Birzeit University
Mathematics Department
Math 1411 Calculus I
First Semester 2017/2018

Student Name(IN ARABIC): .... Number:................Discussion Section #:.....
First Exam (Time: 90 Minutes) Name of discussion teacher:.............

Question 1 (60%). **Choose the most correct answer:**

(1) The graph of the function $f(x) = x - \sin x$ is symmetric about

(a) origin only

(b) y-axis only

(c) y-axis and origin

(d) none of the above

(2) The domain D and range R of the function $f(x) = \sqrt{-x^2+4}$ are

(a) $D = (-\infty, \infty), R = [0, \infty)$

(b) $D = [0, \infty), R = [0, \infty)$

(c) $D = [-2, 2], R = [-2, 2]$

(d) $D = [-2, 2], R = [0, 2]$

(3) The discontinuity of $f(x) = \dfrac{\sin^2(4x)}{x^2}$ at $x = 0$ is

(a) a removable discontinuity

(b) not removable

(4) The graph of $f(x) = \frac{x^2+2x-3}{x-1}$ has

(a) a vertical asymptote $x = 1$

(b) an oblique asymptote $x = 1$

(c) a horizontal asymptote $y = 1$

(d) no asymptotes

(5) The domain of the function $f(x) = \sqrt{1 - \frac{1}{x}}$ is

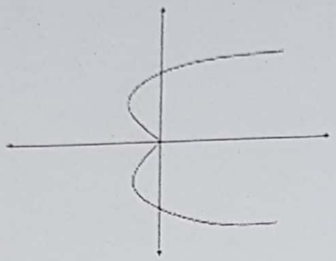
(a) $[1, \infty)$

(b) $(-\infty, 0) \cup (0, \infty)$

(c) $(-\infty, 0) \cup [1, \infty)$

(d) $[-1, 1]$

(6) Determine whether the following graph is a graph of a function



(a) is a function

(b) is not a function

(7) The absolute maximum of the function $f(x) = \frac{x}{\pi} + \tan x$ on $[0, \frac{\pi}{4}]$ is

(a) $\frac{5}{4}$

(b) $\frac{\pi}{4}$

(c) 0

(d) No absolute maximum

(8) If f is a function such that f' is positive and decreasing, then f is

(a) increasing and concave up

(b) increasing and concave down

(c) decreasing and concave up

(d) decreasing and concave down

(9) $\frac{1}{i} =$

(a) i

(b) $-i$

(c) 1

(d) -1

(10) The linearization $L(x)$ of the function $f(x) = \sin(\frac{\pi}{4} + \sin x)$ at $a = 0$ is

(a) $L(x) = \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}}x$

(b) $L(x) = \frac{1}{\sqrt{2}}$

(c) $L(x) = \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}}(x - \frac{1}{\sqrt{2}})$

(d) $L(x) = \frac{1}{\sqrt{2}}x$

(11) $\int_0^2 |\, x^2 - 1 \,|\; dx =$

(a) $\frac{2}{3}$
(b) $\frac{-1}{3}$
(c) 2
(d) -2

(12) If $f(x) = \dfrac{g(x)}{x-3}$, where $g(x)$ is a polynomial. If $f(x)$ has a horizontal asymptote $y = 1$, then a possible value for $g(x)$ is

(a) $g(x) = x^2 - 9$
(b) $g(x) = x + 3$
(c) 3
(d) 0

(13) Suppose that $f(x)$ and $g(x)$ are functions of x that are differentiable at $x = 1$ and that $f(1) = -3$, $f'(1) = 3$, $g(1) = 4$, $g'(1) = 2$. Then $\frac{d}{dx}(\frac{f+g}{f})$ at $x = 1$ is

(a) $\frac{-4}{3}$
(b) -2
(c) $\frac{1}{2}$
(d) $\frac{-3}{4}$

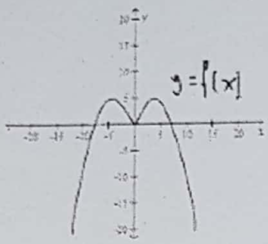
(14) If $2xy - y^2 = 1$, then $\frac{dy}{dx} =$

(a) $\frac{x}{x-y}$
(b) $\frac{y}{x-y}$
(c) $\frac{y}{y-x}$
(d) $\frac{x}{y-x}$

(15) An equation of the tangent line of $f(x) = x + \tan x$ at $x = 0$ is

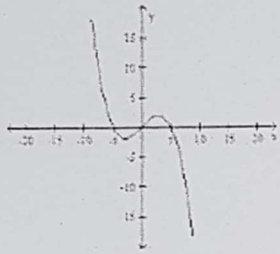
(a) $y = -2(x-1)$
(b) $y = 2(x-1)$
(c) $y = 2x$
(d) $y = 2x - 1$

(16) Consider the graph of $y = f(x)$

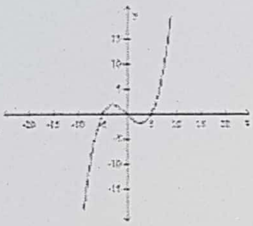


Choose the graph that represents its derivative

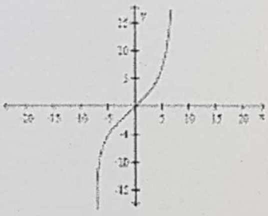
(a)



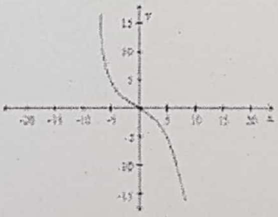
(b)



(c)



(d)



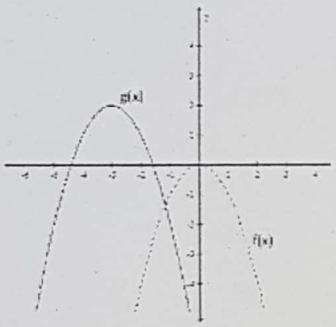
(17) If $z = a + ib$ is a complex number, then the real part of z ($Re(z)$) is given by

(a) $Re(z) = \frac{z.\bar{z}}{2}$
(b) $Re(z) = \frac{z+\bar{z}}{2}$
(c) $Re(z) = \frac{z-\bar{z}}{2}$
(d) $Re(z) = z + \bar{z}$

(18) $(1+i)^8 =$

(a) $8 + 8i$
(b) $8 - 8i$
(c) 16
(d) -16

(19) The figure below shows the graph of $f(x)$ shifted to a new position, the equation of the new graph $g(x)$ is

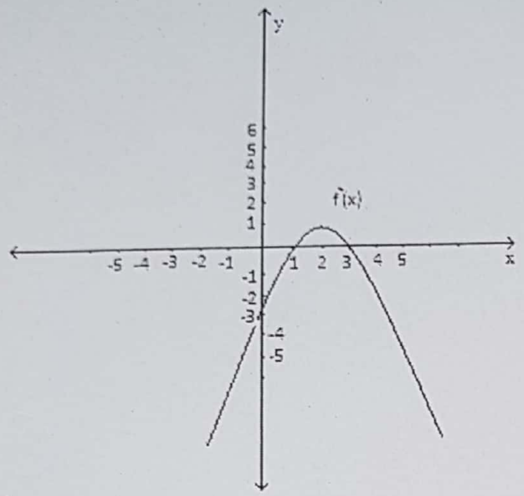


(a) $g(x) = f(x+3) + 2$
(b) $g(x) = f(x-3) + 2$
(c) $g(x) = f(x+3) - 2$
(d) $g(x) = f(x-3) - 2$

(20) The function $f(x) = x^3 - 12x + 2$ has

(a) a local maximum at $(-2, 18)$ and a local minimum at $(2, -14)$
(b) a local maximum at $(2, -14)$ and a local minimum at $(-2, 18)$
(c) a local maximum at $(0, 2)$
(d) no extreme values

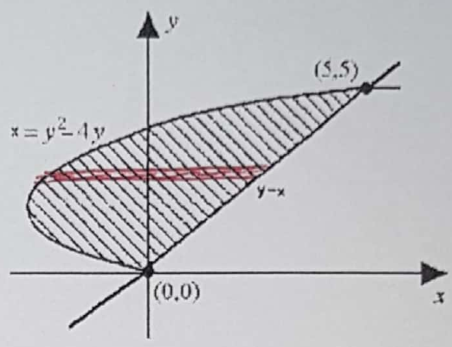
Question 2 (14%). Given the graph of $f'(x)$ below, mark each of the following statements by **True** or **False**



1. (..F...) $f(x)$ is increasing on $(-\infty, 2]$
2. (..T...) $f(x)$ has a local minimum at $x = 1$
3. (..F...) $f(x)$ has a local maximum at $x = 2$
4. (..T...) $f(x)$ is decreasing on $(-\infty, 1] \cup [3, \infty)$
5. (..T...) curve of $f(x)$ is concave up on $(-\infty, 2]$
6. (..T...) curve of $f(x)$ has an inflection point at $x = 2$
7. (..T...) curve of $f(x)$ has a horizontal tangent at $x = 1$ and at $x = 3$

Please turn over

Question 3 (10%). Let R be the region enclosed by the curves $x = y^2 - 4y$, $y = x$ (region in the graph below). Find the area of R.



$$\text{Area} = \int_0^5 \left[y - (y^2 - 4y)\right] dy$$

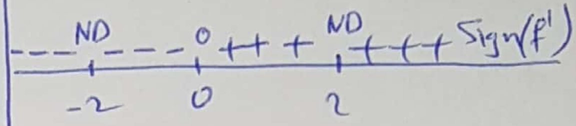
$$= \int_0^5 (5y - y^2)\, dy$$

$$= \left(\frac{5\,y^2}{2} - \frac{y^3}{3}\right)\Big|_0^5$$

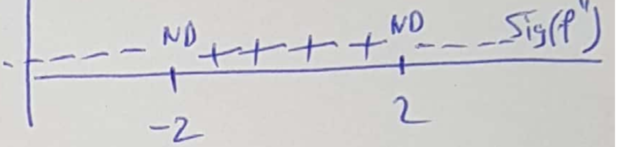
$$= \frac{5 \times 25}{2} - \frac{125}{3} = \frac{125}{6}$$

Question 4 (21%). Let $f(x) = \frac{1-x^2}{x^2-4}$ [$f'(x) = \frac{6x}{(x^2-4)^2}$, $f''(x) = \frac{-6(3x^2+4)}{(x^2-4)^3}$]. Find

$f'(x) = 0 \Rightarrow x = 0$



$f''(x) = 0$ No solution



1. Domain of $f(x) = \mathbb{R} - \{2, -2\}$ or $x \neq \pm 2$
2. $\lim_{x\to\infty} f(x) = -1$
3. $\lim_{x\to-\infty} f(x) = -1$
4. $\lim_{x\to2^+} f(x) = -\infty$
5. $\lim_{x\to2^-} f(x) = \infty$
6. $\lim_{x\to-2^+} f(x) = \infty$
7. $\lim_{x\to-2^-} f(x) = -\infty$
8. Horizontal asymptotes (if any) are : $y = -1$
9. Vertical asymptotes (if any) are: $x = 2$, $x = -2$
10. Oblique asymptotes (if any) are: No
11. Intervals of increasing $x \in [0, 2) \cup (2, \infty)$
12. Intervals of decreasing $x \in (-\infty, -2) \cup (-2, 0]$
13. local maximum points No
14. local minimum points at $x = 0$, $(0, -\frac{1}{4})$
15. when is the graph concave up? $x \in (-2, 2)$
16. when is the graph concave down? $(-\infty, -2) \cup (2, \infty)$
17. inflection points are No

Graph the function using the above information.