Lecture 1: Review of functions

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0.1 Review of functions

[1]

0.2 Functions

The aim of this part is to review some important functions with their domains, ranges and graphs.

Definition 0.2.1 *A function f is a rule that assigns to each point x in the domain a unique point $y = f(x)$ in the range of f. We write $f : D \to R$ where D is the domain of f and R is its range.*

Example 0.2.1 (a) $f(x) = x^2$, $D = (-\infty, \infty)$, $R = [0, \infty)$.

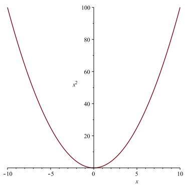

Figure 1: Graph of $y = x^2$

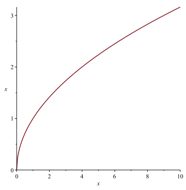

Figure 2: Graph of $y = \sqrt{x}$

(b) $f(x) = \sqrt{1 - x^2}$, $D = [-1, 1], R = [0, 1]$.

(c) The absolute value function $f(x) = |x| = \sqrt{x^2}$, $D = (-\infty, \infty)$, $R = [0, \infty)$.

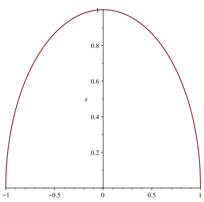

Figure 3: Graph of $y = \sqrt{1 - x^2}$

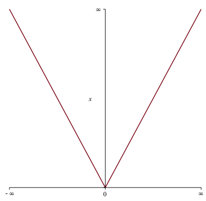

Figure 4: Graph of $y = |x|$

(d) The greatest integer function $f(x) = \lfloor x \rfloor$, $D = (-\infty, \infty)$, $R = 0, \pm 1, \pm 2, ...$

[1] This part is a review of chapter 1 in the textbook

0.3 Trigonometric functions

In this section, we review the six trigonometric functions: $\sin x, \cos x, \tan x, \cot x, \sec x$ and $\csc x$. You are supposed to know the values of these functions at the main values $0, \frac{\pi}{6}, \frac{\pi}{4}, \frac{\pi}{3}, \frac{\pi}{2}, \ldots$

(a) $y = \sin x,\ D = (-\infty, \infty),\ R = [-1, 1]$.

(b) $y = \cos x,\ D = (-\infty, \infty),\ R = [-1, 1]$.

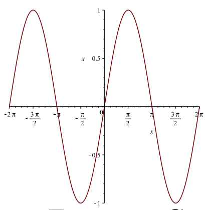

Figure 5: Graph of $y = \sin x$

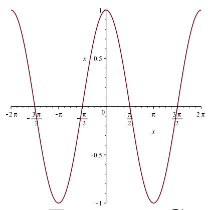

Figure 6: Graph of $y = \cos x$

(c) $y = \tan x = \frac{\sin x}{\cos x},\ D = (-\infty, \infty) \setminus \{\frac{\pi}{2} \pm n\pi\},\ n = 0, 1, 2, ...,\ R = (-\infty, \infty)$

(d) $y = \cot x = \frac{\cos x}{\sin x},\ D = (-\infty, \infty) \setminus \{\pm n\pi\},\ n = 0, 1, 2, ...,\ R = (-\infty, \infty)$

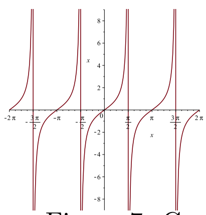

Figure 7: Graph of $y = \tan x$

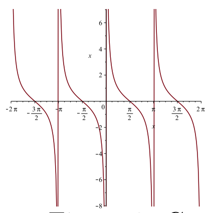

Figure 8: Graph of $y = \cot x$

(e) $y = \sec x = \frac{1}{\cos x},\ D = (-\infty, \infty) \setminus \{\frac{\pi}{2} \pm n\pi\},\ n = 0, 1, 2, ...,\ R = (-\infty, -1] \cup [1, \infty)$

(f) $y = \csc x = \frac{1}{\sin x},\ D = (-\infty, \infty) \setminus \{\pm n\pi\},\ n = 0, 1, 2, ...,\ R = (-\infty, -1] \cup [1, \infty)$

Remark 0.3.1 Since $\sin(x+2\pi) = \sin x$, $\cos(x+2\pi) = \cos x$, $\sec(x+2\pi) = \sec x$ and $\csc(x + 2\pi) = \csc x$, the functions $\sin x, \cos x, \sec x$ and $\csc x$ are called periodic with period 2π. Whereas $\tan x$ and $\cot x$ are periodic with period π since $\tan(x + \pi) = \tan x$ and $\cot(x + \pi) = \cot x$.

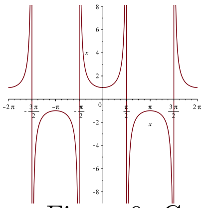

Figure 9: Graph of $y = \sec x$

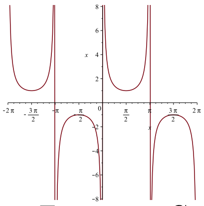

Figure 10: Graph of $y = \csc x$

0.3.1 Trigonometric identities

1. $\sin^2 x + \cos^2 x = 1$.
2. $\sin(2x) = 2\sin x \cos x$.
3. $\cos(2x) = \cos^2 x - \sin^2 x$.
4. $\cos^2 x = \frac{1+\cos(2x)}{2}$.
5. $\sin^2 x = \frac{1-\cos(2x)}{2}$.
6. $\sec^2 x = 1 + \tan^2 x$.
7. $\csc^2 = 1 + \cot^2 x$.
8. $\cos(A+B) = \cos A \cos B - \sin A \sin B$.
9. $\sin(A+B) = \sin A \cos B + \cos A \sin B$.

Example 0.3.1 Using the above identities, we find the following:

(a) $\sin(x+\pi) = -\sin x$, $\cos(x+\pi) = -\cos x$.

(b) $\sin(x+\frac{\pi}{2}) = \cos x$, $\cos(x+\frac{\pi}{2}) = -\sin x$.

0.4 Even and odd functions

Definition 0.4.1 *Let f be a function defined on an interval $I = [-a, a]$, where a is some real number. Then*

- *$f(x)$ is called even if $f(-x) = f(x)$. f is even if its graph is symmetric about the $y-$axis.*
- *$f(x)$ is called odd if $f(-x) = -f(x)$. f is odd if its graph is symmetric about the origin.*

Example 0.4.1 $x^2, x^4, x^6, ..., \cos x, \sec x$ are even. $x, x^3, x^5, ..., \sin x, \tan x, \csc x, \cot x$ are odd.

0.4.1 Exercises

(1) Find the domain and the range of the following functions:

(a) $f(x) = \frac{1}{\sqrt{x}}$.

(b) $f(x) = \tan(\pi x)$.

(c) $f(x) = 1 + |x|$.

(d) $f(x) = \sec^2 x$.

(e) $g(x) = \frac{1}{x^2}$.

(f) $h(x) = \frac{1}{\sqrt{1-x^2}}$.

(2) Sketch the following functions:

(a) $y = \sin(\pi x)$

(b) $y = |x - 1|$

(c) $y = \cos(x) + 1$

(3) Determine whether the following functions are even, odd or neither:

(a) $f(x) = x^2 + 1$.

(b) $f(x) = x^3 + x$.

(c) $g(t) = \frac{1}{t-1}$.

(d) $h(x) = \frac{x}{x^2-1}$.

(4) Prove the following:

(a) If $f(x)$ is even and $g(x)$ is odd then $(g \circ f)(x)$ is even.

(b) If $f(x)$ is even and $g(x)$ is odd then $\frac{f(x)}{g(x)}$ is odd.