Birzeit University
Mathematics Department
Math 1411 Calculus I
First Semester 2017/2018

Student Name(IN ARABIC): .... Number:................Discussion Section #:.....
Second Exam (Time: 90 Minutes) Name of discussion teacher:.............

Question 1 (63%). **Choose the most correct answer:**

(1) If $f(x) = \dfrac{2x-3}{3x+1}$, then

(a) $f^{-1}(x) = \dfrac{3x+1}{2x-3}$.

(b) $f^{-1}(x) = \dfrac{x-3}{x+2}$.

(c) $f^{-1}(x) = \dfrac{x+3}{2-3x}$.

(d) f has no inverse.

(2) The inverse of $f(x) = \sqrt{x+9}, x \geq -9$ is

(a) $f^{-1}(x) = x^2 - 9, x \geq 0$

(b) $f^{-1}(x) = -x^2 + 9, x \geq 0$

(c) $f^{-1}(x) = x^2 - 9, x \leq 0$

(d) $f(x)$ is not one-to-one, and so has no inverse.

(3) The length of the curve $y = \frac{x^2}{4} - \frac{1}{2}\ln(x), 2 \leq x \leq 4$ equals

(a) $2 + \ln 2$

(b) $3 + \frac{\ln 2}{2}$

(c) $3 + \ln 2$

(d) $\frac{\ln 2}{2}$

(4) If $f(x) = e^x + x^e$, then $f'(e) =$

(a) $2e^2$

(b) $1 + e^e$

(c) $e^e + e^{e-1}$

(d) $2e^e$

(5) The solution(s) of $2\ln x = \ln(x+2)$ is

(a) $x = 2$

(b) $x = -1$

(c) $x = 2, -1$

(d) has no solution

(6) The following calculation is correct
$\lim\limits_{x\to 4} \frac{x-4}{x^2-4} = \lim\limits_{x\to 4} \frac{1}{2x} = \frac{1}{8}$

(a) No

(b) Yes

(7) $\lim\limits_{x\to\infty} \frac{\sin(\frac{1}{x})}{e^{(\frac{1}{x})}} =$

(a) 1

(b) e

(c) $\frac{1}{e}$

(d) 0

(8) $\int\limits_0^1 \frac{e^{\sqrt{x}}}{\sqrt{x}}\, dx =$

(a) $e - 1$

(b) 2

(c) $\frac{1}{\sqrt{e}}$

(d) $2(e-1)$

(9) An equation of the curve whose length is given by $L = \int\limits_1^4 \sqrt{1+4x^2}\, dx$ and that passes through the point $(1,1)$ is

(a) $y = x^2, 1 \le x \le 4$

(b) $y = 2x, 1 \le x \le 4$

(c) $y = x^2 + 1, 1 \le x \le 4$

(d) $y = x, 1 \le x \le 4$

Please turn over

(10) If $3^{\log_9(100)} = x^2 + 4^{\log_2(7x)}$, then $x =$

(a) $\dfrac{1}{\sqrt{5}}$

(b) $\dfrac{1}{\sqrt{5}}, \dfrac{-1}{\sqrt{5}}$

(c) $\dfrac{-1}{\sqrt{5}}$

(d) has no real solution

(11) $\lim\limits_{x\to\infty} \dfrac{x}{2^x} =$

(a) $\frac{1}{\ln 2}$

(b) $\ln 2$

(c) 0

(d) DNE

(12) $\lim\limits_{x\to\infty} x\tan(\frac{1}{x}) =$

(a) 1

(b) ∞

(c) $-\infty$

(d) 0

(13) The function $f(x) = a^x, 0 < a < 1$ is

(a) decreasing and concave up

(b) decreasing and concave down

(c) increasing and concave up

(d) increasing and concave down

(14) $\log_7 x =$

(a) $\ln(\frac{5}{7}) \log_5 x$

(b) $\ln(\frac{7}{5}) \log_5 x$

(c) $\frac{\ln 7}{\ln 5} \log_5 x$

(d) $\frac{\ln 5}{\ln 7} \log_5 x$

Please turn over

(15) If $f(x) = x^2 - 4x + 3, x \geq 2$, then $\dfrac{df^{-1}}{dx}$ at $x = 3$ is

(a) -4
(b) $\frac{1}{4}$
(c) 2
(d) $\frac{1}{2}$

(16) $e^{-\ln(-x)} =$

(a) $-x$, for all x
(b) $\frac{-1}{x}$, for all $x \neq 0$
(c) $\frac{-1}{x}$, for all $x < 0$
(d) x, for all $x > 0$

(17) If $e^y = 6x + 3y$, then $\frac{dy}{dx} =$

(a) $\dfrac{6}{e^y - 3}$
(b) $\dfrac{e^x - 3}{6}$
(c) $\dfrac{e^y - 3}{6}$
(d) $\dfrac{6}{e^x - 3}$

(18) $\ln(5x^2 - 15x) + \ln \frac{1}{5x} =$

(a) $\ln(x - 15)$
(b) $\ln(25x^2(x - 3))$
(c) $\ln 5x(5x^2 - 15x)$
(d) $\ln(x - 3)$

(19) $\int_0^1 9x^2 2^{x^3}\, dx =$

(a) $\frac{6}{\ln 2}$
(b) $\frac{1}{\ln 2}$
(c) $\ln 2$
(d) $\frac{3}{\ln 2}$

(20) $\int \frac{\cos x \, dx}{1 + 2\sin x} =$

(a) $\ln \sqrt{|\, 1 + 2\sin x \,|} + C$

(b) $\ln(1 + 2\sin x)^2 + C$

(c) $\ln |\, 1 + 2\sin x \,| + C$

(d) $\ln \sqrt{1 + 2\sin x} + C$

(21) The domain (D) and range (R) of the inverse of $f(x) = \sqrt{4 - x}$ are

(a) $D = (-\infty, 0],\ R = [-4, \infty)$

(b) $D = [-4, \infty),\ R = (-\infty, 0]$

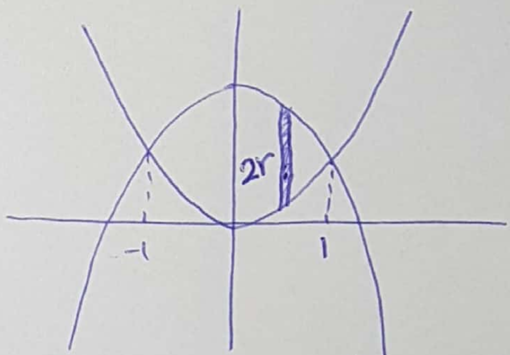
(c) $D = [0, \infty),\ R = (-\infty, 4]$

(d) $D = (-\infty, 4],\ R = [0, \infty)$

Question 2 (18%). Answer the following questions and show your work

1. A solid lies between the planes perpendicular to the x-axis at $x = -1$ and $x = 1$. The cross sections perpendicular to the x-axis are circular disks whose diameters run from the parabola $y = x^2$ to the parabola $y = 2 - x^2$. Find the volume of the solid

$$V = \int_{-1}^{1} A(x)\,dx$$



$$A(x) = \pi r^2 = \pi\left(\tfrac{1}{2}(2 - x^2 - x^2)\right)^2$$

$$= \pi(1 - x^2)^2 = \pi(1 - 2x^2 + x^4)$$

$$r = \tfrac{1}{2}(2 - x^2 - x^2) = 1 - x^2$$

$$V = \int_{-1}^{1} \pi(1 - 2x^2 + x^4)\,dx = \pi\left(x - \frac{2x^3}{3} + \frac{x^5}{5}\right)\Bigg|_{-1}^{1}$$

$$= \pi\left[1 - \frac{2}{3} + \frac{1}{5} - \left(-1 + \frac{2}{3} - \frac{1}{5}\right)\right]$$

$$= \pi\left(2 - \frac{4}{3} + \frac{2}{5}\right) = \frac{16\pi}{15}$$

2. Evaluate $\int \frac{dx}{1+e^{-x}}$ $= \int \frac{e^x dx}{e^x+1}$

let $u = e^x+1 \Rightarrow du = e^x dx$

$$\int \frac{dx}{e^x+1} = \int \frac{du}{u} = \ln|u| + C$$
$$= \ln|e^x+1| + C = \ln(e^x+1) + C.$$

3. Find $\lim_{x\to\infty} x^{(\frac{1}{x})}$, let $y = x^{\frac{1}{x}} \Rightarrow \ln y = \frac{1}{x}\ln x$.

$$\lim_{x\to\infty} \ln y = \lim_{x\to\infty} \frac{\ln x}{x} \quad \left(\frac{\infty}{\infty}\right)$$
$$= \lim_{x\to\infty} \frac{1}{x} = 0$$

$$\therefore \lim_{x\to\infty} x^{\frac{1}{x}} = e^0 = 1.$$

4. [*Bonus* : 5%] For what value(s) of $x > 0$ does $x^{(x^x)} = (x^x)^x$.

Taking ln of both sides

$$x^x \ln x = x \ln x^x = x^2 \ln x.$$

$$\Rightarrow x^x \ln x - x^2 \ln x = 0 \Rightarrow (x^x - x^2)\ln x = 0$$

$\Rightarrow$ either $\ln x = 0$ or $x^x = x^2$

$\Rightarrow$ $x = 1$ or $x \ln x = 2\ln x \Rightarrow (x-2)\ln x = 0$

$\Rightarrow x = 2$ or $x = 1$.

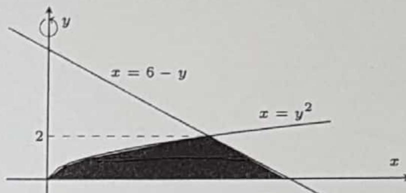
so solutions are $x = 1$, $x = 2$.

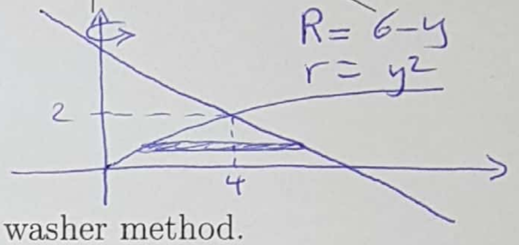
Question 3 (24%). Consider the region in the first quadrant enclosed between $x = y^2$, $x = 6 - y$, and x-axis. Find the volume of the solid of revolution in the cases below. (**Do not evaluate the integrals**)

(a) The axis of revolution is the y-axis. Use the washer method.

$$V = \int_0^2 \pi (R^2 - r^2)\, dy$$

$$= \int_0^2 \pi \left((6-y)^2 - (y^2)^2 \right) dy$$





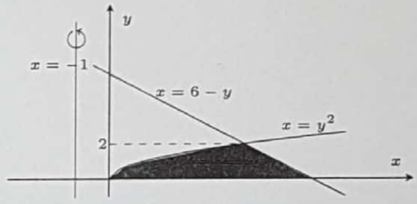
$R = 6 - y$

$r = y^2$

(b) The axis of revolution is $x = -1$. Use the washer method.

$$V = \int_0^2 \pi (R^2 - r^2)\, dy$$

$$= \int_0^2 \pi \left((7-y)^2 - (1+y^2)^2 \right) dy$$



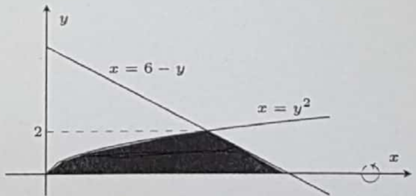
$R = 6 - y - (-1) = 7 - y$

$r = y^2 - (-1) = y^2 + 1$

(c) The axis of revolution is the x-axis. Use the shell method.

$$V = \int_0^2 2\pi (\text{radius})(\text{height})\, dy$$

$$= \int_0^2 2\pi (y)(6 - y - y^2)\, dy$$



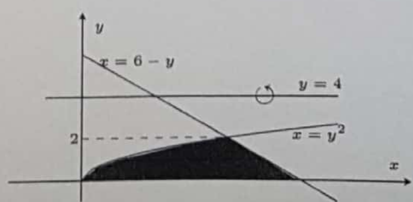
radius $= y$

height $= 6 - y - y^2$

(d) The axis of revolution is $y = 4$. Use the shell method.

$$V = \int_0^2 2\pi (\text{radius})(\text{height})\, dy$$

$$= \int_0^2 2\pi (4-y)(6 - y - y^2)\, dy$$



radius $= 4 - y$

height $= 6 - y - y^2$