 & 6 & 0 \\ 12 & 6 & 1 & 2 \\ 12 & 0 & 0 & 1 \end{bmatrix}$

E) BA is undefined.

Ans: E

59. Use the matrices to find FB^T, if possible.

$$B = \begin{bmatrix} 1 & 1 & 3 & 0 \\ 5 & 2 & 1 & 2 \\ 3 & 3 & 0 & 1 \end{bmatrix} \qquad F = \begin{bmatrix} 1 & 0 & -1 & 3 \\ 2 & -1 & 3 & -4 \end{bmatrix}$$

A) $\begin{bmatrix} 1 & 0 & -3 & 0 \\ 10 & -2 & 3 & -8 \\ 3 & 3 & 0 & 1 \end{bmatrix}$

B) $\begin{bmatrix} -2 & 10 \\ 10 & 3 \\ 6 & -1 \end{bmatrix}$

C) $\begin{bmatrix} -2 & 10 & 6 \\ 10 & 3 & -1 \end{bmatrix}$

D) $\begin{bmatrix} 1 & 10 & 3 \\ 0 & -2 & 3 \\ -3 & 3 & 0 \\ 0 & -8 & 1 \end{bmatrix}$

E) FB^T is undefined.

Ans: C

60. Use the matrices to find *BE*, if possible.

$$B = \begin{bmatrix} 2 & 1 & 3 & 0 \\ 4 & 2 & 1 & 2 \\ 4 & 2 & 1 & 1 \end{bmatrix} \qquad E = \begin{bmatrix} 1 & 1 & 4 \\ 6 & 1 & 0 \end{bmatrix}$$

A) $\begin{bmatrix} 2 & 1 & 12 \\ 24 & 2 & 0 \end{bmatrix}$

B) $\begin{bmatrix} 2 & 1 & 12 & 0 \\ 24 & 2 & 0 & 2 \\ 4 & 2 & 1 & 1 \end{bmatrix}$

C) $\begin{bmatrix} 22 & 11 & 8 & 6 \\ 16 & 8 & 19 & 2 \end{bmatrix}$

D) $\begin{bmatrix} 15 & 12 & 11 \\ 13 & 28 & 27 \end{bmatrix}$

E) *BE* is undefined.

Ans: E

61. Use the matrices to find AE^T , if possible.

$$A = \begin{bmatrix} 2 & 1 & 2 \\ 3 & 2 & 1 \\ 4 & 1 & 4 \end{bmatrix} \qquad E = \begin{bmatrix} 1 & 1 & 4 \\ 5 & 2 & 0 \end{bmatrix}$$

A) $\begin{bmatrix} 11 & 12 \\ 9 & 19 \\ 21 & 22 \end{bmatrix}$

B) $\begin{bmatrix} 2 & 1 & 8 \\ 15 & 4 & 0 \end{bmatrix}$

C) $\begin{bmatrix} 2 & 1 & 8 \\ 15 & 4 & 0 \\ 4 & 1 & 4 \end{bmatrix}$

D) $\begin{bmatrix} 11 & 9 & 21 \\ 12 & 19 & 22 \end{bmatrix}$

E) AE^T is undefined.

Ans: A

62. Use matrix A to find A^2, if possible.

$$A = \begin{bmatrix} 2 & 1 & 3 \\ 4 & 2 & 1 \\ 5 & 1 & 4 \end{bmatrix}$$

A) $\begin{bmatrix} 4 & 1 & 9 \\ 16 & 4 & 1 \\ 25 & 1 & 16 \end{bmatrix}$

B) $\begin{bmatrix} 4 & 2 & 6 \\ 8 & 4 & 2 \\ 10 & 2 & 8 \end{bmatrix}$

C) $\begin{bmatrix} 14 & 13 & 23 \\ 13 & 21 & 26 \\ 23 & 26 & 42 \end{bmatrix}$

D) $\begin{bmatrix} 23 & 10 & 19 \\ 21 & 9 & 18 \\ 34 & 11 & 32 \end{bmatrix}$

E) A^2 is undefined.

Ans: D

63. Use matrix A to find A^3, if possible.

$$A = \begin{bmatrix} 1 & 1 & 3 \\ 4 & 3 & 2 \\ 4 & 0 & 4 \end{bmatrix}$$

A) $\begin{bmatrix} 17 & 4 & 17 \\ 24 & 13 & 26 \\ 20 & 4 & 28 \end{bmatrix}$

B) $\begin{bmatrix} 1 & 1 & 27 \\ 64 & 27 & 8 \\ 64 & 0 & 64 \end{bmatrix}$

C) $\begin{bmatrix} 101 & 29 & 127 \\ 180 & 63 & 202 \\ 148 & 32 & 180 \end{bmatrix}$

D) $\begin{bmatrix} 3 & 3 & 9 \\ 12 & 9 & 6 \\ 12 & 0 & 12 \end{bmatrix}$

E) A^3 is undefined.

Ans: C

64. Use matrix F to find F^2, if possible.

$$F = \begin{bmatrix} 1 & 1 & 0 & 4 \\ 3 & 0 & 3 & -3 \end{bmatrix}$$

A) $\begin{bmatrix} 1 & 1 & 0 & 16 \\ 9 & 0 & 9 & 9 \end{bmatrix}$

B) $\begin{bmatrix} 2 & 2 & 0 & 8 \\ 6 & 0 & 6 & -6 \end{bmatrix}$

C) $\begin{bmatrix} 10 & 1 & 9 & 25 \end{bmatrix}$

D) $\begin{bmatrix} 18 & -9 \\ -9 & 27 \end{bmatrix}$

E) F^2 is undefined.

Ans: E

65. $(CD)E$ and $C(DE)$ are equal.

$$C = \begin{bmatrix} 6 & 3 \\ 1 & 3 \end{bmatrix} \quad D = \begin{bmatrix} 5 & 2 \\ 3 & 6 \end{bmatrix} \quad E = \begin{bmatrix} 1 & 0 & 4 \\ 5 & 2 & 0 \end{bmatrix}$$

A) true
B) false
Ans: A

66. $CD = DC$.

$$C = \begin{bmatrix} 6 & 3 \\ 1 & 3 \end{bmatrix} \quad D = \begin{bmatrix} 5 & 2 \\ 3 & 6 \end{bmatrix}$$

A) true
B) false
Ans: B

67. $\left(\frac{1}{2}A\right)B$ and $\frac{1}{2}(AB)$ are equal.

$$A = \begin{bmatrix} 1 & 0 & 2 \\ 3 & 2 & 2 \\ 4 & 0 & 3 \end{bmatrix} \quad B = \begin{bmatrix} 1 & 1 & 3 & 1 \\ 4 & 3 & 1 & 1 \\ 3 & 2 & 1 & 1 \end{bmatrix}$$

A) true
B) false
Ans: A

68. Use the matrices below. Perform the indicated operations.

AB

$$A = \begin{bmatrix} 2 & 5 & 4 \\ 1 & 5 & 3 \\ 2 & -3 & -1 \end{bmatrix} \qquad B = \begin{bmatrix} -1 & 3 & 1 \\ -4 & 8 & 2 \\ -7 & -3 & -3 \end{bmatrix}$$

A) $\begin{bmatrix} -2 & 15 & 4 \\ -4 & 40 & 6 \\ -14 & 9 & 3 \end{bmatrix}$

B) $\begin{bmatrix} 10 & 34 & 24 \\ 2 & 34 & 20 \\ -17 & 33 & 5 \end{bmatrix}$

C) $\begin{bmatrix} -50 & 34 & 0 \\ -42 & 34 & 2 \\ 17 & -15 & -1 \end{bmatrix}$

D) $\begin{bmatrix} 3 & 7 & 4 \\ 4 & 14 & 6 \\ -23 & -41 & -34 \end{bmatrix}$

E) $\begin{bmatrix} 6 & 34 & 0 \\ -42 & 34 & 20 \\ 17 & -15 & 11 \end{bmatrix}$

Ans: C

69. Use the matrices below. Perform the indicated operations.

BA

$$A = \begin{bmatrix} 2 & 5 & 5 \\ 1 & 5 & 3 \\ 1 & -3 & -1 \end{bmatrix} \qquad B = \begin{bmatrix} -1 & 3 & 1 \\ -4 & 8 & 3 \\ -7 & -3 & -3 \end{bmatrix}$$

A) $\begin{bmatrix} -2 & 15 & 5 \\ -4 & 40 & 9 \\ -7 & 9 & 3 \end{bmatrix}$

B) $\begin{bmatrix} 6 & 23 & 13 \\ 3 & 29 & 1 \\ 14 & 29 & 23 \end{bmatrix}$

C) $\begin{bmatrix} -57 & 31 & 2 \\ -42 & 34 & 7 \\ 18 & -18 & -5 \end{bmatrix}$

D) $\begin{bmatrix} 2 & 7 & 3 \\ 3 & 11 & 1 \\ -20 & -41 & -41 \end{bmatrix}$

E) $\begin{bmatrix} 2 & 17 & 3 \\ 3 & 11 & 1 \\ -14 & -41 & 29 \end{bmatrix}$

Ans: D

70. Use the matrices below. Perform the indicated operations.

DC

$$C = \begin{bmatrix} 4 & 0 & 4 \\ 1 & 8 & -1 \\ 4 & 0 & 5 \end{bmatrix} \qquad D = \begin{bmatrix} 4 & 4 & -8 \\ 0 & -1 & 3 \\ -3 & -3 & 6 \end{bmatrix}$$

A) $\begin{bmatrix} 4 & 4 & -8 \\ 7 & -1 & 10 \\ 1 & 1 & -2 \end{bmatrix}$

B) $\begin{bmatrix} 16 & 0 & -32 \\ 0 & -8 & -3 \\ -12 & 0 & 30 \end{bmatrix}$

C) $\begin{bmatrix} -12 & 32 & -28 \\ 11 & -8 & 16 \\ 9 & -24 & 21 \end{bmatrix}$

D) $\begin{bmatrix} 52 & 32 & -20 \\ 11 & 8 & 16 \\ 9 & -24 & 45 \end{bmatrix}$

E) $\begin{bmatrix} -16 & 32 & -28 \\ -1 & -8 & 18 \\ 15 & -24 & 21 \end{bmatrix}$

Ans: C

71. Use the matrices below. Perform the indicated operations.

AZ

$$A = \begin{bmatrix} 2 & 6 & 5 \\ 2 & 4 & 3 \\ 2 & -3 & -1 \end{bmatrix} \qquad Z = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

A) $\begin{bmatrix} 2 & 6 & 5 \\ 2 & 4 & 3 \\ 2 & -3 & -1 \end{bmatrix}$

B) $\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$

C) $\begin{bmatrix} 2 & 2 & 2 \\ 6 & 4 & -3 \\ 5 & 3 & -1 \end{bmatrix}$

D) $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

E) $\begin{bmatrix} 4 & 36 & 25 \\ 4 & 16 & 9 \\ 4 & 9 & 1 \end{bmatrix}$

Ans: B

72. Use the matrices below. Perform the indicated operations.

AI

$$A = \begin{bmatrix} 2 & 5 & 5 \\ 2 & 4 & 4 \\ 1 & -2 & -1 \end{bmatrix} \qquad I = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

A) $\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$

B) $\begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{bmatrix}$

C) $\begin{bmatrix} 2 & 2 & 1 \\ 5 & 4 & -2 \\ 5 & 4 & -1 \end{bmatrix}$

D) $\begin{bmatrix} 2 & 0 & 0 \\ 0 & 4 & 0 \\ 0 & 0 & -1 \end{bmatrix}$

E) $\begin{bmatrix} 2 & 5 & 5 \\ 2 & 4 & 4 \\ 1 & -2 & -1 \end{bmatrix}$

Ans: E

73. Use the matrices below. Perform the indicated operations.

IA

$$A = \begin{bmatrix} 2 & 5 & 4 \\ 1 & 5 & 4 \\ 1 & -2 & -2 \end{bmatrix} \qquad I = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

A) $\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$

B) $\begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{bmatrix}$

C) $\begin{bmatrix} 2 & 1 & 1 \\ 5 & 5 & -2 \\ 4 & 4 & -2 \end{bmatrix}$

D) $\begin{bmatrix} 2 & 5 & 4 \\ 1 & 5 & 4 \\ 1 & -2 & -2 \end{bmatrix}$

E) $\begin{bmatrix} 2 & 0 & 0 \\ 0 & 5 & 0 \\ 0 & 0 & -2 \end{bmatrix}$

Ans: D

74. Use the matrices below. Perform the indicated operations.

IFZ

$$I = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \qquad F = \begin{bmatrix} 4 & 1 & 2 \\ 0 & 2 & -3 \\ 2 & 2 & 0 \end{bmatrix} \qquad Z = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

A) $\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$

B) $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

C) $\begin{bmatrix} 4 & 1 & 2 \\ 0 & 2 & -3 \\ 2 & 2 & 0 \end{bmatrix}$

D) $\begin{bmatrix} 4 & 0 & 2 \\ 1 & 2 & 2 \\ 2 & -3 & 0 \end{bmatrix}$

E) $\begin{bmatrix} 4 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 0 \end{bmatrix}$

Ans: A

75. Is it true for matrices (as it is for real numbers) that multiplication by a zero matrix only sometimes gives is a result of a zero matrix provided the multiplication is defined?

A) false
B) true

Ans: A

76. Find AB and BA if $A = \begin{bmatrix} e & f \\ g & h \end{bmatrix}$ and $B = \dfrac{1}{eh - fg}\begin{bmatrix} h & -f \\ -g & e \end{bmatrix}$, with $eh - fg \neq 0$.

A) $AB = \begin{bmatrix} eh - fg & fe - ef \\ gh - hg & he - gf \end{bmatrix}, BA = \begin{bmatrix} eh - fg & fe - ef \\ gh - hg & he - gf \end{bmatrix}$

B) $AB = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}, BA = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$

C) $AB = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, BA = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

D) $AB = \begin{bmatrix} \dfrac{h}{eh - fg} & \dfrac{-f}{eh - fg} \\ \dfrac{-g}{eh - fg} & \dfrac{h}{eh - fg} \end{bmatrix}, BA = \begin{bmatrix} \dfrac{h}{eh - fg} & \dfrac{-f}{eh - fg} \\ \dfrac{-g}{eh - fg} & \dfrac{h}{eh - fg} \end{bmatrix}$

E) $AB = \begin{bmatrix} \dfrac{eh - fg}{eh - fg} & \dfrac{-ef + ef}{eh - fg} \\ \dfrac{gh - gh}{eh - fg} & \dfrac{-fg + he}{eh - fg} \end{bmatrix}, BA = \begin{bmatrix} \dfrac{eh - fg}{eh - fg} & \dfrac{-ef + ef}{eh - fg} \\ \dfrac{gh - gh}{eh - fg} & \dfrac{-fg + he}{eh - fg} \end{bmatrix}$

Ans: E

77. Using the matrices A and B below, what condition must be satisfied in order that AB and BA are defined?

$A = \begin{bmatrix} m & n \\ o & p \end{bmatrix}$ and $B = \dfrac{1}{mp - no}\begin{bmatrix} p & -n \\ -o & m \end{bmatrix}$

A) $mp - no \neq 1$
B) $mp - no = 1$
C) $mp - no = no - mp$
D) $mp - no = mn - po$
E) $mp - no \neq 0$

Ans: E

78. For $F = \begin{bmatrix} 1 & 0 & -1 & 3 \\ 3 & -1 & 4 & -4 \end{bmatrix}$, are FF^T and F^TF defined?

A) yes
B) no

Ans: A

79. For $F = \begin{bmatrix} 1 & 0 & 0 & 4 \\ 2 & -1 & 3 & -4 \end{bmatrix}$, what size is each product FF^T and F^TF ?

A) FF^T is 2 x 2; F^TF is 2 x 4
B) FF^T is 2 x 2; F^TF is 4 x 4
C) FF^T is 4 x 2; F^TF is 4 x 4
D) FF^T is 4 x 4; F^TF is 2 x 2
E) FF^T is 2 x 4; F^TF is 4 x 2
Ans: B

80. For $F = \begin{bmatrix} 2 & 1 & -1 & 4 \\ 3 & -1 & 3 & -3 \end{bmatrix}$, can $FF^T = F^TF$? Explain.

A) No, because FF^T and F^TF have the same dimensions but different values.
B) Yes, because FF^T and F^TF have the same dimensions.
C) Yes, because FF^T and F^TF have the same values.
D) No, because FF^T and F^TF have different dimensions.
E) Yes, because FF^T and F^TF have the same dimensions and the same values.
Ans: D

81. Substitute the given values of *x*, *y*, and *z* into the matrix equation and use matrix multiplication to determine if the values satisfy the equation.

$$\begin{bmatrix} 1 & 2 & 1 \\ 3 & 4 & -2 \\ 2 & 0 & -1 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 2 \\ -5 \\ 3 \end{bmatrix} \quad x = 3,\ y = -2,\ z = 3$$

A) true
B) false
Ans: A

82. Substitute the given values of *x*, *y*, and *z* into the matrix equation and use matrix multiplication to determine if the values satisfy the equation.

$$\begin{bmatrix} 3 & 1 & 0 \\ 2 & -2 & 1 \\ 1 & 1 & 2 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 10 \\ 15 \\ 8 \end{bmatrix} \quad x = 4,\ y = -2,\ z = 3$$

A) true
B) false
Ans: A

83. Substitute the given values of *x*, *y*, and *z* into the matrix equation and use matrix multiplication to determine if the values satisfy the equation.

$$\begin{bmatrix} 1 & 1 & 2 \\ 4 & 0 & 1 \\ 2 & 1 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 9 \\ 13 \\ 11 \end{bmatrix} \quad x = 3,\ y = 4,\ z = 1$$

A) true
B) false
Ans: A

84. Substitute the given values of *x*, *y*, and *z* into the matrix equation and use matrix multiplication to determine if the values satisfy the equation.

$$\begin{bmatrix} 1 & 0 & 2 \\ 3 & 1 & 0 \\ 1 & 2 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 8 \\ 17 \\ 10 \end{bmatrix} \quad x = 4,\ y = 2,\ z = 3$$

A) true
B) false
Ans: B

85. Use technology to find the product *AB* of the following matrices.

$$A = \frac{1}{2}\begin{bmatrix} 3 & 15 & -9 & -15 & 6 \\ 3 & -9 & 3 & 9 & -6 \\ -1 & -17 & 3 & 23 & -4 \\ 0 & 12 & 0 & -12 & 0 \\ -1 & -5 & 3 & 5 & 2 \end{bmatrix} \qquad B = \begin{bmatrix} 2 & 3 & 0 & 1 & 3 \\ 1 & 0 & 2 & 3 & 1 \\ 0 & 1 & 0 & 2 & 3 \\ 1 & 0 & 2 & 2 & 1 \\ 1 & 0 & 0 & 0 & 3 \end{bmatrix}$$

A) $$AB = \begin{bmatrix} 12 & 0 & 0 & 0 & 0 \\ 0 & 12 & 0 & 0 & 0 \\ 0 & 0 & 12 & 0 & 0 \\ 0 & 0 & 0 & 12 & 0 \\ 0 & 0 & 0 & 0 & 12 \end{bmatrix}$$

B) $$AB = \begin{bmatrix} 24 & 0 & 0 & 0 & 0 \\ 0 & 24 & 0 & 0 & 0 \\ 0 & 0 & 24 & 0 & 0 \\ 0 & 0 & 0 & 24 & 0 \\ 0 & 0 & 0 & 0 & 24 \end{bmatrix}$$

C) $$AB = \begin{bmatrix} 6 & 0 & 0 & 0 & 0 \\ 0 & 6 & 0 & 0 & 0 \\ 0 & 0 & 6 & 0 & 0 \\ 0 & 0 & 0 & 6 & 0 \\ 0 & 0 & 0 & 0 & 6 \end{bmatrix}$$

D) $$AB = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

E) $$AB = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

Ans: C

86. A car dealer can buy midsize cars for 8% under the list price, and he can buy luxury cars for 14% under the list price. The following table gives the list prices for two midsize and two luxury cars.

Midsize	36,000	42,000
Luxury	50,000	56,000

Write these data in a matrix and multiply it on the left by the matrix
$\begin{bmatrix} 0.92 & 0 \\ 0 & 0.86 \end{bmatrix}$

A) $\begin{bmatrix} 36,000 & 42,000 \\ 50,000 & 56,000 \end{bmatrix}$

B) $\begin{bmatrix} 0.92 & 0 \\ 0 & 0.86 \end{bmatrix}$

C) $\begin{bmatrix} 33,120 & 36,120 \\ 46,000 & 48,160 \end{bmatrix}$

D) $\begin{bmatrix} 30,960 & 36,120 \\ 46,000 & 51,520 \end{bmatrix}$

E) $\begin{bmatrix} 33,120 & 38,640 \\ 43,000 & 48,160 \end{bmatrix}$

Ans: C

87. A clothing manufacturer has factories in Atlanta, Chicago, and New York. Sales (in thousands) during the first quarter are summarized in the matrix below.

$$\begin{array}{r c} & \begin{array}{ccc} \text{Atl.} & \text{Chi.} & \text{NY} \end{array} \\ \begin{array}{r} \text{Coats} \\ \text{Shirts} \\ \text{Pants} \\ \text{Ties} \end{array} & \begin{bmatrix} 40 & 63 & 18 \\ 85 & 56 & 42 \\ 6 & 18 & 8 \\ 7 & 10 & 8 \end{bmatrix} \end{array}$$

During this period, the selling price of a coat was \$200, of a shirt \$50, of a pair of pants \$55, and of a tie \$20. Use matrix multiplication to find the total revenue received by each factory.

A) Atlanta \$19,470,000; Chicago \$15,540,000; New York \$9,900,000
B) Atlanta \$12,720,000; Chicago \$16,590,000; New York \$6,300,000
C) Atlanta \$19,470; Chicago \$15,540; New York \$9,900
D) Atlanta \$12,720; Chicago \$16,590; New York \$6,300
E) Atlanta \$1,275,500; Chicago \$1,626,000; New York \$630,000

Ans: B

88. Matrix A below gives the fraction of the Earth's area and the projected fraction of its population for five continents in 2050. Matrix B gives the Earth's area (in square miles) and its projected 2050 population. Find the area and population of each given continent by finding AB.

$$A = \begin{array}{cc} \text{Fraction of Area} & \text{Fraction of Population} \\ \left[\begin{array}{cc} 0.162 & 0.047 \\ 0.119 & 0.086 \\ 0.066 & 0.067 \\ 0.298 & 0.584 \\ 0.202 & 0.216 \end{array}\right] \end{array} \begin{array}{l} \\ \text{North America} \\ \text{South America} \\ \text{Europe} \\ \text{Asia} \\ \text{Africa} \end{array}$$

$$B = \begin{array}{cc} \text{Area} & \text{Population} \\ \left[\begin{array}{cc} 57{,}850{,}000 & 0 \\ 0 & 9{,}322{,}000{,}000 \end{array}\right] \end{array} \begin{array}{l} \\ \text{Area} \\ \text{Population} \end{array}$$

A) $$AB = \begin{bmatrix} 9{,}371{,}700 & 438{,}134{,}000 \\ 6{,}884{,}150 & 801{,}692{,}000 \\ 3{,}818{,}100 & 624{,}574{,}000 \\ 17{,}239{,}300 & 5{,}444{,}048{,}000 \\ 11{,}685{,}700 & 2{,}013{,}552{,}000 \end{bmatrix}$$

B) $$AB = \begin{bmatrix} 1{,}510{,}164{,}000 & 2{,}718{,}950 \\ 1{,}109{,}318{,}000 & 4{,}975{,}100 \\ 615{,}252{,}000 & 3{,}875{,}950 \\ 2{,}777{,}956{,}000 & 33{,}784{,}400 \\ 1{,}883{,}044{,}000 & 12{,}495{,}600 \end{bmatrix}$$

C) $$AB = \begin{bmatrix} 9{,}371{,}700 & 2{,}718{,}950 \\ 1{,}109{,}318{,}000 & 801{,}692{,}000 \\ 3{,}818{,}100 & 624{,}574{,}000 \\ 2{,}777{,}956{,}000 & 33{,}784{,}400 \\ 11{,}685{,}700 & 2{,}013{,}552{,}000 \end{bmatrix}$$

D) $$AB = \begin{bmatrix} 9{,}371{,}700 & 2{,}718{,}950 \\ 1{,}109{,}318{,}000 & 801{,}692{,}000 \\ 615{,}252{,}000 & 3{,}875{,}950 \\ 17{,}239{,}300 & 33{,}784{,}400 \\ 1{,}883{,}044{,}000 & 2{,}013{,}552{,}000 \end{bmatrix}$$

E) $$AB = \begin{bmatrix} 9{,}371{,}700 & 2{,}013{,}552{,}000 \\ 7{,}462{,}650 & 5{,}444{,}048{,}000 \\ 3{,}818{,}100 & 624{,}574{,}000 \\ 17{,}239{,}300 & 801{,}692{,}000 \\ 12{,}264{,}200 & 438{,}134{,}000 \end{bmatrix}$$

Ans: A

89. Suppose the weights (in grams) and lengths (in centimeters) of three groups of laboratory animals are given by matrix A, where column 1 gives the lengths and each row corresponds to one group.

$$A = \begin{bmatrix} 12.5 & 250 \\ 11.8 & 215 \\ 9.8 & 190 \end{bmatrix}$$

If the increase in both weight and length over the next two weeks is 14% for group I, 6% for group II, and 1% for group III, then the increases in the measures during the 2 weeks can be found by computing GA, where

$$G = \begin{bmatrix} 0.14 & 0 & 0 \\ 0 & 0.06 & 0 \\ 0 & 0 & 0.01 \end{bmatrix}$$

What are the increases in respective weights and measures at the end of these 2 weeks?

A) $$GA = \begin{bmatrix} 35 & 1.75 \\ 12.9 & 0.708 \\ 1.9 & 0.098 \end{bmatrix}$$

B) $$GA = \begin{bmatrix} 23.35 & 440 \\ 22.308 & 452.9 \\ 24.398 & 466.9 \end{bmatrix}$$

C) $$GA = \begin{bmatrix} 1.75 & 35 \\ 0.708 & 12.9 \\ 0.098 & 1.9 \end{bmatrix}$$

D) $$GA = \begin{bmatrix} 17.5 & 350 \\ 7.08 & 129 \\ 0.98 & 19 \end{bmatrix}$$

E) $$GA = \begin{bmatrix} 350 & 17.5 \\ 129 & 7.08 \\ 19 & 0.98 \end{bmatrix}$$

Ans: C

90. Suppose the weights (in grams) and lengths (in centimeters) of three groups of laboratory animals are given by matrix *A*, where column 1 gives the lengths and each row corresponds to one group.

$$A = \begin{bmatrix} 12.5 & 250 \\ 11.8 & 215 \\ 9.8 & 190 \end{bmatrix}$$

If the increase in both weight and length over the next two weeks is 15% for group I, 8% for group II, and 3% for group III, then the increases in the measures during the 2 weeks can be found by computing *GA*, where

$$G = \begin{bmatrix} 0.15 & 0 & 0 \\ 0 & 0.08 & 0 \\ 0 & 0 & 0.03 \end{bmatrix}$$

Find the matrix that gives the new weights and measures at the end of this period by computing $(I+G)A$, where *I* is the 3 × 3 identity matrix.

A) $$(I+G)A = \begin{bmatrix} 1.875 & 37.5 \\ 0.944 & 17.2 \\ 0.294 & 5.7 \end{bmatrix}$$

B) $$(I+G)A = \begin{bmatrix} 31.25 & 625 \\ 21.24 & 387 \\ 12.74 & 247 \end{bmatrix}$$

C) $$(I+G)A = \begin{bmatrix} 287.5 & 14.375 \\ 232.2 & 12.744 \\ 195.7 & 10.094 \end{bmatrix}$$

D) $$(I+G)A = \begin{bmatrix} 14.03 & 293.25 \\ 13.824 & 271.08 \\ 10.197 & 200.85 \end{bmatrix}$$

E) $$(I+G)A = \begin{bmatrix} 14.375 & 287.5 \\ 12.744 & 232.2 \\ 10.094 & 195.7 \end{bmatrix}$$

Ans: E

91. Multiplication by a matrix can be used to encode messages (and multiplication by its inverse can be used to decode messages). Given the code

a	b	c	d	e	f	g	h	i	j	k	l	m	n	o
p	q	r	s	t	u	v	w	x	y	z	blank			
1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
17	18	19	20	21	22	23	24	25	26	27				

and the code matrix $A = \begin{bmatrix} 2 & 10 \\ 7 & 12 \end{bmatrix}$, use matrix A to encode the message "The die is cast."

A) $40, 50, 8, 50, 18, 270, 2, 200, 56, 324, 63, 324, 133, 36, 133, 324$

B) $120, 236, 280, 359, 98, 136, 280, 359, 208, 291, 84, 225, 192, 235, 310, 464$

C) $40, 56, 50, 324, 8, 63, 50, 324, 18, 133, 270, 36, 2, 133, 200, 324$

D) $120, 280, 98, 280, 208, 84, 192, 310, 236, 359, 136, 359, 291, 225, 235, 464$

E) $120, 280, 236, 359, 98, 280, 136, 359, 208, 84, 291, 225, 192, 310, 235, 464$

Ans: B

92. Multiplication by a matrix can be used to encode messages (and multiplication by its inverse can be used to decode messages). Given the code

a	b	c	d	e	f	g	h	i	j	k	l	m	n	o
p	q	r	s	t	u	v	w	x	y	z	blank			
1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
17	18	19	20	21	22	23	24	25	26	27				

and the code matrix $A = \begin{bmatrix} 2 & 9 \\ 5 & 13 \end{bmatrix}$, use matrix A to encode the message "To be or not to be."

A) $175, 295, 72, 161, 253, 376, 192, 309, 180, 317, 210, 335, 234, 395, 273, 426, 49, 75$

B) $40, 75, 243, 26, 10, 135, 135, 234, 54, 70, 135, 260, 54, 100, 135, 351, 4, 25$

C) $175, 72, 295, 161, 253, 192, 376, 309, 180, 210, 317, 335, 234, 273, 395, 426, 49, 75$

D) $40, 243, 10, 135, 54, 135, 54, 135, 4, 75, 26, 135, 234, 70, 260, 100, 351, 25$

E) $175, 72, 253, 192, 180, 210, 234, 273, 49, 295, 161, 376, 309, 317, 335, 395, 426, 75$

Ans: A

93. Suppose products A and B are made from plastic, steel, and glass, with the number of units of each raw material required for each product given by the table below.

	Plastic	Steel	Glass
Product A	4	1	0.50
Product B	5	0.50	2

Because of transportation costs to the firm's two plants, X and Y, the unit costs for some of the raw materials are different. The table below gives the unit costs for each of the raw materials at the two plants.

	Plant X	Plant Y
Plastic	10	8
Steel	23	27
Glass	15	13

Using the information just given, find the total cost of producing each of the products at each of the factories.

A) $\begin{bmatrix} 80 & 14 & 21 \\ 227 & 36.5 & 65.5 \\ 125 & 21.5 & 33.5 \end{bmatrix}$

B) $\begin{bmatrix} 40 & 40 \\ 23 & 13.5 \\ 7.5 & 26 \end{bmatrix}$

C) $\begin{bmatrix} 40 & 23 & 7.5 \\ 40 & 13.5 & 26 \end{bmatrix}$

D) $\begin{bmatrix} 70.5 & 65.5 \\ 91.5 & 79.5 \end{bmatrix}$

E) $\begin{bmatrix} 55.5 & 73.5 \\ 87.5 & 77.5 \end{bmatrix}$

Ans: D

94. Consider the original budget matrix. Assume there is a 20% increase in manufacturing, a 3% increase in office, a 6% increase in sales, a 20% increase in shipping, a 6% increase in accounting, and a 4% decrease in management. Find the new budget matrix by developing a matrix *A* to represent these departmental increases and then computing the matrix *BA*.

	Mfg.	Office	Sales	Shp.	Act.	Mgt.
Supplies	0.7	8.5	10.2	1.1	5.6	3.6
Phone	0.5	0.2	6.1	1.3	0.2	1.0
Transp.	2.2	0.4	8.8	1.2	1.2	4.8
Salaries	251.8	63.4	81.6	35.2	54.3	144.2
Utilities	30.0	1.0	1.0	1.0	1.0	1.0
Materials	788.9		0	0		0
	0			0		

Mfg. = Manugacturing; Shp. = Shipping;

Act. = Accounting; Mgt. = Management

$$B=\begin{bmatrix} 0.7 & 8.5 & 10.2 & 1.1 & 5.6 & 3.6 \\ 0.5 & 0.2 & 6.1 & 1.3 & 0.2 & 1.0 \\ 2.2 & 0.4 & 8.8 & 1.2 & 1.2 & 4.8 \\ 251.8 & 63.4 & 81.6 & 35.2 & 54.3 & 144.2 \\ 30.0 & 1.0 & 1.0 & 1.0 & 1.0 & 1.0 \\ 788.9 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

A) $$BA=\begin{bmatrix} 0.840 & 8.755 & 10.812 & 1.320 & 5.936 & 3.744 \\ 0.600 & 0.206 & 6.466 & 1.560 & 0.212 & 1.04 \\ 2.640 & 0.412 & 9.328 & 1.440 & 1.272 & 4.992 \\ 302.160 & 65.302 & 86.496 & 42.240 & 57.558 & 149.968 \\ 36.000 & 1.030 & 1.060 & 1.200 & 1.060 & 1.04 \\ 946.680 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

B) $$BA=\begin{bmatrix} 14 & 25.5 & 61.2 & 22 & 33.6 & 14.4 \\ 10 & 0.6 & 36.6 & 26 & 1.2 & 4 \\ 44 & 1.2 & 52.8 & 24 & 7.2 & 19.2 \\ 5036 & 190.2 & 489.6 & 704 & 325.8 & 576.8 \\ 600 & 3 & 6 & 20 & 6 & 4 \\ 15778 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

C)

$$BA = \begin{bmatrix} 0.560 & 8.245 & 9.588 & 0.880 & 5.264 & 3.744 \\ 0.400 & 0.194 & 5.734 & 1.040 & 0.188 & 1.04 \\ 1.760 & 0.388 & 8.272 & 0.960 & 1.128 & 4.992 \\ 201.440 & 61.498 & 76.704 & 28.160 & 51.042 & 149.968 \\ 24.000 & 0.970 & 0.940 & 0.800 & 0.940 & 1.04 \\ 631.120 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

D)

$$BA = \begin{bmatrix} 0.560 & 8.245 & 9.588 & 0.880 & 5.264 & 3.456 \\ 0.400 & 0.194 & 5.734 & 1.040 & 0.188 & 0.960 \\ 1.760 & 0.388 & 8.272 & 0.960 & 1.128 & 4.608 \\ 201.440 & 61.498 & 76.704 & 28.160 & 51.042 & 138.432 \\ 24.000 & 0.970 & 0.940 & 0.800 & 0.940 & 0.960 \\ 631.120 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

E)

$$BA = \begin{bmatrix} 0.840 & 8.755 & 10.812 & 1.320 & 5.936 & 3.456 \\ 0.600 & 0.206 & 6.466 & 1.560 & 0.212 & 0.960 \\ 2.640 & 0.412 & 9.328 & 1.440 & 1.272 & 4.608 \\ 302.160 & 65.302 & 86.496 & 42.240 & 57.558 & 138.432 \\ 36.000 & 1.030 & 1.060 & 1.200 & 1.060 & 0.960 \\ 946.680 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

Ans: E

95.

Use the indicated row operation to change matrix *A*, where $A = \left[\begin{array}{ccc|c} 1 & -3 & -2 & -7 \\ 3 & 2 & 3 & 0 \\ 4 & 3 & 3 & 1 \end{array}\right]$.

Add -4 times row 1 to row 3 of matrix *A* and place the result in row 3 to get 0 in row 3, column 1.

A) $\left[\begin{array}{ccc|c} 1 & -3 & -2 & -7 \\ 3 & 2 & 3 & 0 \\ 0 & 12 & 8 & 28 \end{array}\right]$

B) $\left[\begin{array}{ccc|c} 1 & -3 & -2 & -7 \\ 3 & 2 & 3 & 0 \\ 0 & -1 & -1 & -3 \end{array}\right]$

C) $\left[\begin{array}{ccc|c} -4 & 12 & 8 & 28 \\ 3 & 2 & 3 & 0 \\ 0 & 15 & 11 & 29 \end{array}\right]$

D) $\left[\begin{array}{ccc|c} -4 & 12 & 8 & 28 \\ 3 & 2 & 3 & 0 \\ 0 & -1 & -1 & -3 \end{array}\right]$

E) $\left[\begin{array}{ccc|c} 1 & -3 & -2 & -7 \\ 3 & 2 & 3 & 0 \\ 0 & 15 & 11 & 29 \end{array}\right]$

Ans: E

96. Write the augmented matrix associated with the system.

$$\begin{cases} x - 4y + 5z = 2 \\ 2x \quad\quad + 2z = 1 \\ x + 2y + z = 1 \end{cases}$$

A) $\left[\begin{array}{ccc|c} 0 & -4 & 5 & 2 \\ 2 & 0 & 2 & 1 \\ 0 & 2 & 0 & 1 \end{array}\right]$

B) $\left[\begin{array}{ccc|c} 1 & 2 & 1 & 2 \\ -4 & 0 & 2 & 1 \\ 5 & 2 & 1 & 1 \end{array}\right]$

C) $\left[\begin{array}{ccc|c} 1 & -4 & 5 & 2 \\ 2 & 0 & 2 & 1 \\ 1 & 2 & 1 & 1 \end{array}\right]$

D) $\left[\begin{array}{ccc|c} 1 & 4 & 5 & 2 \\ 2 & 0 & 2 & 1 \\ 1 & 2 & 1 & 1 \end{array}\right]$

E) $\left[\begin{array}{ccc|c} 2 & 1 & -4 & 5 \\ 1 & 2 & 0 & 2 \\ 1 & 1 & 2 & 1 \end{array}\right]$

Ans: C

97. Write the augmented matrix associated with the system.

$$\begin{cases} x+2y+2z=3 \\ x-2y \quad\quad = 4 \\ \quad y - z = 1 \end{cases}$$

A) $\left[\begin{array}{ccc|c} 1 & 2 & 2 & 3 \\ 1 & -2 & 0 & 4 \\ 0 & 1 & -1 & 1 \end{array}\right]$

B) $\left[\begin{array}{ccc|c} 1 & 2 & 2 & 3 \\ 1 & 2 & 0 & 4 \\ 0 & 1 & 1 & 1 \end{array}\right]$

C) $\left[\begin{array}{ccc|c} 1 & 2 & 2 & 3 \\ 1 & -2 & 1 & 4 \\ 1 & 1 & -1 & 1 \end{array}\right]$

D) $\left[\begin{array}{ccc|c} 0 & 2 & 2 & 3 \\ 0 & -2 & 1 & 4 \\ 1 & 0 & 0 & 1 \end{array}\right]$

E) $\left[\begin{array}{ccc|c} 3 & 1 & 2 & 2 \\ 4 & 1 & -2 & 0 \\ 1 & 0 & 1 & -1 \end{array}\right]$

Ans: A

98. The given matrix is an augmented matrix used in the solution of a system of linear equations. What is the solution of the system?

$$\left[\begin{array}{ccc|c} 1 & 0 & 0 & 2 \\ 0 & 1 & 0 & \frac{1}{5} \\ 0 & 0 & 1 & -6 \end{array}\right]$$

A) $x=2,\ y=\dfrac{1}{5},\ z=6$

B) $x=-6,\ y=\dfrac{1}{5},\ z=2$

C) $x=1,\ y=0,\ z=0$

D) $x=2,\ y=\dfrac{1}{5},\ z=-6$

E) $x=1,\ y=1,\ z=1$

Ans: D

99. The given matrix is an augmented matrix used in the solution of a system of linear equations. What is the solution of the system?

$$\left[\begin{array}{ccc|c} 1 & 0 & 0 & -8 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & \frac{1}{2} \end{array}\right]$$

A) $x = 8,\ y = 2,\ z = \dfrac{1}{2}$

B) $x = -8,\ y = 2,\ z = \dfrac{1}{2}$

C) $x = 1,\ y = 0,\ z = 0$

D) $x = 1,\ y = 1,\ z = 1$

E) $x = \dfrac{1}{2},\ y = 2,\ z = -8$

Ans: B

100. The given matrix is an augmented matrix used in the solution of a system of linear equations. What is the solution of the system?

$$\left[\begin{array}{ccc|c} 1 & 0 & 0 & -7 \\ 0 & 1 & 0 & 8 \\ 0 & 0 & 1 & 0 \end{array}\right]$$

A) $x = 1,\ y = 0,\ z = 0$

B) $x = 0,\ y = 8,\ z = -7$

C) $x = 1,\ y = 1,\ z = 1$

D) $x = -7,\ y = 8,\ z = 0$

E) $x = 7,\ y = 8,\ z = 0$

Ans: D

101. The given matrix is an augmented matrix representing a system of linear equations. Find the solution of the system.

$$\left[\begin{array}{ccc|c} 1 & 5 & 2 & 7 \\ 0 & -2 & 3 & 9 \\ 0 & 1 & 3 & 0 \end{array}\right]$$

A) $x = 7,\ y = 9,\ z = 0$

B) $x = 18,\ y = -3,\ z = 1$

C) $x = 20,\ y = -3,\ z = 1$

D) $x = 19,\ y = 3,\ z = 1$

E) $x = 22,\ y = 3,\ z = 0$

Ans: C

102. The given matrix is an augmented matrix representing a system of linear equations. Find the solution of the system.

$$\left[\begin{array}{ccc|c} 1 & 2 & 5 & -9 \\ 2 & -2 & 4 & -6 \\ 0 & 1 & -3 & 6 \end{array}\right]$$

A) $x = 25,\ y = -6,\ z = -2$
B) $x = -9,\ y = -6,\ z = 6$
C) $x = -43,\ y = 0,\ z = 48$
D) $x = 4,\ y = -12,\ z = -2$
E) $x = 1,\ y = 0,\ z = -2$
Ans: E

103. The given matrix is an augmented matrix representing a system of linear equations. Find the solution of the system.

$$\left[\begin{array}{ccc|c} 1 & 1 & 1 & 4 \\ 3 & -2 & 4 & 4 \\ 1 & 2 & 1 & 4 \end{array}\right]$$

A) $x = 12,\ y = 0,\ z = -8$
B) $x = 4,\ y = 4,\ z = 4$
C) $x = 12,\ y = 8,\ z = 56$
D) $x = 20,\ y = -8,\ z = -8$
E) $x = -8,\ y = 0,\ z = -16$
Ans: A

104. Use row operations on the augmented matrix to solve the given system of linear equations.

$$\begin{cases} x + y - z = 0 \\ x + 2y + 3z = -6 \\ 2x - y - 13z = 17 \end{cases}$$

A) $x = 0,\ y = -6,\ z = 17$
B) $x = 1,\ y = -2,\ z = -1$
C) $x = 169,\ y = -134,\ z = -1$
D) $x = -169,\ y = 134,\ z = 35$
E) $x = 1,\ y = 49,\ z = -35$
Ans: B

105. Use row operations on the augmented matrix to solve the given system of linear equations.

$$\begin{cases} x + 2y - z = 2 \\ 2x + 5y - 2z = 7 \\ -x + y + 5z = -13 \end{cases}$$

A) $x = 19,\ y = 11,\ z = -5$
B) $x = 2,\ y = 7,\ z = -13$
C) $x = -17,\ y = 12,\ z = -5$
D) $x = -9,\ y = 3,\ z = -5$
E) $x = 12,\ y = 9,\ z = 5$
Ans: D

106. Use row operations on the augmented matrix to solve the given system of linear equations.

$$\begin{cases} 6x + 4y - 8z = -8 \\ 6x - 6y + 4z = 4 \\ 8x + 12y + 2z = 2 \end{cases}$$

A) $x = 0,\ y = 0,\ z = 1$
B) $x = -8,\ y = 4,\ z = 2$
C) $x = 0,\ y = 0,\ z = -1$
D) $x = -2,\ y = -2,\ z = -1$
E) $x = 1,\ y = -1,\ z = 3$
Ans: A

107. Use row operations on the augmented matrix to solve the given system of linear equations.

$$\begin{cases} -3x + 6y - 9z = 3 \\ x - y - 2z = 0 \\ 5x + 5y - 7z = 63 \end{cases}$$

A) $x = 3,\ y = 0,\ z = 63$
B) $x = -8,\ y = -6,\ z = -1$
C) $x = 8,\ y = 6,\ z = 1$
D) $x = 6,\ y = 4,\ z = 2$
E) $x = 1,\ y = 0,\ z = -1$
Ans: C

108. Use row operations on the augmented matrix to solve the given system of linear equations.

$$\begin{cases} x_1 + 3x_2 + 2x_3 + 2x_4 = 3 \\ x_1 + x_2 + 3x_3 = 3 \\ 2x_1 + 2x_3 - 3x_4 = 6 \\ x_1 - 3x_2 = 3 \end{cases}$$

A) $x_1 = 3,\ x_2 = 3,\ x_3 = 6,\ x_4 = 3$
B) $x_1 = 3,\ x_2 = 0,\ x_3 = 0,\ x_4 = 0$
C) $x_1 = 2,\ x_2 = 1,\ x_3 = -1,\ x_4 = 1$
D) $x_1 = 4,\ x_2 = -1,\ x_3 = 1,\ x_4 = -1$
E) $x_1 = -3,\ x_2 = 0,\ x_3 = 0,\ x_4 = 0$
Ans: B

109. Use row operations on the augmented matrix to solve the given system of linear equations.

$$\begin{cases} x + 4y - 6z - 3w = 5 \\ x - y + 2w = -3 \\ x + z + w = 3 \\ y + z + w = 2 \end{cases}$$

A) $x = 5,\ y = -3,\ z = 3,\ w = 2$
B) $x = -2,\ y = 3,\ z = 2,\ w = 2$
C) $x = 3,\ y = 2,\ z = 2,\ w = -2$
D) $x = 4,\ y = 1,\ z = 3,\ w = -3$
E) $x = 2,\ y = 3,\ z = 1,\ w = -1$
Ans: C

110. A system of linear equations and a reduced matrix for the system are given. Use the reduced matrix to find the general solution of the system, if one exists.

$$\begin{cases} x + 2y + 3z = 5 \\ 2x - y = 15 \\ x + 2y + 3z = 10 \end{cases} \qquad \left[\begin{array}{ccc|c} 1 & 0 & \frac{3}{5} & 0 \\ 0 & 1 & \frac{6}{5} & 0 \\ 0 & 0 & 0 & 1 \end{array}\right]$$

A) $x = 5,\ y = 15,\ z = 10$
B) $x = 0,\ y = 0,\ z = 1$
C) $x = -1,\ y = 3,\ z = -2$
D) $x = \frac{3}{5},\ y = \frac{6}{5},\ z = 1$
E) no solution
Ans: E

111. A system of linear equations and a reduced matrix for the system are given. Use the reduced matrix to find the general solution of the system, if one exists.

$$\begin{cases} 2x+3y+4z=-6 \\ x+2y+2z=-3 \\ x+y+2z=-6 \end{cases} \quad \left[\begin{array}{ccc|c} 1 & 0 & 2 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{array}\right]$$

A) $x=-6,\ y=-3,\ z=-6$

B) $x=0,\ y=0,\ z=1$

C) $x=2,\ y=0,\ z=0$

D) $x=2,\ y=0,\ z=1$

E) no solution

Ans: E

112. A system of linear equations and a reduced matrix for the system are given. Use the reduced matrix to find the general solution of the system, if one exists.

$$\begin{cases} 2x+y-z=14 \\ x-y-z=8 \\ 3x+3y-z=20 \end{cases} \quad \left[\begin{array}{ccc|c} 1 & 0 & -\frac{2}{3} & \frac{22}{3} \\ 0 & 1 & \frac{1}{3} & -\frac{2}{3} \\ 0 & 0 & 0 & 0 \end{array}\right]$$

A) $x=\dfrac{2+2z}{3}z,\ y=\dfrac{-2-z}{3},$ for any real number z.

B) $x=\dfrac{22+2z}{3},\ y=\dfrac{-2-z}{3},$ for any real number z.

C) $x=\dfrac{24}{3}z,\ y=\dfrac{-3}{3}z,$ for any real number z.

D) $x=\dfrac{22-2z}{3},\ y=\dfrac{-2+z}{3},$ for any real number z.

E) no solution

Ans: B

113. A system of linear equations and a reduced matrix for the system are given. Use the reduced matrix to find the general solution of the system, if one exists.

$$\begin{cases} x - y + z = 12 \\ 3x + 2z = 28 \\ x - 4y + 2z = 20 \end{cases} \quad \left[\begin{array}{ccc|c} 1 & 0 & \frac{2}{3} & \frac{28}{3} \\ 0 & 1 & -\frac{1}{3} & -\frac{8}{3} \\ 0 & 0 & 0 & 0 \end{array}\right]$$

A) $x = -\dfrac{2}{3}z + \dfrac{28}{3},\ y = \dfrac{1}{3}z - \dfrac{8}{3}$, for any real number z.

B) $x = \dfrac{28}{3}z,\ y = \dfrac{8}{3}$z, for any real number z.

C) $x = -\dfrac{2}{3}z - \dfrac{8}{3},\ y = \dfrac{1}{3}z + \dfrac{28}{3}$, for any real number z.

D) $x = -\dfrac{2}{3}z - \dfrac{28}{3},\ y = \dfrac{1}{3}z + \dfrac{8}{3}$, for any real number z.

E) no solution

Ans: A

114. Describe the procedure for finding the general solution of a system of three linear equations involving *x*, *y*, and *z* if its reduced matrix indicate has one row of zeros at the bottom. You may assume that the coefficients in the augmented matrix correspond to variables in alphabetical order from left to right.

A) Use the reduced matrix to find equations for each variable *x*, *y*, and *z*. These equations can be used as the general solution to the system of equations.

B) Use the reduced matrix to find a general equation for *x* in terms of *y* and *z*. This equation can be used as the general solution to the system of equations.

C) Use the reduced matrix to solve for *x* and *y* in terms of *z*. These solutions will be valid for any real value of *z*.

D) Use the reduced matrix to solve for *z* in terms of *x* and *y*. This solution will be valid for any real value of x and y.

E) It is not possible to find a general solution for a system of three linear equations if its reduced matrix indicates an infinite number of solutions.

Ans: C

115. A system of equations may have a unique solution, an infinite number of solutions, or no solution. Use matrices to find the general solution of the following system.

$$\begin{cases} 5x + 5y + 5z = 0 \\ 10x - 5y - 5z = 0 \\ -5x + 10y + 10z = 0 \end{cases}$$

A) $x = 0,\ y = 1,\ z = 0$

B) $x = 1,\ y = z,$ for any real number z.

C) $x = 1,\ y = 1,\ z = 0$

D) $x = 0,\ y = -z,$ for any real number z.

E) no solution

Ans: D

116. A system of equations may have a unique solution, an infinite number of solutions, or no solution. Use matrices to find the general solution of the following system.

$$\begin{cases} 8x - 4y + 12z = 0 \\ 4x + 8y + 8z = 0 \\ 4x - 12y + 4z = 0 \end{cases}$$

A) $x = \frac{8}{5}z,\ y = \frac{1}{5}z,$ for any real number z.

B) $x = 1,\ y = 1,\ z = 0$

C) $x = -\frac{8}{5}z,\ y = -\frac{1}{5}z,$ for any real number z.

D) $x = -\frac{3}{5}z,\ y = -\frac{4}{5}z,$ for any real number z.

E) no solution

Ans: C

117. A system of equations may have a unique solution, an infinite number of solutions, or no solution. Use matrices to find the general solution of the following system, if a solution exists.

$$\begin{cases} x + 3y + 2z = 8 \\ 2x - y - 2z = 4 \\ 3x + 2y = 12 \end{cases}$$

A) $x = \frac{4}{7},\ y = -\frac{6}{7}z,\ z = 0$

B) $x = \frac{20}{7},\ y = \frac{12}{7},\ z = 0$

C) $x = \frac{4}{7}z + \frac{20}{7},\ y = \frac{12}{7} - \frac{6}{7}z,$ for any real number z.

D) $x = \frac{12}{7} + \frac{4}{7}z,\ y = \frac{20}{7} - \frac{6}{7}z,$ for any real number z.

E) no solution

Ans: C

118. A system of equations may have a unique solution, an infinite number of solutions, or no solution. Use matrices to find the general solution of the following system, if a solution exists.

$$\begin{cases} 2x + y - z = 3 \\ x - y + 2z = 2 \\ x + y - z = 0 \end{cases}$$

A) $x = 3,\ y = -7,\ z = -4$
B) $x = 3,\ y = 2,\ z = 0$
C) $x = 3,\ y = 11,\ z = 4$
D) $x = -3,\ y = 7,\ z = -4$
E) no solution
Ans: A

119. A system of equations may have a unique solution, an infinite number of solutions, or no solution. Use matrices to find the general solution of the following system, if a solution exists.

$$\begin{cases} 2x - y + z = 6 \\ 3x + y - 6z = -21 \\ x - y + 2z = 9 \end{cases}$$

A) $x = 6,\ y = -21,\ z = 9$
B) $x = 3,\ y = 12,\ z = 0$
C) $x = 1,\ y = 1,\ z = 0$
D) $x = z - 3,\ y = 3z - 12$
E) no solution
Ans: D

120. A system of equations may have a unique solution, an infinite number of solutions, or no solution. Use matrices to find the general solution of the following system, if a solution exists.

$$\begin{cases} x + y + z = 9 \\ x - y + z = 12 \end{cases}$$

A) $x = \frac{21}{2} - z,\ y = -\frac{3}{2},\ z = z,$ for any real number z.
B) $x = 9,\ y = 12,\ z = 0$
C) $x = \frac{9}{2},\ y = -6,\ z = 0$
D) $x = 0,\ y = 1,\ z = 0$
E) no solution
Ans: A

121. A system of equations may have a unique solution, an infinite number of solutions, or no solution. Use matrices to find the general solution of the following system, if a solution exists.

$$\begin{cases} 3x+2y+z=9 \\ x-\ \ y-z=6 \end{cases}$$

A) $x=9,\ y=6,\ z=0$

B) $x=\frac{21}{5}+\frac{1}{5}z,\ y=-\frac{9}{5}-\frac{4}{5}z,$ for any real number z.

C) $x=\frac{9}{5}+\frac{3}{5}z,\ y=-\frac{6}{5}-\frac{2}{5}z,$ for any real number z.

D) $x=\frac{21}{5}-\frac{1}{5}z,\ y=\frac{9}{5}+\frac{4}{5}z,$ for any real number z.

E) no solution

Ans: B

122. A system of equations may have a unique solution, an infinite number of solutions, or no solution. Use matrices to find the general solution of the following system, if a solution exists.

$$\begin{cases} x-4y+\ \ z=-8 \\ 2x-5y-5z=-18 \end{cases}$$

A) $x=\frac{32}{3}-\frac{25}{3}z,\ y=\frac{2}{3}-\frac{7}{3}z,$ for any real number z.

B) $x=-8z,\ y=-18z,$ for any real number z.

C) $x=-\frac{32}{3}+\frac{25}{3}z,\ y=-\frac{2}{3}+\frac{7}{3}z,$ for any real number z.

D) $x=-\frac{8}{3}+\frac{7}{3}z,\ y=-6+\frac{25}{3}z,$ for any real number z.

E) no solution

Ans: C

123. A system of equations may have a unique solution, an infinite number of solutions, or no solution. Use matrices to find the general solution of the following system, if a solution exists.

$$\begin{cases} 0.1x_1-0.1x_2-0.3x_3=\ \ \ 0 \\ 0.2x_1+0.3x_2+0.1x_3=-1.8 \\ 0.3x_1+0.7x_2+0.5x_3=-3.6 \end{cases}$$

A) $x_1=1.6x_3+3.6,\ x_2=1.4x_3+3.6,$ for any real number x_3.

B) $x_1=0,\ x_2=-1.8,\ x_3=-3.6$

C) $x_1=1.6x_3-3.6,\ x_2=-1.4x_3-3.6,$ for any real number x_3.

D) $x_1=-1.4x_3-3.6,\ x_2=1.6x_3-3.6,$ for any real number x_3.

E) No solution

Ans: C

124. Use technology to solve the system of equations, if a solution exists.

$$\begin{cases} 3x+2y+\ \ z-\ \ w=\ \ 6 \\ \ \ x-\ \ y-2z+2w=\ \ 4 \\ 2x+3y-\ \ z+\ \ \ w=\ \ 2 \\ -x+y+2z-2w=-4 \end{cases}$$

A) $x=6,\ y=4,\ z=2,\ w=-4$

B) $x=\frac{6}{5},\ y=\frac{2}{5},\ z=-w,$ for any real number w.

C) $x=-\frac{6}{5},\ y=\frac{4}{5},\ z=1,\ w=0$

D) $x=\frac{14}{5},\ y=-\frac{6}{5},\ z=w,$ for any real number w.

E) no solution

Ans: D

125. Use technology to solve the system of equations, if a solution exists.

$$\begin{cases} 2x_1+3x_2-\ \ x_3+3x_4+\ \ x_5=\ \ 110 \\ 3x_1+\ \ x_2-4x_3+3x_4-\ \ x_5=\ \ 0 \\ \ \ x_1+\ \ x_2+3x_3-4x_4+2x_5=\ \ 30 \\ \ \ x_1+2x_2-3x_3+2x_4-2x_5=-30 \\ 2x_1\qquad\qquad\qquad +4x_4-5x_5=-35 \end{cases}$$

A) $x_1=10,\ x_2=15,\ x_3=20,\ x_4=25,\ x_5=30$

B) $x_1=5,\ x_2=10,\ x_3=15,\ x_4=20,\ x_5=25$

C) $x_1=2,\ x_2=12,\ x_3=13,\ x_4=19,\ x_5=26$

D) $x_1=110,\ x_2=0,\ x_3=30,\ x_4=-30,\ x_5=-35$

E) no solution

Ans: B

126. Use technology to solve the system of equations, if a solution exists.

$$\begin{cases} \ \ x_1+2x_2-\ \ x_3+\ \ x_4=\ \ 18 \\ \ \ x_1+3x_2+4x_3+\ \ x_4=-12 \\ 2x_1+5x_2+2x_3+2x_4=\ \ 6 \\ 2x_1+3x_2-6x_3+2x_4=\ \ 18 \end{cases}$$

A) $x_1=-1,\ x_2=0,\ x_3=0,\ x_4=1$

B) $x_1=18,\ x_2=-12,\ x_3=6,\ x_4=18$

C) $x_1=-1,\ x_2=0,\ x_3=0,\ x_4=0$

D) $x_1=-1,\ x_2=0,\ x_3=0,\ x_4=x_4$

E) no solution

Ans: E

127. Use technology to solve the system of equations, if a solution exists.

$$\begin{cases} x_1 + 2x_2 + 3x_3 + 4x_4 = 14 \\ x_1 + 2x_2 + 4x_3 + 3x_4 = 14 \\ 2x_1 + 4x_2 + 6x_3 + 8x_4 = 28 \\ x_1 + 4x_2 + 3x_3 = 28 \end{cases}$$

A) $x_1 = 11x_4,\ x_2 = -2x_4,\ x_3 = x_4,$ for any real number x_4.

B) $x_1 = -11,\ x_2 = 2,\ x_3 = 1,\ x_4 = 0$

C) $x_1 = 14,\ x_2 = 14,\ x_3 = 28,\ x_4 = 28$

D) $x_1 = -11x_4,\ x_2 = 2x_4 + 7,\ x_3 = x_4,$ for any real number x_4.

E) no solution

Ans: D

128. Use technology to solve the system of equations, if a solution exists.

$$\begin{cases} 2x_1 - x_2 + x_3 + 3x_4 + 3x_5 = 56 \\ x_1 + x_2 + x_3 + 2x_4 + 2x_5 = 0 \\ 2x_1 + x_2 - x_3 + x_4 + x_5 = -24 \\ 4x_1 - x_2 - 3x_3 + x_4 + x_5 = 8 \end{cases}$$

A) $x_1 = 8 - x_4 - x_5,\ x_2 = -24,\ x_3 = 16 - x_4 - x_5,$ for any real numbers x_4 and x_5.

B) $x_1 = 6,\ x_2 = -24,\ x_3 = 14,\ x_4 = 8,\ x_5 = 0$

C) $x_1 = 56,\ x_2 = 0,\ x_3 = -24,\ x_4 = 8,\ x_5 = 0$

D) $x_1 = 8 + x_4 + x_5,\ x_2 = 24,\ x_3 = 16x_4 + 16x_5,$ for any real numbers x_4 and x_5.

E) no solution

Ans: A

129. Solve the following system of equations for *y*. This gives a formula for solving two equations in two variables for *y*.

$$\begin{cases} a_1x + b_1y = c_1 \\ a_2x + b_2y = c_2 \end{cases}$$

A)
$$y = \frac{a_2c_2 - a_1c_1}{a_2b_2 - a_1b_1}$$

B)
$$y = \frac{a_1c_1 + a_2c_2}{a_2b_1 + a_1b_2}$$

C)
$$y = \frac{a_2c_1 - a_1c_2}{a_2b_1 - a_1b_2}$$

D)
$$y = \frac{a_2b_1 - a_1b_2}{a_2c_1 - a_1c_2}$$

E)
$$y = \frac{a_2b_1 + a_1b_2}{a_2c_1 + a_1c_2}$$

Ans: C

130. A man has $\$376,000$ invested in three properties. One earns 12%, one 10%, and one 8%. His annual income from the properties is $\$36,000$ and the amount invested at 8% is twice that invested at 12%. How much is invested in each property?

A) \$45,120 at 12%, \$37,600 at 10% and \$30,080 at 8%
B) \$118,400 at 12%, \$59,200 at 10% and \$198,400 at 8%
C) \$102,400 at 12%, \$91,200 at 10% and \$182,400 at 8%
D) \$74,000 at 12%, \$148,000 at 10% and \$154,000 at 8%
E) \$80,000 at 12%, \$136,000 at 10% and \$160,000 at 8%

Ans: E

131. A man has $\$305,500$ invested in three properties. One earns 12%, one 10%, and one 8%. His annual income from the properties is $\$29,250$ and the amount invested at 8% is twice that invested at 12%. What is the annual income from each property?
A) $\$3,666,\ \$3,055,\ \$24,440$
B) $\$7,800,\ \$11,050,\ \$10,400$
C) $\$9,984,\ \$7,410,\ \$11,856$
D) $\$7,215,\ \$12,025,\ \$10,010$
E) $\$11,544,\ \$4,810,\ \$12,896$
Ans: B

132. A bank lent $\$1.68$ million for the development of three new products, with one loan at 6%, one loan at 7%, and the third loan at 8%. The amount lent at 8% was equal to the sum of the amounts lent at the other two rates, and the bank's annual income from the loans was $\$123,200$. How much was lent at each rate?
A) $\$280,000$ at 6%, $\$560,000$ at 7%, $\$840,000$ at 8%
B) $\$140,000$ at 6%, $\$840,000$ at 7%, $\$700,000$ at 8%
C) $\$140,000$ at 6%, $\$700,000$ at 7%, $\$840,000$ at 8%
D) $\$280,000$ at 6%, $\$700,000$ at 7%, $\$700,000$ at 8%
E) $\$140,000$ at 6%, $\$560,000$ at 7%, $\$980,000$ at 8%
Ans: A

133. A psychologist studying the effects of nutrition on the behavior of laboratory rats is feeding one group a combination of three foods: I, II, and III. Each of these foods contains three additives, A, B, and C, that are being used in the study. Each additive is a certain percentage of each of the foods as follows:

	Foods		
	I	**II**	**III**
Additive A	10%	30%	60%
Additive B	0%	4%	5%
Additive C	2%	2%	12%

If the diet requires 53 g per day of A, 4.5 g per day of B, and 8.4 g per day of C, find the number of grams per day of each food that must be used.
A) 80 gm of I, 50 gm of II, 50 gm of III
B) 227 gm of I, 65 gm of II, 18 gm of III
C) 218 gm of I, 60 gm of II, 22 gm of III
D) 89 gm of I, 55 gm of II, 46 gm of III
E) 209 gm of I, 55 gm of II, 26 gm of III
Ans: D

134. A brokerage house offers three stock portfolios. Portfolio I consists of 2 blocks of common stock and 1 municipal bond. Portfolio II consists of 4 blocks of common stock, 2 municipal bonds, and 3 blocks of preferred stock. Portfolio III consists of 7 blocks of common stock, 3 municipal bonds, and 3 blocks of preferred stock. A customer wants 23 blocks of common stock, 10 municipal bonds, and 9 blocks of preferred stock. How many units of each portfolio should be offered?

A) 4 portfolio I, 2 portfolio II, 1 portfolio III

B) 2 portfolio I, 2 portfolio II, 1 portfolio III

C) 1 portfolio I, 0 portfolio II, 3 portfolio III

D) 1 portfolio I, 1 portfolio II, 2 portfolio III

E) 4 portfolio I, 3 portfolio II, 0 portfolio III

Ans: C

135. Each ounce of substance A supplies 2% of the required nutrition a patient needs. Substance B supplies 15% of the required nutrition per ounce and substance C supplies 15% of required nutrition per ounce. If digestive restrictions require that substances A and C be given in equal amounts and that the amount of substance B be one-fifth of these other amounts, find the number of ounces of each substance that should be in the meal to provide 100% nutrition.

A) 6 oz of A, 2 oz of B, 4 oz of C

B) 4 oz of A, 1 oz of B, 4 oz of C

C) 5 oz of A, 3 oz of B, 6 oz of C

D) 3 oz of A, 1 oz of B, 3 oz of C

E) 5 oz of A, 1 oz of B, 5 oz of C

Ans: E

136. The King Trucking Company has an order for three products for delivery. The following table gives the particulars for the products.

	Type I	**Type II**	**Type III**
Unit volume (cubic feet)	10	8	20
Unit weight (pounds)	10	20	40
Unit value (dollars)	100	20	200

If the carrier can carry 6,000 cu ft, can carry 12,000 lb, and is insured for $36,600, how many units of each type can be carried?

A) 156 units of Type I, 390 units of Type II, 66 units of Type III

B) 156 units of Type I, 395 units of Type II, 63 units of Type III

C) 160 units of Type I, 400 units of Type II, 60 units of Type III

D) 260 units of Type I, 400 units of Type II, 10 units of Type III

E) 258 units of Type I, 395 units of Type II, 113 units of Type III

Ans: A

137. A botanist can purchase plant food of four different types, I, II, III, and IV. Each food comes in the same size bag, and the following table summarizes the number of grams of each of three nutrients that each bag contains.

	Foods (grams)			
	I	II	III	IV
Nutrient A	5	5	10	5
Nutrient B	10	5	30	10
Nutrient C	5	15	10	25

The botanist wants to use a food that has these nutrients in a different proportion and determines that he will need a total of $50,000$ g of A, $100,000$ g of B, and $100,000$ g of C. Let *x* = Type I bags, *y* = Type II bags, *z* = Type III bags, and *w* = Type IV bags, where *x*, *y*, *z*, and *w* are non-negative amounts.
Find a general solution that will determine the number of bags of each type of food that should be ordered.

A) $x = 2w,\ y = 500 - 2w,\ z = 25,000 + w,$ where *w* is any non-negative amount $\leq 25,0$

B) $x = w,\ y = 5,000 + 2w,\ z = 250 + w,$ where *w* is any non-negative amount ≤ 250.

C) $x = 5w,\ y = 500 - w,\ z = 2,500 - 2w,$ where *w* is any non-negative amount $\leq 2,500$.

D) $x = 3w,\ y = 5,000 - 2w,\ z = 2,500 - w,$ where *w* is any non-negative amount $\leq 2,50$

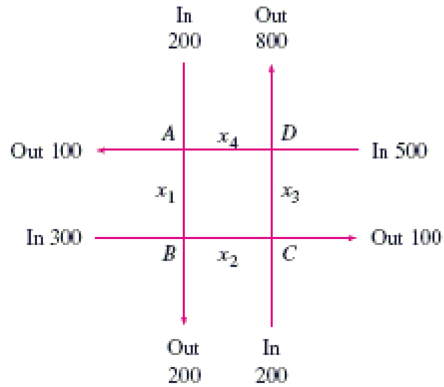
E) $x = 50,000,\ y = 100,000,\ z = 100,000,\ w = 0$

Ans: D

138. In the analysis of traffic flow, a certain city estimates the following situation for the "square" of its downtown district. In the following figure, the arrows indicate the flow of traffic. If x_1 represents the number of cars traveling between intersections *A* and *B*, x_2 represents the number of cars traveling between *B* and *C*, x_3 the number between *C* and *D*, and x_4 the number between *D* and *A*, we can formulate equations based on the principle that the number of vehicles entering an intersection equals the number leaving it. That is, for intersection *A* we obtain

$$200 + x_4 = 100 + x_1$$

Formulate equations for the traffic at *B*, *C*, and *D*.



A) *B*: $x_1 + 100 = x_2 + 200$; *C*: $800 + x_2 = x_3 + 800$; *D*: $x_3 + 300 = x_4 + 300$

B) *B*: $x_1 + 300 = x_2 + 300$; *C*: $200 + x_2 = x_3 + 100$; *D*: $x_3 + 200 = x_4 + 100$

C) *B*: $x_1 + 300 = x_2 + 200$; *C*: $200 + x_2 = x_3 + 100$; *D*: $x_3 + 500 = x_4 + 800$

D) *B*: $x_1 + 100 = x_2 + 200$; *C*: $300 + x_2 = x_3 + 300$; *D*: $x_3 + 200 = x_4 + 200$

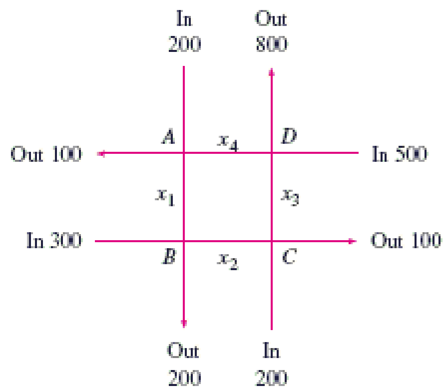
E) *B*: $x_1 + 100 = x_2 + 100$; *C*: $100 + x_2 = x_3 + 200$; *D*: $x_3 + 300 = x_4 + 300$

Ans: C

139. In the analysis of traffic flow, a certain city estimates the following situation for the "square" of its downtown district. In the following figure, the arrows indicate the flow of traffic. If x_1 represents the number of cars traveling between intersections *A* and *B*, x_2 represents the number of cars traveling between *B* and *C*, x_3 the number between *C* and *D*, and x_4 the number between *D* and *A*, we can formulate equations based on the principle that the number of vehicles entering an intersection equals the number leaving it. That is, for intersection *A* we obtain

$$200 + x_4 = 100 + x_1$$

Formulate equations for the traffic at *B*, *C*, and *D*. Solve the system of these four equations.



A) $x_1 = x_4 + 100,\ x_2 = x_4 + 200,\ x_3 = x_4 + 100,\ x_4 = x_4$

B) $, x_1 = x_4 + 100,\ x_2 = x_4 + 200,\ x_3 = x_4 + 300,\ x_4 = x_4$

C) $x_1 = x_4 + 100,\ x_2 = x_4 + 300,\ x_3 = x_4 + 100,\ x_4 = x_4$

D) $x_1 = x_4 + 100,\ x_2 = x_4 + 100,\ x_3 = x_4,\ x_4 = x_4$

E) $x_1 = x_4 + 100,\ x_2 = x_4 + 100,\ x_3 = x_4 + 300,\ x_4 = x_4$

Ans: B

140. Three different bacteria are cultured in one dish and feed on three nutrients. Each individual of species I consumes 1 unit of each of the first and second nutrients and 2 units of the third nutrient. Each individual of species II consumes 2 units of the first nutrient and 2 units of the third nutrient. Each individual of species III consumes 2 units of the first nutrient, 3 units of the second nutrient, and 5 units of the third nutrient. Let $x_1,\ x_2,$ and x_3 represent species I, II, and III respectively. If the culture is given $20,400$ units of the first nutrient, $27,600$ units of the second nutrient, and $48,000$ units of the third nutrient, how many of each species can be supported such that all of the nutrients are consumed?

A) $x_1 = 27,600 - 3x_3,\ x_2 = -3,600 + \frac{1}{2}x_3,\ x_3$ between $7,200$ and $9,200$

B) $x_1 = 20,400,\ x_2 = 27,600,\ x_3 = 48,000$

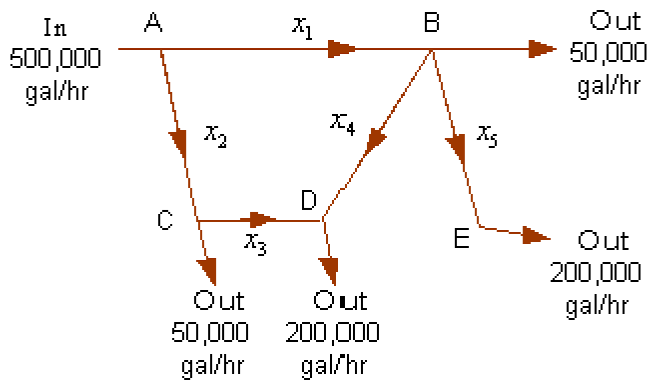
C) $x_1 = 27,600,\ x_2 = 3,600,\ x_3 = 0$

D) $x_1 = 2,760 + 2x_3,\ x_2 = 3,600 - x_3,\ x_3$ between $1,380$ and $3,600$

E) $x_1 = 2,760,\ x_2 = 360,\ x_3 = 4,600$

Ans: A

141. An irrigation system allows water to flow in the pattern shown in the figure below. Water flows into the system at *A* and exits at *B*, *C*, *D*, and *E* with the amounts shown. Using the fact that at each point the water entering equals the water leaving, formulate an equation for water flow at each of the five points and solve the system.



A) $x_1 = 500,\ x_2 = 500,\ x_3 = 450,\ x_4 = -200,\ x_5 = 200$ (in thousands of gallons)

B) $x_1 = 500,\ x_2 = 500,\ x_3 = 500,\ x_4 = -300,\ x_5 = 200$ (in thousands of gallons)

C) $x_1 = 200 - x_4,\ x_2 = 250 + x_4,\ x_3 = 450 + x_4,\ x_4 = 0,\ x_5 = 200$ (in thousands of gallons

D) $x_1 = 200,\ x_2 = 200,\ x_3 = 300,\ x_4 = -100,\ x_5 = 200$ (in thousands of gallons)

E) $x_1 = 250 + x_4,\ x_2 = 250 - x_4,\ x_3 = 200 - x_4,\ x_5 = 200,\ x_4 \le 200$ (in thousands of gallo

Ans: E

142. A trust account manager has $220,000 to be invested. The investment choices have current yields of 8%, 7%, and 10%. Suppose that the investment goal is to earn interest of $15,700, and risk factors make it prudent to invest some money in all three investments. Let *x* denote the amount invested at 8 %, *y* denote the amount invested at 7%, and *z* denote the amount invested at 10%. Find a general description for the amounts invested at the three rates.

A) $x = 220{,}000 - z,\ y = 15{,}700 - z,$ with $0 < z \le 15{,}700$

B) $x = 30{,}000 - 3z,\ y = 190{,}000 + 2z,$ with $0 < z \le 10{,}000$

C) $x = 190{,}000 - 2z,\ y = 30{,}000 + 3z,$ with $0 < z \le 95{,}000$

D) $x = 50{,}000 + 2z,\ y = 180{,}000 - 3z,$ with $0 < z \le 60{,}000$

E) $x = 40{,}000 + z,\ y = 210{,}000 - z,$ with $0 < z \le 210{,}000$

Ans: B

143. A trust account manager has $220,000 to be invested. The investment choices have current yields of 8%, 7%, and 10%. Suppose that the investment goal is to earn interest of $16,600, and risk factors make it prudent to invest some money in all three investments. If $25,000 is invested at 10%, how much will be invested at each of the other rates?
    A) $65,000 at 8%, $130,000 at 7%
    B) $35,000 at 8%, $160,000 at 7%
    C) $45,000 at 8%, $150,000 at 7%
    D) $25,000 at 8%, $170,000 at 7%
    E) $55,000 at 8%, $140,000 at 7%
    Ans: C

144. A trust account manager has $220,000 to be invested. The investment choices have current yields of 8%, 7%, and 10%. Suppose that the investment goal is to earn interest of $15,800, and risk factors make it prudent to invest some money in all three investments. What is the minimum amount that will be invested at 7%, and in this case how much will be invested at the other rates?
    A) $40,000 at 8%, $0 at 7%, $180,000 at 10%
    B) $30,000 at 8%, $170,000 at 7%, $20,000 at 10%
    C) $35,000 at 8%, $175,000 at 7%, $10,000 at 10%
    D) $40,000 at 8%, $180,000 at 7%, $0 at 10%
    E) $30,000 at 8%, $190,000 at 7%, $0 at 10%
    Ans: D

145. A trust account manager has $220,000 to be invested. The investment choices have current yields of 8%, 7%, and 10%. Suppose that the investment goal is to earn interest of $16,000, and risk factors make it prudent to invest some money in all three investments. What is the maximum amount that will be invested at 7%, and in this case how much will be invested at the other rates?
    A) $220,000 at 7%, $0 at 8%, $0 at 10%
    B) $190,000 at 7%, $10,000 at 8%, $20,000 at 10%
    C) $200,000 at 7%, $0 at 8%, $20,000 at 10%
    D) $20,000 at 7%, $0 at 8%, $200,000 at 10%
    E) $190,000 at 7%, $20,000 at 8%, $10,000 at 10%
    Ans: C

146. A brokerage house offers three stock portfolios. Portfolio I consists of 2 blocks of common stock and 1 municipal bond. Portfolio II consists of 4 blocks of common stock, 2 municipal bonds, and 3 blocks of preferred stock. Portfolio III consists of 2 blocks of common stock, 1 municipal bond, and 3 blocks of preferred stock. A customer wants 48 blocks of common stock, 24 municipal bonds, and 18 blocks of preferred stock. Let x equal the number of units of Portfolio I, y equal the number of units of Portfolio II, and z equal the number of units of Portolio III. If the numbers of the three portfolios offered must be integers, find the general solution that can be used to determine all possible offerings.

A) $x = z + 12,\ y = z + 6,\ z = 0$
B) $x = 48,\ y = 24,\ z = 18$
C) $x = -z + 6,\ y = z + 12,\ z = 0$
D) $x = z + 15,\ y = -z + 5,\ z \le 5$
E) $x = z + 12,\ y = -z + 6,\ z \le 6$
Ans: E

147. If A is a 8×8 matrix and B is its inverse, what does the product AB equal?

A) 8×8 zero matrix
B) 8×8 identity matrix
C) 64×64 identity matrix
D) 64×64 zero matrix
E) matrix A
Ans: B

148. If $C = \begin{bmatrix} 2 & -5 & 12 \\ -1 & 6 & -13 \\ 1 & -3 & 3 \end{bmatrix}$ and $D = \frac{1}{28}\begin{bmatrix} 21 & 21 & 7 \\ 10 & 6 & -14 \\ 3 & -1 & -7 \end{bmatrix}$, are C and D inverse matrices?

A) yes
B) no
Ans: A

149. If $A = \begin{bmatrix} 2 & 3 & 2 \\ 0 & 0 & 4 \\ 2 & 0 & 2 \end{bmatrix}$ and $B = \begin{bmatrix} 0 & -\frac{1}{4} & \frac{1}{2} \\ \frac{1}{3} & 0 & -\frac{1}{3} \\ 0 & \frac{1}{4} & 0 \end{bmatrix}$, does $B = A^{-1}$?

A) yes
B) no
Ans: A

150. If $D = \begin{bmatrix} 0 & 2 & -5 \\ -3 & 0 & 4 \\ 0 & -1 & 0 \end{bmatrix}$ and $C = -\frac{1}{2}\begin{bmatrix} 2 & 4 & 2 \\ 0 & 0 & 6 \\ 2 & 0 & 2 \end{bmatrix}$, does $C = D^{-1}$?

A) yes
B) no
Ans: B

151. Find the inverse matrix, if one exists.

$\begin{bmatrix} 3 & 4 \\ 2 & 3 \end{bmatrix}$

A) $\begin{bmatrix} 3 & 4 \\ -2 & -3 \end{bmatrix}$
B) $\begin{bmatrix} -4 & 3 \\ -3 & 2 \end{bmatrix}$
C) $\begin{bmatrix} -3 & 2 \\ 4 & -3 \end{bmatrix}$
D) $\begin{bmatrix} 3 & -4 \\ -2 & 3 \end{bmatrix}$
E) no inverse exists
Ans: D

152. Find the inverse matrix, if one exists.

$\begin{bmatrix} 4 & 16 \\ 2 & 8 \end{bmatrix}$

A) $\begin{bmatrix} 8 & 16 \\ -2 & -4 \end{bmatrix}$
B) $\begin{bmatrix} 4 & -16 \\ -2 & 8 \end{bmatrix}$
C) $\begin{bmatrix} -4 & 2 \\ -16 & 8 \end{bmatrix}$
D) $\begin{bmatrix} 2 & -4 \\ 8 & -16 \end{bmatrix}$
E) no inverse exists
Ans: E

153. Find the inverse matrix, if one exists.

$$\begin{bmatrix} 10 & 0 \\ 2 & 1 \end{bmatrix}$$

A) $\begin{bmatrix} \frac{1}{10} & 0 \\ -\frac{1}{5} & 1 \end{bmatrix}$

B) $\begin{bmatrix} 1 & 0 \\ -\frac{1}{5} & \frac{1}{10} \end{bmatrix}$

C) $\begin{bmatrix} -10 & 0 \\ -2 & -1 \end{bmatrix}$

D) $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

E) no inverse exists

Ans: A

154. Find the inverse of matrix A and check it by calculating AA^{-1}.

$$A = \begin{bmatrix} 8 & 0 & 0 \\ 0 & 8 & 0 \\ 0 & 0 & 8 \end{bmatrix}$$

A) $\begin{bmatrix} -\frac{1}{8} & 0 & 0 \\ 0 & -\frac{1}{8} & 0 \\ 0 & 0 & -\frac{1}{8} \end{bmatrix}$

B) $\begin{bmatrix} \frac{1}{8} & 0 & 0 \\ 0 & \frac{1}{8} & 0 \\ 0 & 0 & \frac{1}{8} \end{bmatrix}$

C) $\begin{bmatrix} 8 & 0 & 0 \\ 0 & 8 & 0 \\ 0 & 0 & 8 \end{bmatrix}$

D) $\begin{bmatrix} -8 & 0 & 0 \\ 0 & -8 & 0 \\ 0 & 0 & -8 \end{bmatrix}$

E) $\begin{bmatrix} \frac{1}{8} & 0 & 1 \\ 1 & \frac{1}{8} & 0 \\ 0 & 1 & \frac{1}{8} \end{bmatrix}$

Ans: B

155. Find the inverse of matrix A and check it by calculating AA^{-1}.

$$A = \begin{bmatrix} 3 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{bmatrix}$$

A) $\begin{bmatrix} -3 & 0 & 0 \\ 0 & -2 & 0 \\ 0 & 0 & -2 \end{bmatrix}$

B) $\begin{bmatrix} 0 & \frac{1}{3} & \frac{1}{3} \\ \frac{1}{2} & 0 & \frac{1}{2} \\ \frac{1}{2} & \frac{1}{2} & 0 \end{bmatrix}$

C) $\begin{bmatrix} \frac{1}{3} & 0 & 0 \\ 0 & \frac{1}{2} & 0 \\ 0 & 0 & \frac{1}{2} \end{bmatrix}$

D) $\begin{bmatrix} 3 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{bmatrix}$

E) $\begin{bmatrix} -\frac{1}{3} & 0 & 0 \\ 0 & -\frac{1}{2} & 0 \\ 0 & 0 & -\frac{1}{2} \end{bmatrix}$

Ans: C

156. Find the inverse of matrix A and check it by calculating AA^{-1}.

$$A = \begin{bmatrix} 0 & 2 & 1 \\ 3 & 0 & 1 \\ 2 & 1 & 1 \end{bmatrix}$$

A) $\begin{bmatrix} 1 & 1 & -2 \\ 1 & 2 & -3 \\ -3 & -4 & 6 \end{bmatrix}$

B) $\begin{bmatrix} 3 & 4 & -6 \\ -1 & -2 & 3 \\ -1 & -1 & 2 \end{bmatrix}$

C) $\begin{bmatrix} 1 & 2 & -3 \\ -1 & -1 & 2 \\ 3 & 4 & -6 \end{bmatrix}$

D) $\begin{bmatrix} 0 & -2 & -1 \\ -3 & 0 & -1 \\ -2 & -1 & -1 \end{bmatrix}$

E) $\begin{bmatrix} -1 & -1 & 2 \\ -1 & -2 & 3 \\ 3 & 4 & -6 \end{bmatrix}$

Ans: E

157. Find the inverse matrix, if one exists.

$$\begin{bmatrix} 2 & 1 & 4 \\ 1 & 5 & 4 \\ 1 & 1 & 1 \end{bmatrix}$$

A) $\begin{bmatrix} \frac{1}{11} & \frac{3}{11} & -\frac{16}{11} \\ \frac{3}{11} & -\frac{2}{11} & -\frac{4}{11} \\ -\frac{4}{11} & -\frac{1}{11} & \frac{9}{11} \end{bmatrix}$

B) $\begin{bmatrix} \frac{1}{2} & 1 & \frac{1}{4} \\ 1 & \frac{1}{5} & \frac{1}{4} \\ 1 & 1 & 1 \end{bmatrix}$

C) $\begin{bmatrix} \frac{4}{11} & \frac{1}{11} & -\frac{9}{11} \\ \frac{3}{11} & \frac{2}{11} & \frac{4}{11} \\ \frac{1}{11} & \frac{3}{11} & -\frac{16}{11} \end{bmatrix}$

D) $\begin{bmatrix} -\frac{1}{11} & -\frac{3}{11} & \frac{16}{11} \\ -\frac{3}{11} & \frac{2}{11} & \frac{4}{11} \\ \frac{4}{11} & \frac{1}{11} & -\frac{9}{11} \end{bmatrix}$

E) no inverse exists

Ans: D

158. Find the inverse matrix, if one exists.

$$\begin{bmatrix} 1 & 2 & 3 \\ -1 & 9 & 6 \\ -1 & 5 & 3 \end{bmatrix}$$

A) $\begin{bmatrix} -1 & 3 & -5 \\ -1 & 2 & -3 \\ \frac{4}{3} & -\frac{7}{3} & \frac{11}{3} \end{bmatrix}$

B) $\begin{bmatrix} 1 & \frac{1}{2} & \frac{1}{3} \\ -1 & \frac{1}{9} & \frac{1}{6} \\ -1 & \frac{1}{5} & \frac{1}{3} \end{bmatrix}$

C) $\begin{bmatrix} \frac{4}{3} & -\frac{7}{3} & \frac{11}{3} \\ -1 & 2 & -3 \\ -1 & 3 & -5 \end{bmatrix}$

D) $\begin{bmatrix} 1 & -2 & 3 \\ 1 & -3 & 5 \\ \frac{4}{3} & -\frac{7}{3} & \frac{11}{3} \end{bmatrix}$

E) no inverse exists

Ans: A

159. Find the inverse matrix, if one exists.

$$\begin{bmatrix} 3 & -1 & 8 \\ -1 & 0 & -2 \\ -1 & -3 & 4 \end{bmatrix}$$

A) $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

B) $\begin{bmatrix} 1 & 0 & 2 \\ -2 & 1 & -6 \\ -6 & 0 & -12 \end{bmatrix}$

C) $\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$

D) $\begin{bmatrix} -3 & 1 & -8 \\ 1 & 0 & 2 \\ 1 & 3 & -4 \end{bmatrix}$

E) no inverse exists

Ans: E

160. Find the inverse matrix, if one exists.

$$\begin{bmatrix} 3 & 4 & -4 \\ 4 & 2 & 1 \\ 2 & 6 & -9 \end{bmatrix}$$

A) $\begin{bmatrix} 0 & 0 & \frac{1}{2} \\ 0 & 1 & -2 \\ \frac{1}{2} & -\frac{1}{2} & -\frac{1}{2} \end{bmatrix}$

B) $\begin{bmatrix} 1 & 3 & -18 \\ 0 & -10 & 19 \\ 0 & 0 & 0 \end{bmatrix}$

C) $\begin{bmatrix} 0 & 0 & \frac{1}{2} \\ 0 & 1 & -2 \\ 1 & -\frac{1}{2} & -\frac{5}{2} \end{bmatrix}$

D) $\begin{bmatrix} 0 & 0 & \frac{1}{2} \\ 0 & 1 & -2 \\ 1 & -\frac{1}{2} & -\frac{1}{2} \end{bmatrix}$

E) no inverse exists

Ans: E

161.

Use a graphing calculator to find the inverse of $B=\begin{bmatrix} 2 & 3 & 0 & 1 & 4 \\ 1 & 0 & 2 & 3 & 1 \\ 0 & 1 & 0 & 2 & 3 \\ 1 & 0 & 2 & 2 & 1 \\ 1 & 0 & 0 & 0 & 4 \end{bmatrix}$.

A) $\begin{bmatrix} \frac{4}{13} & \frac{3}{13} & -\frac{3}{26} & 0 & -\frac{1}{13} \\ \frac{20}{13} & -\frac{11}{13} & -\frac{41}{26} & 1 & -\frac{5}{13} \\ -\frac{12}{13} & \frac{4}{13} & \frac{9}{26} & 0 & \frac{3}{13} \\ -\frac{20}{13} & \frac{11}{13} & \frac{27}{13} & -1 & \frac{5}{13} \\ \frac{5}{13} & -\frac{6}{13} & -\frac{7}{26} & 0 & \frac{2}{13} \end{bmatrix}$

B) $\begin{bmatrix} \frac{4}{13} & \frac{20}{13} & -\frac{12}{13} & -\frac{20}{13} & \frac{5}{13} \\ \frac{3}{13} & -\frac{11}{13} & \frac{4}{13} & \frac{11}{13} & -\frac{6}{13} \\ -\frac{3}{26} & -\frac{41}{26} & \frac{9}{26} & \frac{27}{13} & -\frac{7}{26} \\ 0 & 1 & 0 & -1 & 0 \\ -\frac{1}{13} & -\frac{5}{13} & \frac{3}{13} & \frac{5}{13} & \frac{2}{13} \end{bmatrix}$

C) $\begin{bmatrix} -\frac{1}{13} & -\frac{5}{13} & \frac{3}{13} & \frac{5}{13} & \frac{2}{13} \\ 0 & 1 & 0 & -1 & 0 \\ -\frac{3}{26} & -\frac{41}{26} & \frac{9}{26} & \frac{27}{13} & -\frac{7}{26} \\ \frac{3}{13} & -\frac{11}{13} & \frac{4}{13} & \frac{11}{13} & -\frac{6}{13} \\ \frac{4}{13} & \frac{20}{13} & -\frac{12}{13} & -\frac{20}{13} & \frac{5}{13} \end{bmatrix}$

D) $\begin{bmatrix} \frac{5}{13} & -\frac{20}{13} & -\frac{12}{13} & \frac{20}{13} & \frac{4}{13} \\ -\frac{6}{13} & \frac{11}{13} & \frac{4}{13} & -\frac{11}{13} & \frac{3}{13} \\ -\frac{7}{26} & \frac{27}{13} & \frac{9}{26} & -\frac{41}{26} & -\frac{3}{26} \\ 0 & -1 & 0 & 1 & 0 \\ \frac{2}{13} & \frac{5}{13} & \frac{3}{13} & -\frac{5}{13} & -\frac{1}{13} \end{bmatrix}$

E) $\begin{bmatrix} \frac{5}{13} & -\frac{6}{13} & -\frac{7}{26} & 0 & \frac{2}{13} \\ -\frac{20}{13} & \frac{11}{13} & \frac{27}{13} & -1 & \frac{5}{13} \\ -\frac{12}{13} & \frac{4}{13} & \frac{9}{26} & 0 & \frac{3}{13} \\ \frac{20}{13} & -\frac{11}{13} & -\frac{41}{26} & 1 & -\frac{5}{13} \\ \frac{4}{13} & \frac{3}{13} & -\frac{3}{26} & 0 & -\frac{1}{13} \end{bmatrix}$

Ans: B

162. The inverse of matrix *A* is given. Use the inverse to solve for *X*.

$A^{-1} = \begin{bmatrix} 3 & 2 \\ 1 & 1 \end{bmatrix}$ Solve $AX = \begin{bmatrix} 8 \\ 5 \end{bmatrix}$.

A) $X = \begin{bmatrix} -2 \\ 7 \end{bmatrix}$

B) $X = \begin{bmatrix} 29 \\ 21 \end{bmatrix}$

C) $X = \begin{bmatrix} 34 \\ 13 \end{bmatrix}$

D) $X = \begin{bmatrix} 3 \\ -1 \end{bmatrix}$

E) $X = \begin{bmatrix} 40 \\ 10 \end{bmatrix}$

Ans: C

163. The inverse of matrix A is given. Use the inverse to solve for X.

$A^{-1} = \begin{bmatrix} 1 & -2 \\ 1 & 3 \end{bmatrix}$ Solve $AX = \begin{bmatrix} 5 \\ -1 \end{bmatrix}$.

A) $X = \begin{bmatrix} 7 \\ 2 \end{bmatrix}$

B) $X = \begin{bmatrix} 10 \\ -1 \end{bmatrix}$

C) $X = \begin{bmatrix} 2 \\ 3 \end{bmatrix}$

D) $X = \begin{bmatrix} 3 \\ 8 \end{bmatrix}$

E) $X = \begin{bmatrix} -7 \\ -8 \end{bmatrix}$

Ans: A

164. The inverse of matrix A is given. Use the inverse to solve for X.

$A^{-1} = \begin{bmatrix} 3 & 0 & -3 \\ 1 & 1 & 2 \\ 1 & 2 & 1 \end{bmatrix}$ Solve $AX = \begin{bmatrix} -1 \\ 2 \\ 5 \end{bmatrix}$.

A) $X = \begin{bmatrix} 4 \\ 12 \\ 12 \end{bmatrix}$

B) $X = \begin{bmatrix} 12 \\ -9 \\ 8 \end{bmatrix}$

C) $X = \begin{bmatrix} -18 \\ 5 \\ 11 \end{bmatrix}$

D) $X = \begin{bmatrix} -18 \\ 11 \\ 8 \end{bmatrix}$

E) $X = \begin{bmatrix} 4 \\ -9 \\ 8 \end{bmatrix}$

Ans: D

165. Use the inverse to solve.

$$\begin{bmatrix} 0 & 2 & 1 \\ 3 & 0 & 1 \\ 1 & 1 & 1 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 3 \\ 3 \\ 8 \end{bmatrix}$$

A) $x = 14,\ y = 17,\ z = 20$
B) $x = -15,\ y = -10,\ z = 33$
C) $x = 33,\ y = 14,\ z = 14$
D) $x = 0,\ y = 0,\ z = 8$
E) $x = -10,\ y = -15,\ z = 33$
Ans: E

166. Use inverse matrices to solve the system of linear equations.

$$\begin{cases} x + 2y = 8 \\ 3x + 4y = 16 \end{cases}$$

A) $x = 40,\ y = 88$
B) $x = 32,\ y = 20$
C) $x = 0,\ y = 4$
D) $x = -8,\ y = 16$
E) $x = 8,\ y = 0$
Ans: C

167. Use inverse matrices to solve the system of linear equations.

$$\begin{cases} 3x - 4y = 24 \\ 2x + 3y = -1 \end{cases}$$

A) $x = \frac{168}{17},\ y = -\frac{1}{17}$
B) $x = 4,\ y = -3$
C) $x = 76,\ y = 45$
D) $x = 68,\ y = 51$
E) $x = \frac{74}{17},\ y = \frac{93}{17}$
Ans: B

168. Use inverse matrices to solve the system of linear equations.

$$\begin{cases} 5x - 2y = 7 \\ 3x + 3y = 21 \end{cases}$$

A) $x = 3,\ y = 4$
B) $x = -7,\ y = 84$
C) $x = 77,\ y = -42$
D) $x = -4,\ y = -3$
E) $x = 21,\ y = 126$
Ans: A

169. Use inverse matrices to find the solution of the system of equations.

$$\begin{cases} x + y + z = 6 \\ 2x + y + z = 5 \\ 2x + 2y + z = 8 \end{cases}$$

A) $x = -11,\ y = -13,\ z = 20$
B) $x = 0,\ y = 0,\ z = 8$
C) $x = 19,\ y = 25,\ z = 30$
D) $x = -1,\ y = 3,\ z = 4$
E) $x = -7,\ y = -1,\ z = -6$
Ans: D

170. Use inverse matrices to find the solution of the system of equations.

$$\begin{cases} 2x - y - 2z = 6 \\ 3x - y + z = -2 \\ x + y - z = 6 \end{cases}$$

A) $x = 22,\ y = 10,\ z = 14$
B) $x = 3,\ y = -2,\ z = -1$
C) $x = 1,\ y = 2,\ z = -3$
D) $x = -6,\ y = -6,\ z = 6$
E) $x = 2,\ y = 26,\ z = -2$
Ans: C

171. Use inverse matrices to find the solution of the system of equations.

$$\begin{cases} x - 2y + z = 0 \\ 2x + y - 2z = 8 \\ 3x + 2y - 3z = 4 \end{cases}$$

A) $x = 10,\ y = -32,\ z = 22$
B) $x = -12,\ y = 0,\ z = 4$
C) $x = -22,\ y = 16,\ z = -42$
D) $x = 12,\ y = 0,\ z = -4$
E) $x = -10,\ y = -16,\ z = -22$
Ans: E

172. Use inverse matrices and the graphing calculator to find the solution of the system of equations.

$$\begin{cases} 2x_1 + 3x_2 + x_4 + 3x_5 = 0 \\ x_1 + 2x_3 + 3x_4 + x_5 = 27 \\ x_2 + 2x_4 + 3x_5 = -14 \\ x_1 + 2x_3 + 2x_4 + x_5 = 23 \\ x_1 + 3x_5 = -23 \end{cases}$$

A) $\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{bmatrix} = \begin{bmatrix} -35 \\ -18 \\ 50 \\ 5 \\ 69 \end{bmatrix}$

B) $\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{bmatrix} = \begin{bmatrix} -5 \\ -4 \\ -4 \\ 9 \\ -10 \end{bmatrix}$

C) $\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{bmatrix} = \begin{bmatrix} 35 \\ 18 \\ 4 \\ -5 \\ -69 \end{bmatrix}$

D) $\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{bmatrix} = \begin{bmatrix} 9 \\ 50 \\ 10 \\ 5 \\ -4 \end{bmatrix}$

E) $\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{bmatrix} = \begin{bmatrix} 4 \\ 5 \\ 10 \\ 4 \\ -9 \end{bmatrix}$

Ans: E

173. Evaluate the determinant.

$$\begin{vmatrix} 4 & 5 \\ -1 & -4 \end{vmatrix}$$

A) 24
B) –11
C) 16
D) –21
E) 11
Ans: B

174. Use determinants to decide whether the matrix has an inverse.

$$\begin{bmatrix} 3 & 4 \\ 1 & -1 \end{bmatrix}$$

A) true
B) false
Ans: A

175. Evaluate the determinant.

$$\begin{vmatrix} 3 & -2 \\ 2 & 4 \end{vmatrix}$$

A) 16
B) 8
C) –14
D) 2
E) –2
Ans: A

176. Evaluate the determinant.

$$\begin{vmatrix} -1 & 3 \\ 4 & -8 \end{vmatrix}$$

A) 20
B) 29
C) –28
D) –4
E) –35
Ans: D

177. Use technology to find the determinant.

$$\begin{vmatrix} 3 & 2 & 1 \\ -1 & 1 & 2 \\ 0 & 1 & 1 \end{vmatrix}$$

A) –4
B) –3
C) –2
D) 5
E) 4
Ans: C

178. Use technology to find the determinant.

$$\begin{vmatrix} 2 & 2 & 2 \\ 3 & 1 & 1 \\ -1 & 0 & -1 \end{vmatrix}$$

A) –8
B) –4
C) 4
D) 0
E) 8
Ans: C

179. Use determinants to decide whether the matrix has an inverse.

$$\begin{bmatrix} 6 & 1 & 0 \\ 0 & 5 & 0 \\ 0 & 0 & -1 \end{bmatrix}$$

A) true
B) false
Ans: A

180. Use technology to find the determinant.

$$\begin{vmatrix} 3 & 2 & 5 & 2 \\ -1 & 2 & 0 & 5 \\ 4 & 3 & 0 & -1 \\ 0 & 4 & 2 & 1 \end{vmatrix}$$

A) –229
B) 395
C) 433
D) 505
E) –425
Ans: B

181. We have encoded messages by assigning the letters of the alphabet the numbers 1–26, with a blank assigned the number 27, and by using an encoding matrix *A* that converts the numbers to the coded message. To decode any message encoded by *A*, we must find the inverse of *A*, denoted by A^{-1}, and multiply A^{-1} times the coded message. Use this code and the given encoding matrix *A* to find the matrix that can be used to decode the given message.

Code matrix is

$$A = \begin{bmatrix} 2 & 5 \\ 1 & 3 \end{bmatrix}$$

The message is

138, 79, 72, 43, 143, 85, 57, 33, 101, 57

A) $\begin{bmatrix} 20 & 14 & 27 & 9 & 13 \\ 19 & 1 & 4 & 6 & 18 \end{bmatrix}$

B) $\begin{bmatrix} 19 & 1 & 4 & 20 & 12 \\ 20 & 14 & 27 & 1 & 12 \end{bmatrix}$

C) $\begin{bmatrix} 16 & 1 & 20 & 18 & 15 \\ 12 & 14 & 27 & 15 & 20 \end{bmatrix}$

D) $\begin{bmatrix} 20 & 11 & 19 & 20 & 13 \\ 1 & 5 & 27 & 9 & 5 \end{bmatrix}$

E) $\begin{bmatrix} 19 & 1 & 4 & 6 & 18 \\ 20 & 14 & 27 & 9 & 13 \end{bmatrix}$

Ans: E

182.

If the code matrix is $A = \begin{bmatrix} 0 & 2 & 1 \\ 3 & 1 & 1 \\ 2 & 1 & 1 \end{bmatrix}$, find the matrix that can be used to decode the message 47, 36, 35, 28, 92, 69, 63, 93, 74, 56, 71, 57, 17, 17, 16. (Because A and A^{-1} are 3 $\times$ 3 matrices, use triples of numbers.)

A) $\begin{bmatrix} 1 & 23 & 19 & 14 & 15 \\ 14 & 5 & 27 & 27 & 15 \\ 19 & 18 & 9 & 2 & 11 \end{bmatrix}$

B) $\begin{bmatrix} 8 & 4 & 7 & 15 & 6 \\ 5 & 9 & 27 & 18 & 21 \\ 1 & 14 & 6 & 27 & 14 \end{bmatrix}$

C) $\begin{bmatrix} 2 & 11 & 15 & 1 & 23 \\ 15 & 27 & 18 & 14 & 5 \\ 15 & 6 & 27 & 19 & 18 \end{bmatrix}$

D) $\begin{bmatrix} 1 & 23 & 19 & 14 & 1 \\ 14 & 5 & 27 & 27 & 3 \\ 19 & 18 & 9 & 2 & 11 \end{bmatrix}$

E) $\begin{bmatrix} 2 & 11 & 14 & 14 & 5 \\ 1 & 27 & 27 & 19 & 18 \\ 3 & 9 & 1 & 23 & 19 \end{bmatrix}$

Ans: D

183. We have encoded messages by assigning the letters of the alphabet the numbers 1–26, with a blank assigned the number 27, and by using an encoding matrix A that converts the numbers to the coded message. To decode any message encoded by A, we must find the inverse of A, denoted by A^{-1}, and multiply A^{-1} times the coded message. Use this code and the given encoding matrix A to find the matrix that can be used to decode the given message.

The code matrix is $A = \begin{bmatrix} 1 & 2 & 0 \\ 4 & 7 & 1 \\ 6 & 10 & 3 \end{bmatrix}$. The message is 33, 132, 214, 22, 110, 203, 34, 132, 201.

A) $\begin{bmatrix} 19 & 5 & 2 \\ 23 & 20 & 1 \\ 5 & 27 & 18 \end{bmatrix}$

B) $\begin{bmatrix} 1 & 12 & 2 \\ 16 & 5 & 1 \\ 16 & 27 & 18 \end{bmatrix}$

C) $\begin{bmatrix} 1 & 12 & 16 \\ 16 & 5 & 9 \\ 16 & 27 & 5 \end{bmatrix}$

D) $\begin{bmatrix} 19 & 1 & 16 \\ 21 & 18 & 9 \\ 7 & 27 & 5 \end{bmatrix}$

E) $\begin{bmatrix} 19 & 5 & 3 \\ 23 & 20 & 21 \\ 5 & 27 & 16 \end{bmatrix}$

Ans: C

184. Suppose that a government study showed that 80% of urban families remained in an urban area in the next generation (and 20% moved to a rural area), whereas 70% of rural families remained in a rural area in the next generation (and 30% moved to an urban area). This means that if $\begin{bmatrix} u_0 \\ r_0 \end{bmatrix}$ represents the current percents of urban families u_0 and of rural families r_0, then $\begin{bmatrix} u \\ r \end{bmatrix} = \begin{bmatrix} 0.80 & 0.30 \\ 0.20 & 0.70 \end{bmatrix} \begin{bmatrix} u_0 \\ r_0 \end{bmatrix}$ represents the percents of urban families u and rural families r one generation from now. Currently the population is 70% urban and 30% rural. Find the percents in each group one generation before this one.

A) $u = 65\%,\ r = 35\%$
B) $u = 116\%,\ r = 76\%$
C) $u = 80\%,\ r = 20\%$
D) $u = 65\%,\ r = 76\%$
E) $u = 20\%,\ r = 65\%$
Ans: C

185. Set up the system of equations and then solve it by using inverse matrices.

Transportation The Ace Freight Company has an order for two products to be delivered to two stores of a company. The table below gives information regarding the two products.

	Product I	**Product II**
Unit volume (cu ft)	20	30
Unit Weight (lb)	100	400

If truck A can carry 2400 cu ft and 23,000 lb, how many of each product can it carry?

A) 652 of I, 106 of II
B) 540 of I, 440 of II
C) 3300 of I, 1400 of II
D) 54 of I, 44 of II
E) 330 of I, 140 of II
Ans: D

186. Set up the system of equations and then solve it by using inverse matrices.

Transportation The Ace Freight Company has an order for two products to be delivered to two stores of a company. The table below gives information regarding the two products.

	Product I	**Product II**
Unit volume (cu ft)	20	30
Unit Weight (lb)	100	400

If truck B can carry 2350 cu ft and 21,000 lb, how many of each product can it carry?

A) 62 of I, 37 of II
B) 620 of I, 370 of II
C) 3140 of I, 1310 of II
D) 608 of I, 98 of II
E) 314 of I, 131 of II
Ans: A

187. Set up the system of equations and then solve it by using inverse matrices.

Nutrition A biologist has a 40% solution and a 10% solution of the same plant nutrient. How many cubic centimeters of each solution should be mixed to obtain 15 cc of a 28% solution?

A) 12 cc of 40%, 3 cc of 10%
B) 2 cc of 40%, 13 cc of 10%
C) 6 cc of 40%, 9 cc of 10%
D) 3 cc of 40%, 12 cc of 10%
E) 9 cc of 40%, 6 cc of 10%
Ans: E

188. A manufacturer of table saws has three models: Deluxe, Premium, and Ultimate, which must be painted, assembled, and packaged for shipping. The table gives the number of hours required for each of these operations for each type of table saw. If the manufacturer has 98 hours available per day for painting, 157 hours for assembly, and 36 hours for packaging, how many of each type of saw can be produced each day?

	Deluxe	Premium	Ultimate
Painting	1.6	2	2.4
Assembly	2	3	4
Packaging	0.5	0.5	1

A) 487 deluxe, 121 premium, 116 ultimate
B) 3 deluxe, 13 premium, 28 ultimate
C) 147 deluxe, 471 premium, 99 ultimate
D) 389 deluxe, 46 premium, 129 ultimate
E) 71 deluxe, 419 premium, 66 ultimate
Ans: B

189. Set up the system of equations and then solve it by using inverse matrices.

Transportation Ace Trucking Company has an order for three products, A, B, and C to be delivered. The table gives the volume in cubic feet, the weight in pounds, and the value for insurance in dollars for a unit of each of the products. If the carrier can carry 8000 cu ft and 12,400 lb and is insured for $52,600, how many units of each product can be carried?

	Product A	**Product B**	**Product C**
Unit volume (cu ft)	24	20	40
Weight (lb)	40	30	60
Value ($)	150	180	200

A) 84 A units, 130 B units, 530 C units
B) 37 A units, 54 B units, 175 C units
C) 100 A units, 120 B units, 80 C units
D) 214 A units, 230 B units, 300 C units
E) 333 A units, 95 B units, 99 C units
Ans: C

190. Set up the system of equations and then solve it by using inverse matrices.

Investment A company offers three mutual fund plans for its employees. Plan I consists of 4 blocks of common stock and 2 municipal bonds. Plan II consists of 8 blocks of common stock, 4 municipal bonds, and 6 blocks of preferred stock. Plan III consists of 14 blocks of common stock, 6 municipal bonds, and 6 blocks of preferred stock. An employee wants to combine these plans so that she has 82 blocks of common stock, 40 municipal bonds, and 36 blocks of preferred stock. How many units of each plan does she need?

A) 7 of Plan I, 7 of Plan II, 1 of Plan III
B) 6 of Plan I, 18 of Plan II, 26 of Plan III
C) 14 of Plan I, 2 of Plan II, 1 of Plan III
D) 7 of Plan I, 5 of Plan II, 1 of Plan III
E) 3 of Plan I, 3 of Plan II, 3 of Plan III

Ans: D

191. Set up the system of equations and then solve it by using inverse matrices.

Bee ancestry Because a female bee comes from a fertilized egg and a male bee comes from an unfertilized egg, the number (n_{t+2}) of ancestors of a male $t+2$ bee generations before the present generation is the sum of the number of ancestors t and $t+1$ generations $(n_t$ and $n_{t+1})$ before the present. If the numbers of ancestors of a male bee in a given generation t and in the previous generation are given by $N = \begin{bmatrix} n_t \\ n_{t+1} \end{bmatrix}$, then there is a matrix M such that the numbers of ancestors in the two generations preceding generation t are given by $MN = \begin{bmatrix} 0 & 1 \\ 1 & 1 \end{bmatrix}\begin{bmatrix} n_t \\ n_{t+1} \end{bmatrix} = \begin{bmatrix} n_{t+1} \\ n_t + n_{t+1} \end{bmatrix}$.

For a given male bee, the numbers of ancestors 7 and 8 generations back are given by $\begin{bmatrix} 21 \\ 34 \end{bmatrix}$.

Find the numbers of ancestors 6 and 7 generations back by multiplying both sides of $MN = \begin{bmatrix} 21 \\ 34 \end{bmatrix}$ by the inverse of M.

A) $\begin{bmatrix} 34 \\ 55 \end{bmatrix}$

B) $\begin{bmatrix} 13 \\ 21 \end{bmatrix}$

C) $\begin{bmatrix} 55 \\ 21 \end{bmatrix}$

D) $\begin{bmatrix} 13 \\ 34 \end{bmatrix}$

E) $\begin{bmatrix} 21 \\ 55 \end{bmatrix}$

Ans: B

192. The following technology matrix for a simple economy describes the relationship of certain industries to each other in the production of 1 unit of product.

$$\begin{array}{cc} \begin{matrix} \text{A} & \text{M} & \text{F} & \text{U} \end{matrix} & \\ \begin{bmatrix} 0.36 & 0.03 & 0.10 & 0.04 \\ 0.06 & 0.42 & 0.35 & 0.33 \\ 0.18 & 0.30 & 0.10 & 0.41 \\ 0.10 & 0.20 & 0.31 & 0.15 \end{bmatrix} & \begin{matrix} \text{Agriculture} \\ \text{Manufacturing} \\ \text{Fuels} \\ \text{Utilities} \end{matrix} \end{array}$$

For each 100 units of manufactured products produced, how many units of fuels are required?

A) 35
B) 99
C) 71
D) 170
E) 30

Ans: E

193. The following technology matrix for a simple economy describes the relationship of certain industries to each other in the production of 1 unit of product.

$$\begin{array}{cc} \begin{matrix} \text{A} & \text{M} & \text{F} & \text{U} \end{matrix} & \\ \begin{bmatrix} 0.36 & 0.03 & 0.10 & 0.04 \\ 0.06 & 0.42 & 0.25 & 0.33 \\ 0.18 & 0.15 & 0.10 & 0.41 \\ 0.10 & 0.20 & 0.31 & 0.15 \end{bmatrix} & \begin{matrix} \text{Agriculture} \\ \text{Manufacturing} \\ \text{Fuels} \\ \text{Utilities} \end{matrix} \end{array}$$

How many units of utilities are required to produce 50 units of agricultural products?

A) 5
B) 38
C) 20
D) 26.5
E) 46.5

Ans: A

194. For the economy, what industry is most dependent on utilities?

$$\begin{array}{ccccc} A & M & F & U & \\ \left[\begin{array}{cccc} 0.36 & 0.03 & 0.10 & 0.04 \\ 0.06 & 0.42 & 0.25 & 0.33 \\ 0.18 & 0.15 & 0.10 & 0.41 \\ 0.10 & 0.20 & 0.31 & 0.09 \end{array}\right. & & & \left.\right] & \begin{array}{l} \text{Agriculture} \\ \text{Manufacturing} \\ \text{Fuels} \\ \text{Utilities} \end{array} \end{array}$$

A) Agriculture
B) Utilities
C) Fuels
D) Manufacturing
E) All industries equally depend on utilities.
Ans: C

195. The following technology matrix describes the relationship of certain industries within the economy to each other. (A&F, agriculture and food; RM, raw materials; M, manufacturing; F, fuels industry; U, utilities; SI, service industries)

$$\begin{array}{cc} \begin{array}{cccccc} \text{A\&F} & \text{RM} & \text{M} & \text{F} & \text{U} & \text{SI} \end{array} & \\ \begin{bmatrix} 0.410 & 0.006 & 0 & 0.002 & 0 & 0.006 \\ 0.025 & 0.493 & 0.190 & 0.024 & 0.030 & 0.150 \\ 0.015 & 0.006 & 0.082 & 0.009 & 0.001 & 0.116 \\ 0.097 & 0.096 & 0.040 & 0.053 & 0.008 & 0.093 \\ 0.028 & 0.129 & 0.039 & 0.058 & 0.138 & 0.409 \\ 0.043 & 0.008 & 0.010 & 0.012 & 0.002 & 0.095 \end{bmatrix} & \begin{array}{c} \text{A\&F} \\ \text{RM} \\ \text{M} \\ \text{F} \\ \text{U} \\ \text{SI} \end{array} \end{array}$$

For each 1000 units of raw materials produced, how many units of agricultural and food products were required?
A) 424
B) 618
C) 25
D) 6
E) 0
Ans: D

196. The following technology matrix describes the relationship of certain industries within the economy to each other. (A&F, agriculture and food; RM, raw materials; M, manufacturing; F, fuels industry; U, utilities; SI, service industries)

$$\begin{array}{cc} \begin{array}{cccccc} \text{A\&F} & \text{RM} & \text{M} & \text{F} & \text{U} & \text{SI} \end{array} & \\ \begin{bmatrix} 0.410 & 0.008 & 0 & 0.002 & 0 & 0.006 \\ 0.025 & 0.493 & 0.190 & 0.024 & 0.030 & 0.120 \\ 0.015 & 0.006 & 0.082 & 0.009 & 0.001 & 0.116 \\ 0.097 & 0.096 & 0.040 & 0.053 & 0.008 & 0.093 \\ 0.028 & 0.129 & 0.039 & 0.058 & 0.138 & 0.409 \\ 0.043 & 0.006 & 0.010 & 0.012 & 0.002 & 0.095 \end{bmatrix} & \begin{array}{l} \text{A\&F} \\ \text{RM} \\ \text{M} \\ \text{F} \\ \text{U} \\ \text{SI} \end{array} \end{array}$$

For each 1000 units of raw materials produced, how many units of service were required?

A) 120
B) 555
C) 6
D) 839
E) 10
Ans: C

197. The following technology matrix describes the relationship of certain industries within the economy to each other. (A&F, agriculture and food; RM, raw materials; M, manufacturing; F, fuels industry; U, utilities; SI, service industries)

$$\begin{array}{cc} \begin{array}{cccccc} \text{A\&F} & \text{RM} & \text{M} & \text{F} & \text{U} & \text{SI} \end{array} & \\ \begin{bmatrix} 0.410 & 0.008 & 0 & 0.002 & 0 & 0.006 \\ 0.025 & 0.493 & 0.190 & 0.024 & 0.030 & 0.150 \\ 0.015 & 0.006 & 0.082 & 0.015 & 0.001 & 0.116 \\ 0.097 & 0.096 & 0.060 & 0.053 & 0.008 & 0.093 \\ 0.028 & 0.129 & 0.039 & 0.058 & 0.138 & 0.409 \\ 0.043 & 0.008 & 0.010 & 0.012 & 0.002 & 0.095 \end{bmatrix} & \begin{array}{l} \text{A\&F} \\ \text{RM} \\ \text{M} \\ \text{F} \\ \text{U} \\ \text{SI} \end{array} \end{array}$$

How many units of fuels were required to produce 1000 units of manufactured goods?

A) 6
B) 1.5
C) 15
D) 397
E) 60
Ans: E

198. The following technology matrix describes the relationship of certain industries within the economy to each other. (A&F, agriculture and food; RM, raw materials; M, manufacturing; F, fuels industry; U, utilities; SI, service industries)

$$\begin{array}{cc} \begin{array}{cccccc} \text{A\&F} & \text{RM} & \text{M} & \text{F} & \text{U} & \text{SI} \end{array} & \\ \begin{bmatrix} 0.410 & 0.008 & 0 & 0.002 & 0 & 0.006 \\ 0.025 & 0.493 & 0.190 & 0.024 & 0.030 & 0.150 \\ 0.015 & 0.006 & 0.082 & 0.009 & 0.001 & 0.116 \\ 0.097 & 0.096 & 0.040 & 0.053 & 0.005 & 0.093 \\ 0.028 & 0.129 & 0.039 & 0.059 & 0.138 & 0.409 \\ 0.043 & 0.008 & 0.010 & 0.012 & 0.002 & 0.095 \end{bmatrix} & \begin{array}{l} \text{A\&F} \\ \text{RM} \\ \text{M} \\ \text{F} \\ \text{U} \\ \text{SI} \end{array} \end{array}$$

How many units of fuels were required to produce 1000 units of power (utilities' goods)?

A) 5.9
B) 5
C) 59
D) 0.5
E) 159
Ans: B

199. The following technology matrix describes the relationship of certain industries within the economy to each other. (A&F, agriculture and food; RM, raw materials; M, manufacturing; F, fuels industry; U, utilities; SI, service industries)

$$\begin{array}{cc} \begin{array}{cccccc} \text{A\&F} & \text{RM} & \text{M} & \text{F} & \text{U} & \text{SI} \end{array} & \\ \begin{bmatrix} 0.493 & 0.008 & 0 & 0.002 & 0 & 0.006 \\ 0.025 & 0.082 & 0.190 & 0.024 & 0.030 & 0.150 \\ 0.015 & 0.006 & 0.053 & 0.009 & 0.001 & 0.116 \\ 0.097 & 0.096 & 0.040 & 0.138 & 0.008 & 0.093 \\ 0.028 & 0.129 & 0.039 & 0.058 & 0.095 & 0.409 \\ 0.043 & 0.008 & 0.010 & 0.012 & 0.002 & 0.41 \end{bmatrix} & \begin{array}{l} \text{A\&F} \\ \text{RM} \\ \text{M} \\ \text{F} \\ \text{U} \\ \text{SI} \end{array} \end{array}$$

Which industry is most dependent on its own goods for its operations?

A) agriculture and food
B) service
C) utilities
D) fuels
E) manufacturing
Ans: A

200. The following technology matrix describes the relationship of certain industries within the economy to each other. (A&F, agriculture and food; RM, raw materials; M, manufacturing; F, fuels industry; U, utilities; SI, service industries)

$$\begin{array}{c c}
\begin{array}{cccccc} \text{A\&F} & \text{RM} & \text{M} & \text{F} & \text{U} & \text{SI} \end{array} & \\
\begin{bmatrix} 0.493 & 0.008 & 0 & 0.002 & 0 & 0.006 \\ 0.025 & 0.082 & 0.190 & 0.024 & 0.030 & 0.150 \\ 0.015 & 0.006 & 0.053 & 0.009 & 0.001 & 0.116 \\ 0.097 & 0.096 & 0.040 & 0.138 & 0.008 & 0.093 \\ 0.028 & 0.129 & 0.039 & 0.058 & 0.095 & 0.409 \\ 0.043 & 0.008 & 0.010 & 0.012 & 0.002 & 0.41 \end{bmatrix} & \begin{array}{c} \text{A\&F} \\ \text{RM} \\ \text{M} \\ \text{F} \\ \text{U} \\ \text{SI} \end{array}
\end{array}$$

Which industry is least dependent on its own goods?
A) agriculture and food
B) service
C) utilities
D) fuels
E) manufacturing
Ans: E

201. The following technology matrix describes the relationship of certain industries within the economy to each other. (A&F, agriculture and food; RM, raw materials; M, manufacturing; F, fuels industry; U, utilities; SI, service industries)

$$\begin{array}{c c}
\begin{array}{cccccc} \text{A\&F} & \text{RM} & \text{M} & \text{F} & \text{U} & \text{SI} \end{array} & \\
\begin{bmatrix} 0.410 & 0.008 & 0 & 0.002 & 0 & 0.006 \\ 0.025 & 0.493 & 0.190 & 0.024 & 0.030 & 0.150 \\ 0.015 & 0.006 & 0.082 & 0.009 & 0.001 & 0.116 \\ 0.097 & 0.096 & 0.040 & 0.053 & 0.008 & 0.093 \\ 0.028 & 0.129 & 0.039 & 0.058 & 0.138 & 0.409 \\ 0.043 & 0.008 & 0.010 & 0.012 & 0.002 & 0.095 \end{bmatrix} & \begin{array}{c} \text{A\&F} \\ \text{RM} \\ \text{M} \\ \text{F} \\ \text{U} \\ \text{SI} \end{array}
\end{array}$$

Which is most dependent on the fuels industry?
A) manufacturing
B) utilities
C) agriculture and food
D) fuels
E) service
Ans: C

202. The following technology matrix describes the relationship of certain industries within the economy to each other. (A&F, agriculture and food; RM, raw materials; M, manufacturing; F, fuels industry; U, utilities; SI, service industries)

$$\begin{array}{cc} \begin{array}{cccccc} \text{A\&F} & \text{RM} & \text{M} & \text{F} & \text{U} & \text{SI} \end{array} & \\ \begin{bmatrix} 0.410 & 0.008 & 0 & 0.002 & 0 & 0.006 \\ 0.025 & 0.493 & 0.190 & 0.024 & 0.030 & 0.150 \\ 0.015 & 0.006 & 0.082 & 0.009 & 0.001 & 0.116 \\ 0.097 & 0.096 & 0.040 & 0.053 & 0.008 & 0.093 \\ 0.028 & 0.129 & 0.039 & 0.058 & 0.138 & 0.409 \\ 0.043 & 0.008 & 0.010 & 0.012 & 0.002 & 0.095 \end{bmatrix} & \begin{array}{l} \text{A\&F} \\ \text{RM} \\ \text{M} \\ \text{F} \\ \text{U} \\ \text{SI} \end{array} \end{array}$$

Which industry would be most affected by a rise in the cost of manufacturing?
A) manufacturing
B) utilities
C) fuels
D) service
E) agriculture and food
Ans: D

203. Suppose a primitive economy consists of two industries, farm products and farm machinery. Suppose also that its technology matrix is

$$\begin{array}{ccc} & \begin{array}{cc} \text{P} & \text{M} \end{array} & \\ A = & \begin{bmatrix} 0.5 & 0.1 \\ 0.1 & 0.3 \end{bmatrix} & \begin{array}{l} \text{Products} \\ \text{Machinery} \end{array} \end{array}$$

If surpluses of 94 units of farm products and 5 units of farm machinery are desired, find the gross production of each industry.
A) 198 units of farm products, 49 units of machinery
B) 47 units of farm products, 6 units of machinery
C) 48 units of farm products, 11 units of machinery
D) 195 units of farm products, 35 units of machinery
E) 190 units of farm products, 30 units of machinery
Ans: D

204. Suppose an economy has two industries, agriculture and minerals. Suppose further that the technology matrix for this economy is *A*.

$$\begin{array}{cc} & \begin{array}{cc} \text{A} & \text{M} \end{array} \\ A = & \begin{bmatrix} 0.3 & 0.1 \\ 0.1 & 0.2 \end{bmatrix} \end{array} \begin{array}{l} \\ \text{Agriculture} \\ \text{Minerals} \end{array}$$

If surpluses of 50 agricultural units and 70 mineral units are desired, find the gross production to the nearest unit for each industry.

A) agriculture = 28 , minerals = 51
B) agriculture = 22 , minerals = 19
C) agriculture = 85 , minerals = 98
D) agriculture = 60 , minerals = 320
E) agriculture = 60, minerals = 80
Ans: C

205. A primitive economy has a mining industry and a fishing industry, with technology matrix *A*.

$$\begin{array}{cc} & \begin{array}{cc} \text{M} & \text{F} \end{array} \\ A = & \begin{bmatrix} 0.25 & 0.05 \\ 0.05 & 0.40 \end{bmatrix} \end{array} \begin{array}{l} \\ \text{Mining} \\ \text{Fishing} \end{array}$$

If surpluses of 117 units of mining output and 28 units of fishing output are desired, find the gross production of each industry.

A) mining = 160, fishing = 60
B) mining = 86, fishing = 11
C) mining = 466, fishing = 12
D) mining = 31, fishing = 17
E) mining = 154, fishing = 34
Ans: A

206. Suppose the economy of an underdeveloped country has an agricultural industry and an oil industry, with technology matrix *A*.

$$\begin{array}{cc} & \begin{array}{cc} \text{A} & \text{O} \end{array} \\ A = & \begin{bmatrix} 0.3 & 0.1 \\ 0.2 & 0.1 \end{bmatrix} \begin{array}{l} \text{Agricultural products} \\ \text{Oil products} \end{array} \end{array}$$

If surpluses of 0 units of agricultural products and 680 units of oil products are desired, find the gross production of each industryto the nearest unit.

A) agricultural = 68, oil = 68
B) agricultural = 111, oil = 780
C) agricultural = 6,800, oil = 20,400
D) agricultural = 68, oil = 612
E) agricultural = 111, oil = 780
Ans: E

207. Suppose a simple economy with only an agricultural industry and a steel industry has the following technology matrix.

$$\begin{array}{cc} & \begin{array}{cc} \text{A} & \text{S} \end{array} \\ A = & \begin{bmatrix} 0.5 & 0.2 \\ 0.1 & 0.6 \end{bmatrix} \begin{array}{l} \text{Agriculture} \\ \text{Steel} \end{array} \end{array}$$

If surpluses of 14 units of agricultural products and 32 units of steel are desired, find the gross production to the nearest unit of each industry.

A) agriculture = 1, steel = 11
B) agriculture = 7, steel = 52
C) agriculture = 13, steel = 21
D) agriculture = 67, steel = 97
E) agriculture = 4, steel = 81
Ans: D

208. One sector of an economy consists of a mining industry and a manufacturing industry, with technology matrix *A*.

$$\begin{array}{cc} & \begin{array}{cc} \text{M} & \text{Mfg} \end{array} \\ A = & \begin{bmatrix} 0.2 & 0.4 \\ 0.3 & 0.3 \end{bmatrix} \end{array} \begin{array}{l} \\ \text{Mining} \\ \text{Manufacturing} \end{array}$$

Surpluses of 32 units of mining output and 263 units of manufacturing output are desired. Find the gross production of each industry.

A) mining = 1593, manufacturing = 717
B) mining = 290, manufacturing = 500
C) mining = 112, manufacturing = 88
D) mining = 80, manufacturing = 175
E) mining = 188, manufacturing = 456
Ans: B

209. An underdeveloped country has an agricultural industry and a manufacturing industry, with technology matrix *A*.

$$\begin{array}{cc} & \begin{array}{cc} \text{A} & \text{M} \end{array} \\ A = & \begin{bmatrix} 0.20 & 0.10 \\ 0.25 & 0.45 \end{bmatrix} \end{array} \begin{array}{l} \\ \text{Agriculture} \\ \text{Manufacturing} \end{array}$$

Surpluses of 4 units of agricultural products and 642 units of manufactured products are desired. Find the gross production of each industry.

A) agriculture = 61, manufacturing = 352
B) agriculture = 960, manufacturing = 1960
C) agriculture = 160, manufacturing = 1240
D) agriculture = 65, manufacturing = 290
E) agriculture = 149, manufacturing = 1235
Ans: C

210. A simple economy has an electronic components industry and a computers industry. Each unit of electronic components output requires inputs of 0.4 unit of electronic components and 0.2 unit of computers. Each unit of computer industry output requires inputs of 0.7 unit of electronic components and 0.1 unit of computers. Write the technology matrix for this simple economy. (E, electronics components industry; C, computer industry)

A) $$\begin{array}{cc} \begin{array}{cc} E & C \end{array} & \\ \begin{bmatrix} 0.4 & 0.7 \\ 0.2 & 0.1 \end{bmatrix} & \begin{array}{c} E \\ C \end{array} \end{array}$$

B) $$\begin{array}{cc} \begin{array}{cc} E & C \end{array} & \\ \begin{bmatrix} 0.6 & –0.2 \\ –0.7 & 0.9 \end{bmatrix} & \begin{array}{c} E \\ C \end{array} \end{array}$$

C) $$\begin{array}{cc} \begin{array}{cc} E & C \end{array} & \\ \begin{bmatrix} 0.4 & 0.2 \\ 0.7 & 0.1 \end{bmatrix} & \begin{array}{c} E \\ C \end{array} \end{array}$$

D) $$\begin{array}{cc} \begin{array}{cc} E & C \end{array} & \\ \begin{bmatrix} –1.0 & 2.0 \\ 7.0 & –4.0 \end{bmatrix} & \begin{array}{c} E \\ C \end{array} \end{array}$$

E) $$\begin{array}{cc} \begin{array}{cc} E & C \end{array} & \\ \begin{bmatrix} 0.2 & 0.4 \\ 0.1 & 0.7 \end{bmatrix} & \begin{array}{c} E \\ C \end{array} \end{array}$$

Ans: C

211. A simple economy has an electronic components industry and a computers industry. Each unit of electronic components output requires inputs of 0.3 unit of electronic components and 0.2 unit of computers. Each unit of computer industry output requires inputs of 0.6 unit of electronic components and 0.2 unit of computers. If surpluses of 649 units of electronic components and 10 units of computers are desired, find the gross production to the nearest unit of each industry.

A) electronics = 452, computers = 381
B) electronics = 1,185, computers = 901
C) electronics = 1,175, computers = 869
D) electronics = 2,130, computers =6,440
E) electronics = 197, computers = 391
Ans: B

212. An economy has an agricultural industry and a textile industry. Each unit of agricultural output requires 0.4 unit of agricultural input and 0.2 unit of textiles input. Each unit of textiles output requires 0.1 unit of agricultural input and 0.2 unit of textiles input. Write the technology matrix for this economy. (A, agricultural industry; T, textile industry)

A) $$\begin{array}{cc} \begin{array}{cc} A & T \end{array} & \\ \begin{bmatrix} 3.3 & 3.3 \\ 1.7 & 6.7 \end{bmatrix} & \begin{array}{c} A \\ T \end{array} \end{array}$$

B) $$\begin{array}{cc} \begin{array}{cc} A & T \end{array} & \\ \begin{bmatrix} 0.4 & 0.2 \\ 0.2 & 0.1 \end{bmatrix} & \begin{array}{c} A \\ T \end{array} \end{array}$$

C) $$\begin{array}{cc} \begin{array}{cc} A & T \end{array} & \\ \begin{bmatrix} 0.2 & 0.2 \\ 0.1 & 0.4 \end{bmatrix} & \begin{array}{c} A \\ T \end{array} \end{array}$$

D) $$\begin{array}{cc} \begin{array}{cc} A & T \end{array} & \\ \begin{bmatrix} 0.6 & 0.2 \\ 0.1 & 0.8 \end{bmatrix} & \begin{array}{c} A \\ T \end{array} \end{array}$$

E) $$\begin{array}{cc} \begin{array}{cc} A & T \end{array} & \\ \begin{bmatrix} 0.4 & 0.2 \\ 0.1 & 0.2 \end{bmatrix} & \begin{array}{c} A \\ T \end{array} \end{array}$$

Ans: E

213. An economy has an agricultural industry and a textile industry. Each unit of agricultural output requires 0.4 unit of agricultural input and 0.1 unit of textiles input. Each unit of textiles output requires 0.1 unit of agricultural input and 0.2 unit of textiles input. If surpluses of 6 units of agricultural products and 187 units of textiles are desired, find the gross production of each industry.

A) agriculture = 250, textiles =1060
B) agriculture = 50, textiles = 240
C) agriculture = 17, textiles = 149
D) agriculture = 21, textiles = 38
E) agriculture = 30, textiles = 237

Ans: B

214. An economy has a manufacturing industry and a banking industry. Each unit of manufacturing output requires inputs of 0.5 unit of manufacturing and 0.2 unit of banking. Each unit of banking output requires inputs of 0.4 unit each of manufacturing and banking. Write the technology matrix for this economy. (M, manufacturing industry; B, banking industry)

A)
$$\begin{array}{cc} \begin{array}{cc} M & B \end{array} & \\ \begin{bmatrix} 0.5 & 0.4 \\ 0.2 & 0.4 \end{bmatrix} & \begin{array}{c} M \\ B \end{array} \end{array}$$

B)
$$\begin{array}{cc} \begin{array}{cc} M & B \end{array} & \\ \begin{bmatrix} 0.4 & -0.2 \\ -0.4 & 0.5 \end{bmatrix} & \begin{array}{c} M \\ B \end{array} \end{array}$$

C)
$$\begin{array}{cc} \begin{array}{cc} M & B \end{array} & \\ \begin{bmatrix} 0.5 & -0.2 \\ -0.4 & 0.6 \end{bmatrix} & \begin{array}{c} M \\ B \end{array} \end{array}$$

D)
$$\begin{array}{cc} \begin{array}{cc} M & B \end{array} & \\ \begin{bmatrix} 0.5 & 0.2 \\ 0.4 & 0.4 \end{bmatrix} & \begin{array}{c} M \\ B \end{array} \end{array}$$

E)
$$\begin{array}{cc} \begin{array}{cc} M & B \end{array} & \\ \begin{bmatrix} 3.3 & -1.7 \\ -3.3 & 4.2 \end{bmatrix} & \begin{array}{c} M \\ B \end{array} \end{array}$$

Ans: D

215. An economy has a manufacturing industry and a banking industry. Each unit of manufacturing output requires inputs of 0.5 unit of manufacturing and 0.2 unit of banking. Each unit of banking output requires inputs of 0.3 unit each of manufacturing and banking. If surpluses of 141 units of manufacturing and 101 units of banking are desired, find the gross production of each industry.

A) manufacturing = 410, banking = 320
B) manufacturing = 50, banking = 28
C) manufacturing = 246, banking = 91
D) manufacturing = 271, banking = 28
E) manufacturing = 91, banking = 113

Ans: A

216. Suppose the development department of a firm charges 10% of its total monthly costs to the promotional department, and the promotional department charges 5% of its total monthly costs to the development department. The direct costs of the development department are $23,400, and the direct costs of the promotional department are $9,600. The solution to the matrix equation $\begin{bmatrix} 23,400 \\ 9,600 \end{bmatrix} + \begin{bmatrix} 0 & 0.05 \\ 0.1 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} x \\ y \end{bmatrix}$ or the column matrix $\begin{bmatrix} x \\ y \end{bmatrix}$ gives the total costs for each department. Note that this equation can be written in the form $D + AX = X$, or $X - AX = D$, which is the form for an open economy. Find the total costs for each department in this (micro)economy.

A) development = 23,035, promotional = 7,296
B) development = 22,920, promotional = 7,260
C) development = 24,000, promotional = 12,000
D) development = 96,000, promotional = 468,000
E) development = 480, promotional = 2,340
Ans: C

217. Suppose the shipping department of a firm charges 20% of its total monthly costs to the printing department and that the printing department charges 10% of its total monthly costs to the shipping department. If the direct costs of the shipping department are $19,400 and the direct costs of the printing department are $11,800, find the total costs for each department.

A) shipping = 18,220, printing = 7,920
B) shipping = 59,000, printing = 194,000
C) shipping = 18,592, printing = 8,082
D) shipping = 1,180, printing = 3,880
E) shipping = 21,000, printing = 16,000
Ans: E

218. The sales department of an auto dealership charges 10% of its total monthly costs to the service department, and the service department charges 20% of its total monthly costs to the sales department. During a given month, the direct costs are $87,100 for sales and $49,600 for service. Find the total costs of each department.

A) sales = 9,920, service = 8,710
B) sales = 99,000, service =59,500
C) sales = 77,180, service = 40,890
D) sales = 78,755, service = 41,724
E) sales = 496,000, service = 435,500
Ans: B

219. Suppose that an economy can be represented by the technology matrix below. If surpluses of 181 units of fishing output, 93 units of agricultural goods, and 40 units of mining goods are desired, find the gross production to the nearest unit of each industry.

$$\begin{array}{cc} & \begin{array}{ccc} \text{F} & \text{A} & \text{M} \end{array} \\ A = & \begin{bmatrix} 0.5 & 0.1 & 0.1 \\ 0.3 & 0.5 & 0.2 \\ 0.1 & 0.3 & 0.4 \end{bmatrix} \end{array} \begin{array}{l} \text{Fishing} \\ \text{Agriculture} \\ \text{Mining} \end{array}$$

A) fishing = 365, agriculture = 44, mining = 28
B) fishing = 104, agriculture = 109, mining = 62
C) fishing = 77, agriculture = 16, mining = 22
D) fishing = 635, agriculture = 795, mining = 570
E) fishing = 363, agriculture = 50, mining = 47
Ans: D

220. Suppose that the economy of a small nation has an electronics industry, a steel industry, and an auto industry, with the following technology matrix.

$$\begin{array}{cc} & \begin{array}{ccc} \text{E} & \text{S} & \text{A} \end{array} \\ A = & \begin{bmatrix} 0.6 & 0.2 & 0.2 \\ 0.1 & 0.4 & 0.5 \\ 0.1 & 0.2 & 0.2 \end{bmatrix} \end{array} \begin{array}{c} \text{Electronics} \\ \text{Steel} \\ \text{Autos} \end{array}$$

If the nation wishes to have surpluses of 118 units of electronics production, 273 units of steel production, and 200 automobiles, find the gross production of each industry.
A) electronics = 59, steel = 307, autos = 166
B) electronics = 164, steel = 2516, autos = 1434
C) electronics = 47, steel = 52, autos = 94
D) electronics = 165, steel = 221, autos = 106
E) electronics = 1304, steel = 1284, autos = 734
Ans: E

221. Suppose an economy has the technology matrix below. If surpluses of 525 units of electronics, 31 units of steel, and 140 autos are desired, find the gross production for each industry.

$$\begin{array}{cc} & \begin{array}{ccc} \text{E} & \text{S} & \text{A} \end{array} \\ A = & \begin{bmatrix} 0.6 & 0.2 & 0.2 \\ 0.1 & 0.4 & 0.5 \\ 0.1 & 0.2 & 0.2 \end{bmatrix} \end{array} \begin{array}{l} \text{Electronics} \\ \text{Steel} \\ \text{Autos} \end{array}$$

A) electronics = 2150, steel =985, autos = 690
B) electronics = 176, steel = 104, autos = 53
C) electronics = 770, steel = 2035, autos = 1720
D) electronics = 349, steel = 135, autos = 87
E) electronics = 1398, steel = 229, autos = 58
Ans: A

222. Suppose that an economy has the technology matrix below. If surpluses of 35 units of manufactured goods and 104 units of agricultural goods are desired, find the gross production of each industry.

$$A = \begin{array}{c} \\ \\ \\ \\ \end{array}\begin{array}{ccc} \text{S} & \text{M} & \text{A} \\ \end{array}$$
$$A = \begin{bmatrix} 0.4 & 0.1 & 0.1 \\ 0.2 & 0.5 & 0.2 \\ 0.2 & 0.1 & 0.3 \end{bmatrix} \begin{array}{l} \text{Service} \\ \text{Manufacturing} \\ \text{Agriculture} \end{array}$$

A) service = 14, manufacturing = 38, agriculture = 35
B) service = 14, manufacturing = 3, agriculture = 69
C) service = 60, manufacturing = 170, agriculture = 190
D) service = 29, manufacturing = 20, agriculture = 154
E) service = 91, manufacturing = 65, agriculture = 429
Ans: C

223. Suppose the technology matrix for a closed model of a simple economy is given by

$$A = \begin{array}{c} \\ \end{array}\begin{array}{ccc} \text{P} & \text{M} & \text{H} \end{array}$$
$$A = \begin{bmatrix} \frac{1}{2} & \frac{1}{10} & \frac{4}{5} \\ \frac{1}{10} & \frac{3}{10} & 0 \\ \frac{2}{5} & \frac{3}{5} & \frac{1}{5} \end{bmatrix} \begin{array}{l} \text{Products} \\ \text{Machinery} \\ \text{Households} \end{array}$$

Find the gross productions for the industries.
A) products = $\frac{49}{34}$ households, machinery = $\frac{7}{34}$ households
B) products = $\frac{28}{17}$ households, machinery = $\frac{4}{17}$ households
C) products = $\frac{15}{34}$ households, machinery = $\frac{7}{34}$ households
D) products = $\frac{7}{17}$ households, machinery = $\frac{1}{17}$ households
E) products = $\frac{21}{34}$ households, machinery = $\frac{3}{34}$ households
Ans: B

224. Suppose the technology matrix for a closed model of a simple economy is given by matrix *A*. Find the gross productions for the industries.

$$A = \begin{array}{c} \begin{array}{ccc} G & I & H \end{array} \\ \begin{bmatrix} \frac{2}{5} & \frac{1}{10} & \frac{3}{10} \\ \frac{2}{5} & \frac{3}{10} & \frac{1}{5} \\ \frac{1}{5} & \frac{3}{5} & \frac{1}{2} \end{bmatrix} \end{array} \begin{array}{l} \text{Government} \\ \text{Industry} \\ \text{Households} \end{array}$$

A) government = $\frac{16}{19}$ households, industry = $\frac{20}{19}$ households

B) government = $\frac{13}{19}$ households, industry = $\frac{21}{19}$ households

C) government = $\frac{25}{38}$ households, industry = $\frac{18}{19}$ households

D) government = $\frac{10}{19}$ households, industry = $\frac{22}{19}$ households

E) government = $\frac{23}{38}$ households, industry = $\frac{12}{19}$ households

Ans: E

225. Suppose the technology matrix for a closed model of an economy is given by matrix *A*. Find the gross productions for the industries.

$$A = \begin{array}{c} \begin{array}{ccc} S & M & H \end{array} \\ \begin{bmatrix} \frac{1}{5} & \frac{1}{10} & \frac{1}{5} \\ \frac{3}{5} & \frac{1}{2} & \frac{1}{5} \\ \frac{1}{5} & \frac{2}{5} & \frac{3}{5} \end{bmatrix} \end{array} \begin{array}{l} \text{Shipping} \\ \text{Manufacturing} \\ \text{Households} \end{array}$$

A) shipping = $\frac{14}{17}$ households, manufacturing = $\frac{6}{17}$ households

B) shipping = $\frac{17}{6}$ households, manufacturing = $\frac{17}{14}$ households

C) shipping = $\frac{11}{34}$ households, manufacturing = $\frac{10}{17}$ households

D) shipping = $\frac{6}{17}$ households, manufacturing = $\frac{14}{17}$ households

E) shipping = $\frac{4}{17}$ households, manufacturing = $\frac{15}{17}$ households

Ans: D

226. A closed model for an economy has a manufacturing industry, utilities industry, and households industry. Each unit of manufacturing output uses 0.6 unit of manufacturing input, 0.4 unit of utilities input, and 0.1 unit of households input. Each unit of utilities output requires 0.3 unit of manufacturing input, 0.5 unit of utilities input, and 0.2 unit of households input. Each unit of household output requires 0.2 unit each of manufacturing and utilities input and 0.5 unit of households input. Write the augmented matrix for this closed model of the economy.

A) $\left[\begin{array}{ccc|c} -0.4 & 0.4 & 0.1 & 0 \\ 0.3 & -0.5 & 0.2 & 0 \\ 0.2 & 0.2 & -0.5 & 0 \end{array}\right]$

B) $\left[\begin{array}{ccc|c} -0.4 & 0.3 & 0.2 & 0 \\ 0.4 & -0.5 & 0.2 & 0 \\ 0.1 & 0.2 & -0.5 & 0 \end{array}\right]$

C) $\left[\begin{array}{ccc|c} 0.6 & 0.3 & 0.2 & 0 \\ 0.4 & 0.5 & 0.2 & 0 \\ 0.1 & 0.2 & 0.5 & 0 \end{array}\right]$

D) $\left[\begin{array}{ccc|c} 0.6 & 0.4 & 0.1 & 0 \\ 0.3 & 0.5 & 0.2 & 0 \\ 0.2 & 0.2 & 0.5 & 0 \end{array}\right]$

E) $\left[\begin{array}{ccc|c} 0.4 & 0.3 & 0.2 & 0 \\ 0.4 & 0.5 & 0.2 & 0 \\ 0.1 & 0.2 & 0.5 & 0 \end{array}\right]$

Ans: B

227. A closed model for an economy has a manufacturing industry, utilities industry, and households industry. Each unit of manufacturing output uses 0.5 unit of manufacturing input, 0.4 unit of utilities input, and 0.1 unit of households input. Each unit of utilities output requires 0.4 unit of manufacturing input, 0.5 unit of utilities input, and 0.1 unit of households input. Each unit of household output requires 0.3 unit each of manufacturing and utilities input and 0.4 unit of households input. Find the gross production for each industry.

A) manufacturing = 6 households, utilities = 5 households
B) manufacturing = 2 households, utilities = 2 households
C) manufacturing = households, utilities = households
D) manufacturing = 4 households, utilities = 4 households
E) manufacturing = 3 households, utilities = 3 households

Ans: E

228. A closed model for an economy identifies government, the profit sector, the nonprofit sector, and households as its industries. Each unit of government output requires 0.4 unit of government input, 0.2 unit of profit sector input, 0.2 unit of nonprofit sector input, and 0.4 unit of households input. Each unit of profit sector output requires 0.2 unit of government input, 0.4 unit of profit sector input, 0.2 unit of nonprofit sector input, and 0.5 unit of households input. Each unit of nonprofit sector output requires 0.2 unit of government input, 0.2 unit of profit sector input, 0.2 unit of nonprofit sector input, and 0.6 unit of households input. Each unit of households output requires 0.04 unit of government input, 0.2 unit of profit sector input, 0.2 unit of nonprofit sector input, and 0.25 unit of households input. Write the augmented matrix for this closed model of the economy.

A) $\left[\begin{array}{cccc|c} 0.4 & 0.2 & 0.2 & 0.04 & 0 \\ 0.2 & 0.4 & 0.2 & 0.2 & 0 \\ 0.2 & 0.2 & 0.2 & 0.2 & 0 \\ 0.4 & 0.5 & 0.6 & 0.25 & 0 \end{array}\right]$

B) $\left[\begin{array}{cccc|c} 0.6 & 0.2 & 0.2 & 0.04 & 0 \\ 0.2 & 0.6 & 0.2 & 0.2 & 0 \\ 0.2 & 0.2 & 0.8 & 0.2 & 0 \\ 0.4 & 0.5 & 0.6 & 0.75 & 0 \end{array}\right]$

C) $\left[\begin{array}{cccc|c} -0.6 & 0.2 & 0.2 & 0.04 & 0 \\ 0.2 & -0.6 & 0.2 & 0.2 & 0 \\ 0.2 & 0.2 & -0.8 & 0.2 & 0 \\ 0.4 & 0.5 & 0.6 & -0.75 & 0 \end{array}\right]$

D) $\left[\begin{array}{cccc|c} -0.6 & 0.2 & 0.2 & 0.4 & 0 \\ 0.2 & -0.6 & 0.2 & 0.5 & 0 \\ 0.2 & 0.2 & -0.8 & 0.6 & 0 \\ 0.04 & 0.2 & 0.2 & -0.75 & 0 \end{array}\right]$

E) $\left[\begin{array}{cccc|c} 0.4 & 0.2 & 0.2 & 0.4 & 0 \\ 0.2 & 0.4 & 0.2 & 0.5 & 0 \\ 0.2 & 0.2 & 0.2 & 0.6 & 0 \\ 0.04 & 0.2 & 0.2 & 0.25 & 0 \end{array}\right]$

Ans: C

229. A closed model for an economy identifies government, the profit sector, the nonprofit sector, and households as its industries. Each unit of government output requires $\frac{3}{10}$ unit of government input, $\frac{1}{5}$ unit of profit sector input, $\frac{1}{5}$ unit of nonprofit sector input, and $\frac{3}{10}$ unit of households input. Each unit of profit sector output requires $\frac{1}{5}$ unit of government input, $\frac{3}{10}$ unit of profit sector input, $\frac{1}{10}$ unit of nonprofit sector input, and $\frac{2}{5}$ unit of households input. Each unit of nonprofit sector output requires $\frac{1}{10}$ unit of government input, $\frac{1}{10}$ unit of profit sector input, $\frac{1}{5}$ unit of nonprofit sector input, and $\frac{3}{5}$ unit of households input. Each unit of households output requires $\frac{1}{20}$ unit of government input, $\frac{2}{5}$ unit of profit sector input, $\frac{2}{5}$ unit of nonprofit sector input, and $\frac{3}{20}$ unit of households input. Find the gross production for each industry.

A) government = $\frac{263}{666}$ households, profit = $\frac{29}{37}$ households, nonprofit = $\frac{232}{333}$ households

B) government = $\frac{4}{9}$ households, profit = $\frac{4}{5}$ households, nonprofit = $\frac{32}{45}$ households

C) government = $\frac{107}{666}$ households, profit = $\frac{8}{37}$ households, nonprofit = $\frac{64}{333}$ households

D) government = $\frac{666}{107}$ households, profit = $\frac{37}{8}$ households, nonprofit = $\frac{333}{64}$ households

E) government = $\frac{666}{263}$ households, profit = $\frac{37}{29}$ households, nonprofit = $\frac{333}{232}$ households

Ans: A

230. Card tables are made by joining 4 legs and a top using 4 bolts. The legs are each made from a steel rod. The top has a frame made from 4 steel rods. A cover and four special clamps that brace the top and hold the legs are joined to the frame using a total of 8 bolts. The parts-listing matrix for the card table assembly is given by

$$A = \begin{array}{c} \begin{array}{ccccccc} \scriptstyle CT & \scriptstyle L & \scriptstyle T & \scriptstyle R & \scriptstyle Co & \scriptstyle Cl & \scriptstyle B \end{array} \\ \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 4 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 4 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 4 & 0 & 0 & 0 & 0 \\ 4 & 0 & 8 & 0 & 0 & 0 & 0 \end{bmatrix} \end{array} \begin{array}{l} \\ \text{Card table} \\ \text{Legs} \\ \text{Top} \\ \text{Rods} \\ \text{Cover} \\ \text{Clamps} \\ \text{Bolts} \end{array}$$

If an order is received for 10 card tables, 4 legs, 1 top, 2 covers, 6 clamps, and 12 bolts, how many of each primary assembly item are required to fill the order?

A) 10 rods, 44 covers, 11 clamps, 88 bolts
B) 72 rods, 11 covers, 42 clamps, 44 bolts
C) 88 rods, 13 covers, 358 clamps, 140 bolts
D) 88 rods, 13 covers, 50 clamps, 140 bolts
E) 72 rods, 11 covers, 150 clamps, 44 bolts

Ans: D

231. A log carrier has a body made from a 4-ft length of reinforced cloth having a patch on each side and a dowel slid through each end to act as handles. The parts-listing matrix for the log carrier is given by

$$A = \begin{array}{c} \begin{array}{ccccc} \scriptstyle LC & \scriptstyle B & \scriptstyle H & \scriptstyle C & \scriptstyle P \end{array} \\ \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ 2 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 2 & 0 & 0 & 0 \end{bmatrix} \end{array} \begin{array}{l} \\ \text{Log carrier} \\ \text{Body} \\ \text{Handles} \\ \text{Cloth} \\ \text{Patch} \end{array}$$

How many of each primary assembly items are required to fill an order for 520 log carriers and 25 handles?

A) 520 handles, 520 cloths, 1065 patches
B) 1015 handles, 520 cloths, 1040 patches
C) 1065 handles, 520 cloths, 1040 patches
D) 1065 handles, 520 cloths, 2130 patches
E) 1015 handles, 520 cloths, 2130 patches

Ans: C

