Birzeit University
Mathematics Department

MATH 141 Final Exam. First Semester 2008/2009

Student Name: ______________________

Student Number: ______________________

Section Number: ______________________

Discussion Instructor: ______________________

Question # 1(78%)

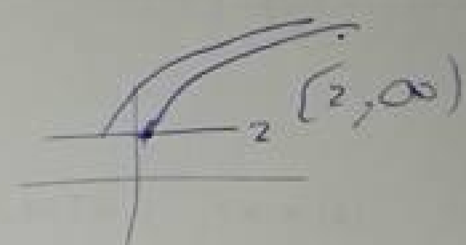
Circle the letter that corresponds to the correct answer:

1- The domain of the function $f(x)=\sqrt{\dfrac{x}{x-1}}$ is

(a) $[0,1]$ (b) $[0,1)$ (c) $(-\infty,0]\cup(1,\infty)$ (d) $(-\infty,1)\cup(1,\infty)$

2- The range of the function $f(x)=2+\sqrt{x-1}$ is

(a) $[0,\infty)$ (b) $[1,\infty)$ (c) $[2,\infty)$ (d) $(-\infty,\infty)$



3- Find the limit $\lim\limits_{x\to 4}\left(\dfrac{\sqrt{2x+1}-3}{x-4}\right)$

(a) ∞ (b) $\dfrac{1}{3}$ (c) 0 (d) does not exist

4- Find the limit $\lim\limits_{x\to 0}\left(\dfrac{\cos x}{x}\right)$ $\frac{1}{0}=\infty$

(a) 0 (b) 1 (c) ∞ (d) does not exist

5- Find the limit $\lim\limits_{x\to 0}\left(\dfrac{\cos x-1}{x^2}\right)$ L-Hôpitell

(a) 0 (b) -1 (c) ∞ (d) $-\dfrac{1}{2}$

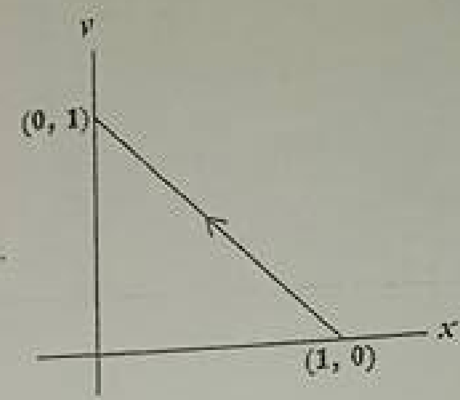
6- Find the limit $\lim_{x \to \infty} \left(\dfrac{2x - \sin x}{3x + \sin x} \right)$

(a) 0 (b) $\frac{2}{3}$ (c) ∞ (d) -1

7- Find the maximum value of the function $f(x) = x + \cos x$ on the interval $[0, \pi]$.

(a) $\pi - 1$ (b) 2 (c) 1 (d) $\frac{\pi}{2}$

8- Find the parametric equations that represent the graph on the right



(a) $x = t, \quad y = 1 - t, \quad t \in [0, 1]$ (b) $x = \cos^2 t, \quad y = \sin^2 t, \quad t \in \left[0, \frac{\pi}{2}\right]$

(c) $x = \cos t, \quad y = \sin t, \quad t \in \left[0, \frac{\pi}{2}\right]$ (d) $x = t, \quad y = 1 - t, \quad t \in [1, 2]$

9- Find the statement that is always true.

(a) If $f'(a) = 0$, then f has an extreme value at $x = a$.

(b) If $f'(a) = 0$ and $f''(a) < 0$, then f has a local maximum at $x = a$.

(c) If $f(x)$ is continuous at $x = a$, the f is differentiable at $x = a$.

(d) If $f''(a) > 0$, then f has a local minimum at $x = a$.

10- Find the slope of the tangent line to the curve $x = 2 + \sin t$, $y = 3 + \cos t$, at $t = \frac{\pi}{4}$.

(a) 1 (b) 0 (c) -1 (d) not defined

$dy = -\sin t\, dt \qquad dx = \cos t\, dt$

$\frac{dy}{dx} = \frac{-\sin t\, dt}{\cos t\, dt} = -\tan t$

$\left.\frac{dy}{dx}\right|_{t=\frac{\pi}{4}} = -\tan\frac{\pi}{4} = \boxed{-1}$

- Use differentials to find an estimate value of $2\sqrt{1.01}-(1.01)^4$

(a) 0.98 (b) 0.969 (c) 0.97 (d) 0.9701

12- If $f'(2)=5, f'(4)=2, g(x)=\sqrt{x}$, find $(f \circ g)'(4)$

(a) 1 (b) $\frac{5}{4}$ (c) $\frac{5}{2}$ (d) $\frac{1}{2}$

13- Find the vertical asymptotes of the curve $f(x)=\frac{\sin x}{x^2-x}$

(a) $x=0$ (b) $x=1$ (c) $x=0, \quad x=1$ (d) f has no vertical asymptotes

14- Find the integral $\int x\sqrt{x-1}\,dx$

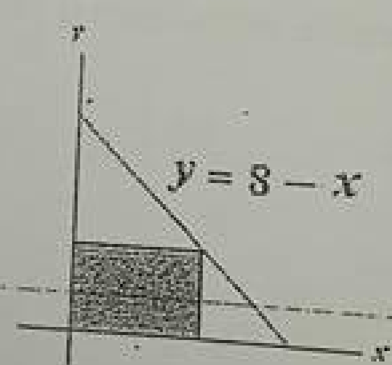
(a) $\frac{1}{3}x^2(x-1)^{3/2}+C$

(b) $\sqrt{\frac{x^4}{4}-\frac{x^3}{3}}+C$

(c) $\frac{2}{5}(x-1)^{5/2}+\frac{2}{3}(x-1)^{3/2}+C$

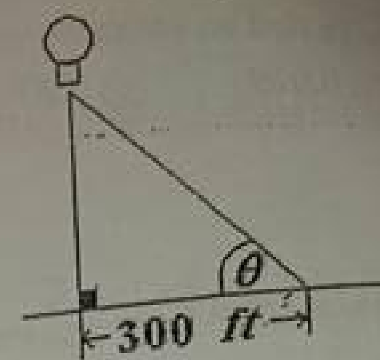
(d) $\sqrt{\frac{x^3}{2}-\frac{x^2}{2}}+C$

15- Find the largest area of a rectangle in the first quadrant with a base on the x-axis and a vertex on the line $y=8-x$



(a) 10 (b) 12 (c) 14 (d) 16

16- A person is watching a balloon that is rising vertically at $75 ft/s$ from a point $300\ ft$ away from the point where the balloon is released. At what rate is the angle of elevation θ changing when the hight of the balloon is $400\ ft$



(a) 0.09 (b) 2.5 (c) 2.7 (d) $\frac{27}{2500}$

17- Find the linearization of the function $f(x)=x+\cos x$ at $x=0$

(a) $L(x)=1+x$ (b) $L(x)=1-x$ (c) $L(x)=1+\frac{x}{2}$ (d) $L(x)=1+2x$

18- Find the length of the curve $x=\frac{1}{3}(y^2+2)^{3/2},\ 0\le y\le 1$

(a) $\frac{1}{2}$ (b) $\frac{1}{3}$ (c) $\frac{4}{3}$ (d) $\frac{2}{3}$

19- If P is a uniform partition of the interval $\left[0,\frac{\pi}{2}\right]$, find the limit $\lim_{n\to\infty}\sum_{k=1}^{n}(\cos c_k)(\Delta x_k)$

(a) 1 (b) 2 (c) $\frac{3}{2}$ (d) $\frac{\pi}{4}$

20- If $\int_1^5 f(x)dx=-2$, and $\int_2^5 f(x)dx=-6$, then find $\int_1^2 2f(x)dx$

(a) 2 (b) 4 (c) 6 (d) 8

21. Find the derivative of $\int_{1}^{x} \frac{\sin t}{t}\, dt$

(a) $\frac{\sin x}{x}$

(b) $\frac{\sin x}{x} - \sin 1$

(c) $\frac{\cos x}{x^2} - \cos 1$

(d) $\frac{x\cos x - \sin x}{x^2}$

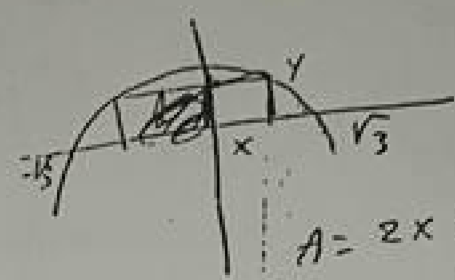
22. Find the area of the largest rectangle inscribed in the first quadrant with the left hand corner at the origin and the upper right hand corner on the curve $y = 3 - x^2$

(a) 2

(b) 1

(c) 0

(d) $\sqrt{3}$



$y = 2$

$x = 1$

$A = 2x\,y$

$A = 2x(3 - x^2)$

$A = 6x - 2x^3$

$\frac{dA}{dx} = 6 - 6x^2 = 0$

$x = 1, -1$

23. If $u = 2 - x$, then $\int_{0}^{2} (2-x)^2\, dx =$

(a) $\int_{2}^{0} u^2\, du$

(b) $\int_{2}^{1} u^2\, du$

(c) $\int_{0}^{2} u^2\, du$

(d) $\int_{4}^{2} u^2\, du$

$x = 2$

$u = 2 - 2$

$4 - 4x + x^2$

$4x - 2x^2 + \frac{x^3}{3} \Big|_0^2$

$8 - 8 + \frac{8}{3}$

24. The average value of the function $f(x) = |x|$ over the interval $[-2, 3]$ is

(a) $\frac{5}{2}$

(b) $\frac{1}{2}$

(c) $\frac{13}{2}$

(d) $\frac{13}{10}$

25. Find the function $y = f(x)$ whose curve passes through the point $(1, 4)$ and whose derivative at each point is $3\sqrt{x}$

(a) $y = 2x^{\frac{3}{2}} - 15$

(b) $y = 2x^{\frac{3}{2}} + 2$

(c) $y = \frac{9}{2}x^{\frac{3}{2}} - 35$

(d) $y = \frac{9}{2}x^{\frac{3}{2}} - \frac{1}{2}$

$4 - 4x + x^2$

$4x - 2x^2 + \frac{x^3}{3} \quad \frac{1}{3 - -2}\left(\frac{13}{2}\right)$

$8 - 8 + \frac{8}{3}$

$\frac{13}{10}$

$\frac{dy}{dx} = 3\sqrt{x}$

$\int dy = 3\int \sqrt{x}$

$= 3 \cdot \frac{2}{3} x^{\frac{3}{2}} + C$

$= 2x^{\frac{3}{2}} + C$

$4 = 2 + C$

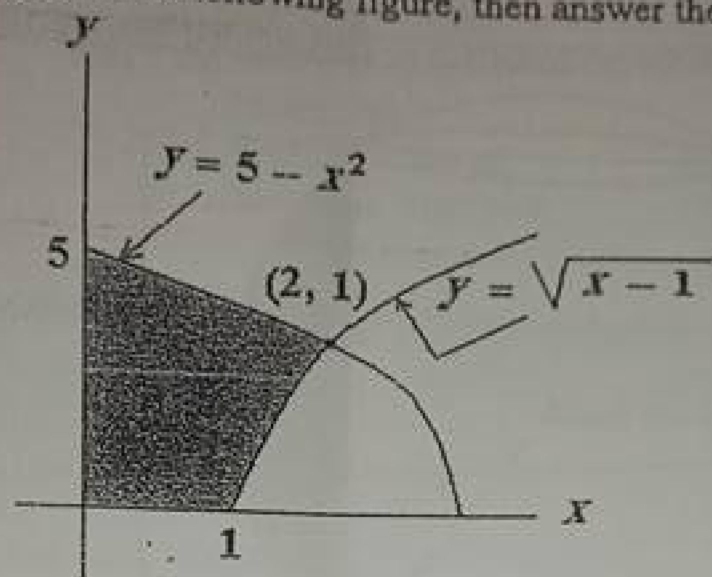
$\int_{-2}^{0} -x\, dx + \int_{0}^{3} x\, dx$

$\frac{-x^2}{2}\Big|_{-2}^{0} + \frac{x^2}{2}\Big|_{0}^{3}$

$\frac{4}{2} + \frac{9}{2} = \frac{13}{2}$

Question # 2(12%)

Consider the shaded region in the following figure, then answer the questions that follow



(a) Write a definite integral that represents the area of the shaded region. (Do not evaluate the integral.)

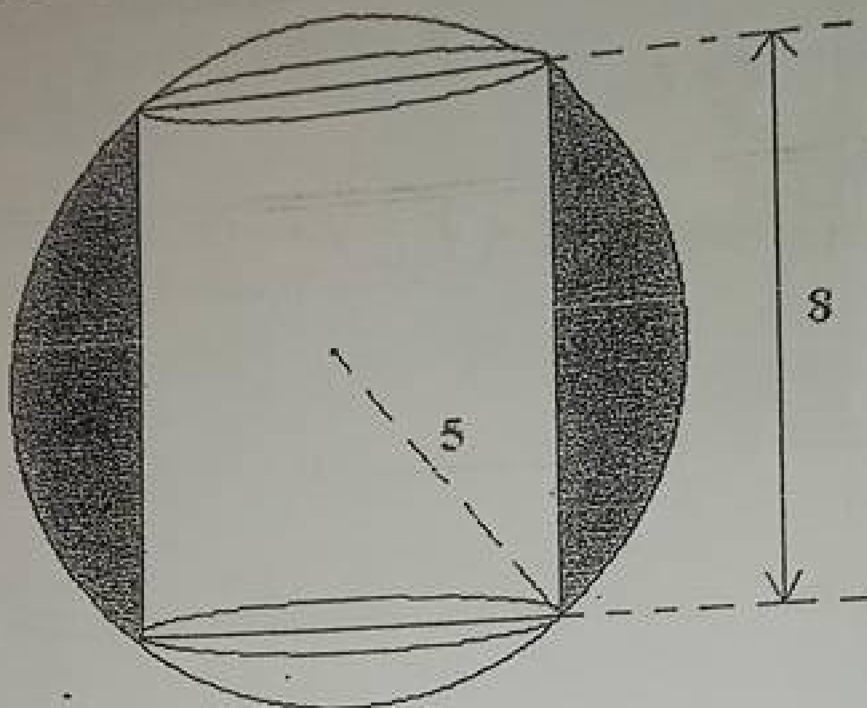
(b) Write a definite integral that represents the volume of the solid generated by revolving the shaded region about the given axis in each of the three parts:

(1) y-axis (use shell method)

(2) y-axis (use disk method)

(3) $y = -1$ (use washer method)

Question # 3(10%)
A cylindrical hole of height 8 is drilled all the way through the center of a sphere of radius 5.

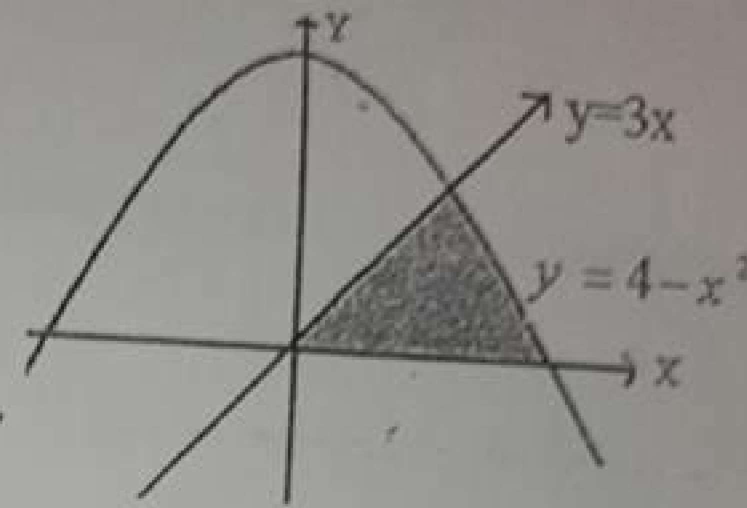


(a) Find the volume of the remaining solid.

(b) Find the total surface area of the remaining solid.

stion #4(12%)

ind the volume of the solid generated by revolving the shaded region arund (Don't evaluate the integral)



a) X- axis

b) Y axis

c)The line X=2

d) the line Y=4